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AN HEURISTIC STUDY OF THE PROPAGATION OF SOUND TO LONG RANGES BELOW AND WITHIN ACOUSTIC SURFACE DUCTS

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This paper arises from a requirement to obtain a simple and reliable formula for mean propagation losses in the ocean along paths which start within an isothermal surface duct and end, at long horizontal ranges, in the region below this duct. It is well known that ray theory, which has been very successful as a prediction tool for in-duct propagation, gives no useful information about sound levels in this region. Rather, it predicts a region of silence ('shadow zone' - to borrow the terminology of geometrical optics), whose lack of physical reality has been proved repeatedly in practice.

Accordingly, while retaining the basic framework of ray acoustics, we look for refinements which more closely approximate the physical situation, and admit mechanisms which will allow energy to be transmitted into the shadow zone. Three such mechanisms exist:

(a) Ray theory assumes the sound to be of vanishingly small wavelength; the finite frequencies of physical sound transmissions, however, are subject to diffractive effects, in particular the leakage of energy across the shadow's edge, and through the base of the duct near the skip points of the limiting ray.

(b) The classical model of the ocean velocity profile assumes stratification in horizontal planes; turbulence, convective mixing, and saline 'finger cells' in practice all violate this condition. The simplest model which takes these effects into account is to make the base of the duct a progressive internal wave, which can cause refraction to below the duct of seemingly entrapped energy. This we adopt.

(c) In ray theory, also, we consider the effect of the air-sea interface to be total internal reflection from a smooth horizontal plane; clearly, surface waves make this picture incomplete, and so we consider as the third insonifying mechanism the scattering of sound into the shadow zone on reflection from the sea surface.

Each of these effects, it must be stressed, constitutes a small perturbation on the ray theory picture, which is still the basic framework within which we work. Accordingly, we neglect all interactions between these insonifying mechanisms, and consider them as independent.

A complete treatment of the effect of diffraction is already available in the context of electromagnetism, in the shape of the theory of geometrical diffraction, developed by Keller, Seckler and Jones. It is straightforward, though tedious, to apply this to the problem of sound propagation in and below the surface duct; when this is done, the most convenient form for the results is as excess loss above spherical spreading per kiloyard beyond the limiting ray, which is plotted against frequency for a representative selection of ocean surface temperatures and thermocline temperature gradients in Figure 1. The extent of this loss makes it a priori unlikely that diffractive leakage is a significant contributor to below duct sound levels except at very low frequencies.

The contribution of refraction from internal waves is also small; evidence to back up this assertion is supplied by Shulkin, who carried out experiments both in the laboratory and at sea. There are several theoretical arguments by which these results may be justified, ranging from a detailed calculation of the amplitude-frequency spectrum of permissible internal waves to the observation that the mechanism is frequency independent. At low frequencies the effective boundary of the duct as seen by the wave is plane (the converse of Nyquist's sampling criterion), so the effect of internal waves is included in the diffractive term. This gives an upper limit to the possible insonification of the shadow zone by refraction from internal waves which is set at a very low level.

We are left with the mechanism of scattering from the sea surface. At first sight, it appears that to make progress with this it is necessary to evaluate the scattering matrix of the sea surface. Much effort has gone into this, but no firm conclusions have been reached either in acoustics or in the more complex but more intensively investigated analogue of electromagnetic theory. However, it has proved possible to sidestep this problem, and deduce several fundamental properties of the field without evaluating this matrix, purely by making the assumption that - at a large enough range from the source - the directional form of the spectrum of energy incident on the sea surface, and issuing from the sea surface, is independent of position.

Now we consider a receiver situated in the shadow zone below the duct, and calculate the energy which reaches it from the sea surface. On Figure 2 we show the notation for this calculation. The contribution from the annulus on the sea surface of radius ρ and thickness $d\rho$, centred directly above the receiver, is

$$\int_0^{2\pi} I(r)\,\frac{g(\theta,\phi)\,\rho\,d\rho}{2\pi s^2}\,d\theta$$

where r is the distance of this centre from the source and θ is the angle between the radius vector and the direction of propagation. Defining ϕ_o, the limiting grazing angle at the surface to be the angle mode at the surface by a ray which just escapes the duct, the total energy received at the receiver is

$$I(r)\int_0^{\rho(\phi_o)} \frac{G(\phi)\,\rho\,d\rho}{s^2} \qquad \text{where} \quad 2\pi G(\phi) = \int_0^{2\pi} y(\theta,\phi)\,d\theta$$

Now defining ψ, the equivalent angle for straight line propagation by $\cos\psi = \left(\frac{c_o}{\bar{c}}\right)\cos\phi$, c_o being the velocity of sound at the sea surface and $\bar{c}$ the harmonic mean on a path from surface to receiver, this energy becomes

$$I(r) \int_{\pi/2}^{\psi_o} G_1(\psi)\cot\psi \, d\psi \qquad \text{where } G_1(\psi) \equiv G(\phi) \qquad (1)$$

As the energy at the point on the surface directly above the receiver is a constant multiple of $I(r)$, this implies that the relative level of signal at the receiver and at the surface directly above it

(a) is independent of range

(b) depends on receiver depth and ocean velocity structure (e.g. duct depth)

only through variations in $\psi_o \equiv \cos^{-1}\left(\frac{c_o \cos\phi_o}{\bar{c}}\right)$; this is a slowly varying parameter and so relative levels are insensitive to changes in duct and receiver depth.

(c) depends strongly on $G_1(\psi)$ - the scattering matrix of the sea surface, which is a function of the sea surface statistics and the frequency of insonification only.

Strong evidence for this theory has been discovered in measurements of propagation levels made several years ago. A series of 30 measurements revealed a coefficient of association of 0.93 between increased sea state and decreased signal level differential, and no significant association of this differential with either duct depth or receiver depth. These measurements were, in fact, of records of explosive signals filtered over a typical octave band, and for this particular frequency range it was possible to fit a log-linear equation of the form

$$\text{Differential} = 28 - 2.5n \quad \text{dB, where } n \text{ is the sea-state} \qquad (2)$$

the linearity being almost certainly spurious and only justified by the lack of data at higher sea states.

It is interesting to continue this analysis to use this level differential to predict the range dependence of propagation loss in the surface duct. There are three steps - use of the equation of continuity to develop an easily soluble differential equation, consideration of the near source field to 'calibrate' levels, and allowance for the effects of attenuation. Fitting the quoted levels this gives

$$\text{PROPAGATION LOSS} = 55 + 10\log R + 5\log h + 10^{n/4}(R - 0.3\sqrt{h})\, b(f) + a(f)\, R \quad \text{dB} \qquad (3)$$

where $a(f)$ is the attenuation, R is range in kilometres and h duct depth in metres. $b(f)$, the scattering coefficient, is in this case 6.88. The functional form of $b(f)$ depends on the scattering matrix, but it may be anticipated that it is approximated by an exponential (corresponding to linear variation of the difference in levels in the empirical formula). In practice the variation appears to be slight over the range 1 to 10 kHz.

Equation 3 is less successful than equation 2 as a prediction tool, on the basis of existing data. This is scarcely surprising, as diffractive leakage, volume scattering, and similar effects are excluded in both. For a one range comparison this is fully acceptable, but the cumulative effects of these teams over long ranges will eventually become significant. Accordingly, equation 3 may be used as a lower limit to the propagation loss in the duct, and if a suitable frequency dependent attenuation term is obtained, the actual loss will be found to approach this limit asymptotically as frequency is increased.

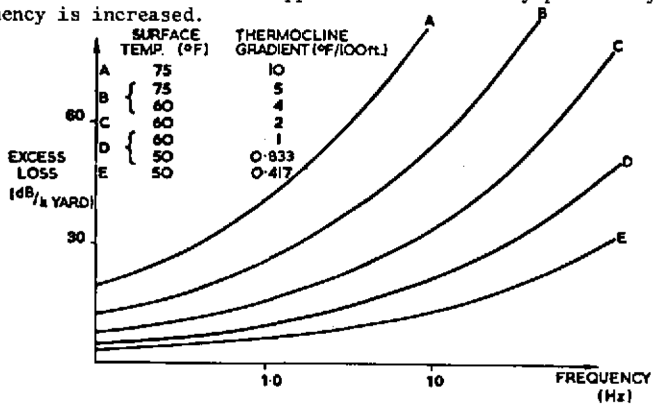

FIG. 1. EXCESS LOSS (ABOVE SPHERICAL SPREADING), AS A FUNCTION OF FREQUENCY (IN dB PER KILOYARD BEYOND LIMITING RANGE)

References

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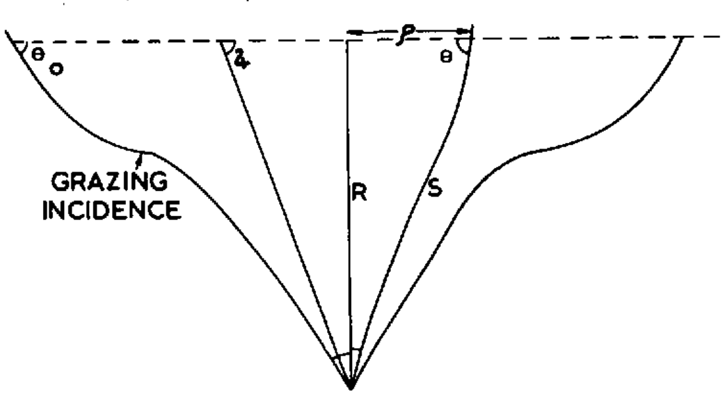
3. Shulkin, M (1969) Propagation of Sound in Imperfect Surface Ducts (PhD thesis).



SCATTERING ENERGY FOR A RECEIVER IN THE SHADOW ZONE