Proceedings of The Institute of Acoustics

# STABILITY CONSIDERATIONS AND PERFORMANCE LIMITS IN MULTIPLE SOURCE CANCELLATION.

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In an earlier paper (Ref 1), the cancellation limits which are inherent in any Active Noise Control system were considered, being illustrated by an analysis of the "one-dimensional" problem of the strictly causal duct. In that paper, the central role played by the generalised correlation and transfer functions of the acoustic field emerged strongly, and "noise" in the form either of the non-coherent part of the perceived uncontrolled field or of the incorrectly replicated control source field was shown to limit performance, the two noise components being equivalent in this.

A natural extension to these considerations is the study of the enhanced potential for cancellation by the simultaneous control of two independent components of a perceived acoustic field, propagating via different paths. The earlier paper suggests that the performance will be improved, the more precisely the individual components can be identified - which in turn suggests the possibility of using hydrophone array techniques to provide such identification. It is to quantify these improvements, and to discuss the stability considerations inherent in simultaneous feedback control, that this paper is directed.

The idealised scenario to be considered is represented schematically in Figure 1.

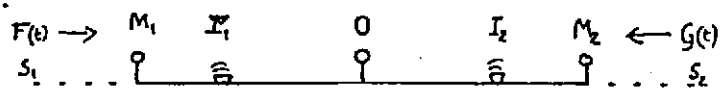


Figure 1. The two-component Active Noise Control scenario.

Here O is the observation point at which the field is to be controlled, $I_1$ and $I_2$ are the injection points at which the controlling fields are introduced, $M_1$ and $M_2$ the points at which the measurements of incident pressure are taken from which the control fields are determined., $S_1$ and $S_2$ are of a different nature, representing the physically inaccessible sources of the component incident fields F(t), G(t). The assumptions of linearity and causality then allow us to write the field at any of the five "observable" points in the form

$$H(t) = \int_r^\infty F(t-\tau)\,T_1(\tau)\,d\tau + \int_s^\infty G(t-\tau)\,T_2(\tau)\,d\tau + N(t) \qquad (1)$$

$T_1$, $T_2$ being transfer functions from $S_1$, $S_2$ respectively to the field point, r,s, positive numbers representing the minimum travel times for acoustic energy from S to the field point, and N(t) the "noise" in the pressure signature at the field point, not attributable to either of the sources at $S_1$ and $S_2$.

STABILITY CONSIDERATIONS AND PERFORMANCE LIMITS IN MULTIPLE SOURCE CANCELLATION

In order to clarify the discussion further, we assume that the T each is derived from a single non-dispersive path, so that equation (1) becomes

$$H(t) = T_1 F(t - r) + T_2 G(t - s) + N(t) \tag{2}$$

the T's now being constants. This formalism is certainly valid for narrow-band cancellation, although it should be pointed out that it is not in any way an axiomatic deduction from (1) via a Fourier Transform - to perform such a transform would require precognition of the field at $t + t'$ at time $t$ - information not available in a causal system.

When active control is imposed, known fields $d_1(t)$, $d_2(t)$ are injected at $I_1$, $I_2$ respectively. We may represent their contributions to the perceived fields at $M_1$, $M_2$, O in the form

$$c(t) = A\, d_1(t - s') + B\, d_2(t - r') \tag{3}$$

and these contributions are entirely predictable in real time, provided that the transfer functions A, B are known, which is true under the same conditions that the T's can be determined. It was intuitively apparent at the outset, and is formally obvious from these equations, that optimal cancellation at O is obtained if we set

$$d_1(t) = -T_1 F(t - r + s')/A \quad ; \quad d_2(t) = -T_2 G(t-s+r')/B \tag{4}$$

but, as previously noted, neither F nor G is physically measurable. Instead, we have, at time t, the histories for all $t' < t$ of the measured quantities

$$( H_i(t') + c_i(t') ), \quad d_i(t') \qquad i = 1,2$$

and our objective is to obtain the best estimate to (4) which is available by a linear combination of these measurements.

In Ref 1, the simpler case with G = O was studied, and it was shown that, in contradistinction to the adaptive beamforming case, taking the measured field at M, time delaying and correctly attenuating it, applying phase reversal and injecting the resultant field at I, does not lead to cancellation but to instability. The difference from the adaptive beamforming scenario, in which this is the basis of the Howells-Applebaum Sidelobe Canceller (Ref 2), is that in active noise control there is physical feedback of the controller field to provide c(t), and unless removed from the measurement this corrupts the next causal estimate of d(t). This instability occurs even more so if two components in the noise field are non-zero, so we consider optimising

$$d_i' = p_{ij} ( H + c)_j + q_{ik} (d)_k \qquad i = 1,2 \tag{5}$$

j, k being summed over the values 1,2, where we have introduced the "prime" notation on the left-hand side of (5) to denote a variable measured at a later time than the unprimed quantities and causally dependent on them.

Causality is paramount in proceeding from (5). To begin with, we note that for i = j the p's and q's refer to components which have already passed through the control point before they reach the relevant inject point, so they cannot provide a causal control of the field at O; hence in an optimal control they are zero. A further consequence of causality is that $q_{11}$, $q_{22}$ should be chosen to remove as totally as possible the contributions which are injected at $I_1$, $I_2$ from the fields sensed at $M_1$, $M_2$ respectively, as these are propagating away from the controlled area and simply add to the noise against which the causal precursors must be determined. Accordingly, if we write that the uncontrolled fields at $M_1$, O, $M_2$ are

$$\begin{array}{lcl} M_1 & : & F + uG'' + N_1 \\ O & : & aF' + bG' + N_o \\ M_2 & : & vF'' + G + N_2 \end{array} \tag{6}$$

the control equations become

$$\begin{array}{ll} & c_1 = -ud_2'' \quad ; \quad c_2 = -vd_1'' \\ \text{with} & c_o = -ad_1' - bd_2' \\ & d_1 = k(M_1 + c_1) \quad ; \quad d_2 = j(M_2 + c_2) \end{array} \tag{7}$$

and after much manipulation, this gives the mean square controlled field at O as

$$P_o = N_o^2 + (1 - jkuv)^{-2}\Big[\{F^2(1-k)^2 + N_1^2k^2\}(a^2 + b^2j^2v^2) + \{G^2(1-j)^2 + N_2^2j^2\}(b^2 + a^2k^2u^2)\Big] \tag{8}$$

The first stability criterion is immediately apparent - namely that jkuv is less than unity. In fact, in practical terms, we require this as small as possible as it represents the feedback loop amplification factor - the factor by which inputs at time t - (r + s) have been multiplied relative to inputs at time t through the operation of the control equations. But, although on physical grounds all of j,k,u,v lie between 0 and 1, in the cases where we wish to apply active control all, for simple measurements, would be nearly unity. Hence the importance of array techniques in discriminating between components at the measurement points and so reducing u,v in the equations (7). For the purposes of the analysis, and to illustrate salient points, we will assume that this has been achieved and that $u^2 = v^2 = e$, $a = b$, and that all the noise contributions, although independent, are equal in the uncontrolled state.

Then equation (8) reduces to

$$P_o = N^2 + a^2(1 - ejk)^{-2}\Big[F^2(1 - k)^2(1+ej^2) + G^2(1-j)^2(1+ek^2) + N^2(j^2+k^2+2ej^2k^2)\Big] \tag{9}$$

and the partial derivative of $P_o$ with respect to j gives the following equation for the optimal value.

$$j = \frac{G^2(1-ek)(1+ek^2) - ekF^2(1-k)^2 - ek^3N^2}{G^2(1-ek)(1+ek^2) + eF^2(1-k)^2 + N^2(1+2ek^2)} \tag{10}$$

and equivalently for k, interchanging F with G and j with k throughout. Although these are soluble analytically, the algebra is horrendous, and it is more practical (and closer to what must be done in implementing Active Control) to use an iterative numerical algorithm to successively estimate each of j, k with the other held fixed at its preceding best estimate, as determined by a minimum $P_o$ output. The conslusions then are:

(i) in all circumstances when F, G N there is significant extra cancellation by varying both j and k than would be predicted by adding the dB effectiveness of the two controls applied independently.
(ii) this is true even for (say) F≫G and vF≫G. In this event k is close to unity and relatively insensitive to the value of j, whereas j is sensitive to small variation in k. Here, for stability, it is imperative to evaluate k before j - particularly if the criterion is the measured value of P being minimised. j alone has small effect on this, but if k is first optimised, an equal further dB gain is achievable by sequential choice of j.
(iii) it is easily seen from (9) that loop noise eventually dominates the output and limits performance - the amplification factor (1 - ejk) raising the feedback contribution greatly whenever F,G are large in comparison with N. Recognise that this is so even in our idealised scenario where the loop itself is not introducing extraneous noise and it is obvious that in real circumstances it limits performance a fortiori. Hence lengthy iterations are rarely worthwhile; only if e is close to unity and F,G are comparable is the convergence of (10) and its dual slow, and in that event potential gains are small also.
(iv) the role of e is crucial. Figures 2 and 3 illustrate this by plotting cancellation limit as a function of e in the two cases F = G and G = eF, this latter implying that F dominates the measured field at both sites. The results for a single control operational are shown for comparison in both cases.

Finally we note that this central role for e is a natural extension of the importance of the cross-correlation coefficient in the single control case, as the symmetry of the present scenario implies that e is proportional to the "anticorrelation coefficient" 1 - C*.

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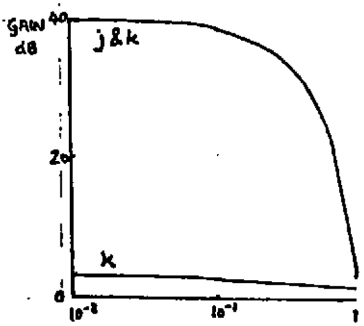


Fig 2. F = G = 100.N

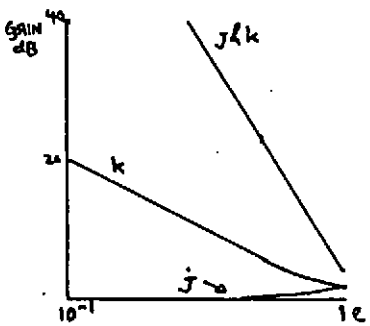


Fig 3. eF = G = 100.N

Figures 2 and 3 Cancellation Performance as a Function of e.