METROLOGY OF THE ACOUSTIC SEAFLOOR RESPONSE: HOW TO ACCURATELY ESTIMATE BACKSCATTER AND ITS INTRINSIC UNCERTAINTY USING SINGLE-BEAM ECHOSOUNDER

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1 INTRODUCTION

Acoustic seafloor response has become the key parameter for many if not most marine surveying communities. Its measurements have been widely extended in hydrography using predominantly bathymetric echosounders, and are used in diverse applications such as habitat mapping, seabed characterisation and classification. In parallel, numerous theoretical models have been developed in recent decades to study its link with physical or heuristic bottom parameters. Measurements of seabed backscatter, its related applications and its theory are three separated but inter-connected domains dealing with the seafloor acoustic response. The metrologic work presented in this paper proposes to link these three approaches of the seabed backscatter.

While analysing how these three domains deal with the backscatter, it appears that the nature of the acoustic seafloor response itself is equivocal. Based on statistical and physical hypotheses, a metrological definition of the backscatter is established in section 2, connecting seafloor response measurements, applications and theory. In particular, the deterministic backscatter parameter derived theoretically is usefully linked to the stochastic nature observed in practice during backscatter measurements. This yields in section 3 a method for accurately estimating backscatter from echosounder data: the best choice of backscatter estimator is justified based on analytical calculations, and ways to represent its uncertainties are proposed. Finally, section 4 shows an example of application of the identified best backscatter estimator and of its uncertainty formulation and estimation using single-beam echosounder data[8].

2 METROLOGIC DEFINITION OF SEABED BACKSCATTER

In most literature on the seafloor acoustic response, it is mainly named, described, modelled, and processed as a single scalar value. This value was observed[2] and proved[9] to be dependent of the incidence angle θ_{i} of an acoustic signal on the seafloor and of its frequency f. We call it in this paper "seabed backscattering strength" or "backscatter" which involves monostatic measurements. Shorten as BS or

$BS(\theta_\text{i}, f)$, its angular variation is known as ARC[10] (angular response curves). Considering an incident intensity I_i scattered by a unit area of seafloor $\mathcal{A}_1 = 1\text{m}^2$ at a reference range $r_1 = 1$ m, the backscattering strength is usually defined as a surface index as the ratio in decibels[14]:

$$BS(\theta_\text{i}, f) = 10 \log_{10} \left(\frac{I_\text{s}(\theta_\text{i}, f)}{I_\text{i}(\theta_\text{i}, f)} \right) \tag{1}$$

where I_s is the intensity scattered by the unit area of the seafloor.

2.1 Three domains dealing with seabed backscatter

Theoretical backscattering strength models are numerous[6] and mainly assume the seafloor as an interface between the sea water and the sediment. This interface can have different properties (flat, rough, penetrable...) depending on the application and which details the user is interested in. In the main literature $BS(\theta_\text{i}, f)$ is considered deterministic i.e. at a given angle, frequency and seafloor nature corresponds only one value of BS. Similarly, in practice, the backscatter is derived from echosounder measurements using a sonar equation yielding the deterministic parameter:

$$BS = EL - SL - DI_\text{Tx} + 2TL - 10 \log_{10}(\mathcal{A}) - G_\text{calib} \tag{2}$$

where EL is the received echo level, SL is the source level of the echosounder, DI_Tx is the transmission directivity index, TL is the transmission loss between the echosounder and the seabed, G_calib is the calibration gain of the echosounder, $\mathcal{A}$ is the instantaneous insonified area on the seafloor.

However, when observing BS measurements, a large variability appears[4]. It is due to two main sources: external phenomena with no relation to the seafloor (changes in echosounder characteristics, environment modifications, etc.) and specific characteristics of the seafloor (temporal changes of the seabed, geographical variations, spatial variabilities in the echosounder beam footprint, intrinsic variability of the scattering mechanism). The first source impacts are restricted by frequent measurements of the environment parameters (sound speed, absorption) during surveys and by performing echosounder calibrations as often as possible. In order to limit the impacts of the second source, time and spatial spread of the analysed data can be reduced. However, the intrinsic randomness of the scattering process will remain. This random process, detailed in section 2.2, leads inevitably to a random backscattering strength. Consequently, from this point of view the seafloor acoustic response is a random variable which mean and variance are specific to a seafloor type (at a given incidence angle and frequency).

2.2 Link between deterministic and stochastic backscatter natures

In order to link the two equivocal natures of the backscattering strength described previously, the scattering process is modeled using a point-scattering model[1]. It is based on the assumption that the seafloor area $\mathcal{A}$ insonified instantaneously by the echosounder is composed of individual scatterers that contribute to the final backscattered signal. The complex amplitude A_s of the signal received by the echosounder is then a random variable that depends on the seafloor scatterers characteristics.

Linearly, the deterministic sonar equation can be written as:

$$A_\text{s} = A_\text{i} \cdot C_\text{eq.so} \cdot A \tag{3}$$

where $C_\text{eq.so}$ is a constant including all sonar equation parameters (systems and environment), A_i is the amplitude of the transmitted signal (at the echosounder front), and A is the complex backscattering strength amplitude which can be also be written as: $A = |A|e^{j\varphi}$, with $|A|$ the modulus of the complex amplitude and φ its phase. We saw previously that in the sonar equation and most of the literature, the backscattering strength is defined as the deterministic parameter BS in decibels which linear (non-dB) equivalent is bs. Combining equations 1, 2 and 3 we then can write in the deterministic context:

$$BS = 10\log_{10}(bs) = 10\log_{10}\left(\frac{I_{\text{s}}}{I_{\text{i}}}\right) = 10\log_{10}\left(\frac{|A_{\text{s}}|^2}{C^2_{\text{eq.so}} \cdot |A_{\text{i}}|^2}\right) = 10\log_{10}\left(|A|^2\right) \quad (4)$$

where $|A_{\text{s}}|$ is the amplitude modulus of the scattered signal from the insonified area received by the echosounder, $|A_{\text{i}}|$ the transmitted signal amplitude modulus, $C_{\text{eq.so}}$ is a constant containing sonar equation parameters, and $|A|$ is the backscattering strength amplitude modulus.

Assuming that the number of insonified scatterers is large (wide instantaneous insonified area), that the amplitude and phase of the scatterers are statistically independent of each other, and that the phases are uniformly distributed on $[-\pi, \pi]$, it can be demonstrated analytically[6] that the successive measured and corrected amplitude modulus $|A|$ are realisations of the random variable $|\mathbf{A}|$ which follows a Rayleigh distribution of parameter σ^2. This parameter corresponds to the variance of the real and imaginary parts of $\mathbf{A}$.

In this stochastic context the received backscattered amplitude $\mathbf{A}_{\text{s}}$ is a random variable and the transmitted amplitude A_{i} is still a constant, leading the backscattering strength amplitude $\mathbf{A}$ to be a random variable based on their relation in equation 4. Consequently, the backscattering strength $\mathbf{bs}$ is also a random variable and can be derived as:

$$10\log_{10}(\mathbf{bs}) = 10\log_{10}\left(\frac{|\mathbf{A}_{\text{s}}|^2}{C^2_{\text{eq.so}} \cdot |A_{\text{i}}|^2}\right) = 10\log_{10}\left(|\mathbf{A}|^2\right) \quad (5)$$

The random variable $\mathbf{bs}$ is therefore the square of $|\mathbf{A}|$ which was demonstrated to follow a Rayleigh distribution of parameter σ^2. As $\mathbf{bs} = |\mathbf{A}|^2$, the random variable $\mathbf{bs}$ follows an exponential distribution of parameter two times the parameter of the Rayleigh distribution i.e. $2\sigma^2$.

We propose in this paper to hold with the seabed backscatter BS as a scalar value (useful e.g. to display ARC) but derived from a stochastic variable. Under this hypothesis, BS is thus defined as the expected value of the random variable $\mathbf{bs}$, i.e.:

$$BS = 10\log_{10}(\mathsf{E}[\mathbf{bs}]) \quad (6)$$

The expected value of the exponential distribution being equal to its parameter, the expected value of $\mathbf{bs}$ is thus:

$$\mathsf{E}[\mathbf{bs}] = 2\sigma^2 \quad (7)$$

Therefore, in decibels:

$$BS = 10\log_{10}\left(2\sigma^2\right) = 10\log_{10}\left(\sigma^2\right) + 3 \quad (8)$$

The backscattering strength BS is thus directly linked to the Rayleigh distribution parameter σ^2. This parameter is also the variance of the real and imaginary parts of the complex backscattering strength amplitude which is linked to the number of scatterers in the instantaneous insonified area and their magnitudes distributions[12]. In other words, the seabed backscatter corresponds to the average stochastic scattering ability of the scatterers composing the interface to reflect the incident intensity.

3 BEST ESTIMATOR OF SEABED BACKSCATTER AND FORMULATION OF ITS UNCERTAINTY

In this section we compare different estimators of the backscattering strength, i.e. estimators of $2\sigma^2$, in order to define the most accurate one. Seafloor echo samples amplitude modulus corrected from sonar equation parameters are noted x i.e. $x = |A|$. Eight estimators are derived based on three methods of reduction of the information contained in the seafloor echo. We call reduction method a way to take seafloor echo samples magnitudes x_i of one or several pings (at a given θ_i and f) and deduce from them one value called a descriptor. In literature, several descriptors can be found. Three of them are predominantly used[3] thus studied in this paper:

- the **median** q of the n samples magnitudes x_i i.e $q = \text{median}\{x_i : i \in [\![1, n]\!]\}$
- the **sample mean** μ of the n samples magnitudes x_i i.e. $\mu = \frac{1}{n}\sum_i^n x_i$
- the **square sample mean** r of the n samples magnitudes x_i i.e. $r = \frac{1}{n}\sum_i^n x_i^2$

For each descriptor, three estimation methods of $2\sigma^2$ are compared: 1) the maximum likelihood estimator (MLE) of $2\sigma^2$ derived from the descriptor distribution[7], 2) the raw descriptor as an estimator of $2\sigma^2$, 3) the unbiased descriptor as a estimator of $2\sigma^2$. The third method only apply for median and sample mean descriptor, as the square sample mean is known to be unbiased when used as a Rayleigh distribution parameter estimator. Bias compensated to generate unbiased descriptors from raw descriptors are calculated analytically[11]. This finally yields eight estimators of $2\sigma^2$ to be compared[6].

3.1 Comparaison of backscatter estimators

Comparisons are made by simulating seabed echo samples magnitudes x_i as realizations of the same Rayleigh distribution of parameter σ^2. Backscattering strengths were estimated from these realizations based on the bathymetric process: for each ping, a reduction of the seafloor echo information to one value is made using a backscattering strength estimator taking into account n samples of the seafloor echo. Estimators criteria of comparison are the evolution of their bias and variances according to the number of samples taken into account, and their speed of convergence to their final value. Table 1 shows the resulting bias and variance of the eight estimators using $n = 5$ or $n = 200$ samples. Estimators using the first method (MLE) are noted $2\hat{\sigma}_d^2$ with d the descriptor used. Estimators using raw descriptors d as estimators are noted $\hat{d}$. And estimators using unbiased descriptors as estimators are noted $\hat{d}^2_{\text{ub}}$.

Table 1 shows that the best estimator of $2\sigma^2$ is $\hat{r}$ (or equivalently $2\hat{\sigma}^2_{\text{SSM}}$) as it is unbiased, has the lower variance, and has the fastest speed of convergence i.e. converge the more rapidly toward its final values (compared at $n = 5$ and $n = 200$). Consequently, in order to estimate the backscattering strength accurately, we recommend to use the square sample mean of the seafloor echo samples magnitudes x_i (see equation 9). Table 1 also shows that two estimators defined as the raw value of descriptors are biased. They are analytically calculated[6] and then unbiased to obtain $\hat{q}^2_{\text{ub}}$ and $\hat{\mu}^2_{\text{ub}}$. However, we see in table 1 that even if these estimators are unbiased, their variances are higher than the square sample mean estimator and their speed of convergence is lower (i.e. they still are biased at $n = 5$).

3.2 Analytical formulations of the best backscatter estimator and its uncertainty

Following previous results, the best estimator of the backscattering strength $\widehat{BS}$ can be written as:

$$\widehat{BS} = 10\log_{10}\left(\widehat{bs}\right) = 10\log_{10}\left(2\hat{\sigma}^2_{\text{SSM}}\right) = 10\log_{10}\left(\hat{r}\right) = 10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right) \tag{9}$$

where $\widehat{bs}$ is the linear (non-dB) backscattering strength estimator. Based on equation 9 we can derive analytically its variance and expected value. If x_i are realizations of a Rayleigh distribution then their square

Name of descriptors estimators are based on	Estimators of $2\sigma^2$	Bias estimation (dB) $10\log_{10}\left(\hat{\mathsf{E}}\left[2\hat{\sigma}^2\right]\right) - BS_{\text{th}}$		Variance estimation (dB)	
		n=5	n=200	n=5	n=200
Median q	$2\hat{\sigma}^2_{\text{med}}$	1.3 dB	0.0 dB*	-22.0 dB	-39.9 dB
	$\hat{q}^2$	-1.0 dB	-1.6 dB	-26.5 dB	-43.1 dB
	$\hat{q}^2_{\text{ub}}$	0.7 dB	0.0 dB*	-23.3 dB	-39.9 dB
Sample mean μ	$2\hat{\sigma}^2_{\text{mean}}$	0.3 dB	0.3 dB	-25.9 dB	-41.5 dB
	$\hat{\mu}^2$	-0.7 dB	-1.0 dB	-28.0 dB	-44.1 dB
	$\hat{\mu}^2_{\text{ub}}$	0.4 dB	0.0 dB*	-25.9 dB	-42.0 dB
Square sample mean r	$2\hat{\sigma}^2_{\text{SSM}}$	0.0 dB*	0.0 dB*	-26.8 dB	-42.3 dB
	$\hat{r}$	0.0 dB*	0.0 dB*	-26.8 dB	-42.3 dB

Table 1: Summary of the estimators of $2\sigma^2$ analysed. Bias are estimated comparing the computed expected value of estimators for a small ($n = 5$) and a large ($n = 200$) number of samples to the theoretical backscattering strength BS_{th}. Computed variances are estimated for the same arbitrary numbers of samples. All estimations are made by generating 400 realizations of a ping. (*: bias are not perfectly equal to zero in simulations but are less than the tenth of decibels which is rounded to 0.0 dB.)

sample means $\frac{1}{n}\sum_{i=1}^{n} x_i^2$ are realizations of the γ-distribution $\gamma\left(n, \frac{2\sigma^2}{n}\right)$. Consequently, the backscattering strength estimate as a random variable, noted $\widehat{\mathbf{bs}}$, follows also that γ-distribution i.e. $\widehat{\mathbf{bs}} \sim \gamma\left(n, \frac{2\sigma^2}{n}\right)$. Therefore, the expected value of $\widehat{\mathbf{bs}}$ is:

$$\mathsf{E}\left[\widehat{\mathbf{bs}}\right] = \mathsf{E}\left[\gamma\left(n, \frac{2\sigma^2}{n}\right)\right] = 2\sigma^2 \tag{10}$$

And its variance $\widehat{\mathbf{bs}}$ is:

$$\text{var}\left[\widehat{\mathbf{bs}}\right] = \text{var}\left[\gamma\left(n, \frac{2\sigma^2}{n}\right)\right] = \frac{(2\sigma^2)^2}{n} \tag{11}$$

These two results validate the simulation results described previously: the expected value of $\widehat{\mathbf{bs}}$ is equal to twice the Rayleigh parameter σ^2 therefore the estimator $\widehat{bs} = 2\hat{\sigma}^2_{\text{SSM}} = \hat{r}$ is unbiased, and its variance depends on the number of samples n taken into account. The result $(2\sigma^2)^2/n$ is also the Cramer-Rao bound of twice the Rayleigh distribution parameter estimator[6] thus var$[\widehat{\mathbf{bs}}]$ is the lowest variance reachable with $\widehat{bs}$. This made the best estimator of the backscattering strength also an efficient estimate.

In table 1 simulation, a theoretical backscattering strength was chosen as $BS_{\text{th}} = 10\log_{10}\left(2\sigma^2_{\text{th}}\right) = -10$ dB. Using equation 11, we can write:

$$10\log_{10}\left(\text{var}\left[\widehat{\mathbf{bs}}\left(2\sigma^2_{\text{th}}, n\right)\right]\right) = 10\log_{10}\left(\frac{(2\sigma^2_{\text{th}})^2}{n}\right) = -20 - 10\log_{10}(n) \tag{12}$$

For $n = 5$ and $n = 200$ respectively variances are therefore equal to -27.0 dB and -43.0 dB. These two analytical results correspond to the simulation results given in table 1.

In practice, a useful criterion describing the accuracy of backscatter measurement is its uncertainty. It includes the uncertainties of all the parameters appearing in the sonar equation[5] in addition to the intrinsic uncertainty of the backscattering strength. The latter is noted $T[\widehat{\mathbf{bs}}]$ in decibels. To better fit with the measurements distribution which is asymmetrical when represented in dB, a level of uncertainty is provided for each sides. They are derived from the expected value and variance of $\widehat{\mathbf{bs}}$ respectively for the upper $+$ and lower $-$ sides of the distribution as[13,5]:

$$T_{\pm}\left[\widehat{\mathbf{bs}}\right] = 10\log_{10}\left(1 \pm \frac{\sqrt{\mathrm{var}\left[\widehat{\mathbf{bs}}\right]}}{\mathrm{E}\left[\widehat{\mathbf{bs}}\right]}\right) \tag{13}$$

The uncertainty of the best backscattering strength estimator can therefore be derived analytically from equations 10, 11 and 13 as[7]:

$$T_{\pm}\left[\widehat{\mathbf{bs}}\right] = 10\log_{10}\left(1 \pm \frac{1}{\sqrt{n}}\right) \tag{14}$$

The intrinsic uncertainty of the backscattering strength, under the Rayleigh distribution assumption, then depends only on the number of samples n taken into account.

4 PRACTICAL EXAMPLE

We present an example of backscatter and uncertainty estimations on singlebeam echosounder data acquired in the Bay of Brest (France)[8]. Representations of the results are proposed as $\widehat{BS}(\theta_i)$ values estimated from seafloor echo samples and plotted per incidence angle on the seafloor (cf. figure 1). Uncertainties are represented as error bars above and below these estimations. Two type of uncertainties are shown in figure 1: one estimated from the data (equation 13), and one calculated theoretically using only the number of samples n (equation 14).

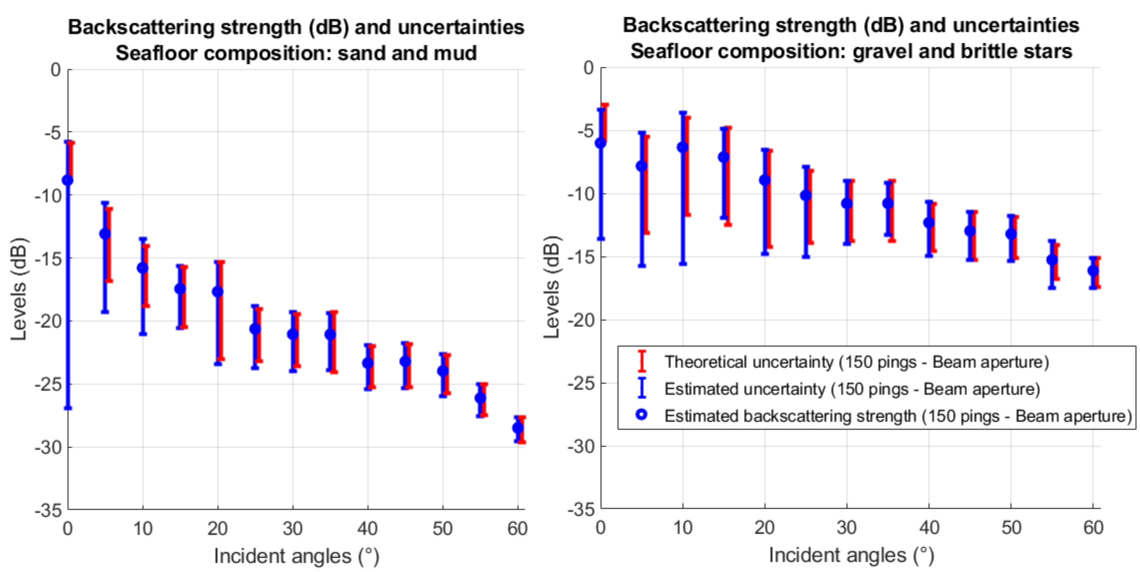

Figure 1: Examples of estimated backscattering strength and uncertainties according to the incidence angles using 150 pings and samples inside the beam aperture. Theoretical uncertainties are plotted in red. Singlebeam echosounder data from project ANR-14-ASTR-0022[8] acquired on two seabed types.

Raw seabed echo data received by echosounder transducers are real signals $s(t)$ that can be written as:

$$s(t) = \mathrm{Re}\left\{A_s(t)e^{j2\pi ft}\right\} \tag{15}$$

where f is the transmit signal central frequency. In order to extract from this raw data the modulus $|A_s|^2(t)$ of the complex amplitude $A_s^2(t)$ scattered by the seafloor we use the Hilbert transform $\mathcal{H}$ i.e. $|A_s|^2(t) = s^2(t) + j\mathcal{H}[s(t)]^2$. These modulus $|A_s|^2(t)$ are then corrected from sonar equation parameters (see equation 4) to obtain values of $|A|^2(t) = x_i^2$. Thus, at each time t corresponds a seabed echo sample corrected magnitude x_i^2 which is equivalent to a realization of the basckscatter random variable **bs**. When provided by main echosounders, this list of bs values derived for one ping from the raw seafloor echo is usually called snippet.

The backscattering strength estimation as described in this paper is only valid if samples magnitudes x_i in equation 9 are independent. Therefore, only samples spaced by the transmitted pulse length are retained in this example. The seafloor is assumed homogeneous for all (150) pings acquired on two different terrains: sandy mud and gravel with brittle stars. Only samples inside the beam aperture are retained.

Figure 1 shows two distinct angular responses shapes corresponding to each seafloor type. This result is expected because of their different composition. Regarding the uncertainty levels, estimated uncertainty levels are mainly close to theory, confirming the validity of our statistical model. At 0° the theoretical negative uncertainty level cannot be represented in dB (equation 14) because $T_- \xrightarrow[n \to 1]{} -\infty$. However, we can observe that at some angles the estimated uncertainty is higher than the theoretical uncertainty and at other angles the contrary appears. This effect could be related to the deviation of the samples magnitudes distribution from the supposed Rayleigh distribution (which parameter σ^2 is the estimated backscatter minus 3dB, see equation 8).

5 CONCLUSION

To conclude, the best backscatter estimator among the eight analysed in this paper is, in term of expected value and variance, the square sample mean of the seafloor echo samples amplitude modulus corrected from sonar equation parameters (x_i). This result was predictable as soon as we demonstrate that the backscattering strength was two times the Rayleigh distribution parameter, as the square sample mean is also twice the maximum likelihood estimator of the Rayleigh distribution parameter σ^2. The analytical variance of this estimator was demonstrated to be $(2\sigma^2)^2/n$ with n the number of samples taken into account.

In practice, this paper results imply that the backscatter should be estimated from echosounder mesurements by 1) extracting the seafloor echo real signal $s(t)$, 2) applying Hilbert transform to calculate the complex amplitude modulus $|A_s|^2(t)$, 3) correcting each seafloor echo samples from sonar equation parameters to obtain snippet values $|A|^2(t) = x_i^2 = bs_i$, 4) retaining only independent samples (i.e. spaced in time by the transmitted pulse length and coming only from pings that insonified areas are not overlapping), 5) using the best estimator $\widehat{BS} = 10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)$. Note that data employed to estimate BS should be acquired on an homogeneous seafloor i.e. on an terrain leading to a unique Rayleigh distribution.

An example of application of backscatter estimation was given at the end of this paper. It allows to compare estimated and theoretical uncertainties. Two trends were observed during the analysis: estimated uncertainty level could be higher or lower than the theoretical ones. The first trend probably originates from the presence of samples that belong to another distribution (or a mixture of distributions). It can be due to changes in the seafloor type, remaining imperfections in the correction of the sonar equations, penetration of the signal in the sediment, etc. The second trend is theoretically not possible because the variance of the backscattering strength estimate reaches the Cramer-Rao bound. Nevertheless, in practice this result could be observed when samples used are too correlated (e.g. insonified areas are not enough separated). Based on these observations, the comparison between theoretical and estimated uncertainties may possibly be employed as an indicator of deviation from the estimated Rayleigh distribution.

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