Proceedings of the Institute of Acoustics 'Spectral Analysis and its Use in Underwater Acoustics': Underwater Acoustics Group Conference, Imperial College, London, 29–30 April 1982

Generation of Multi-Directional Random Seas

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Abstract

The purpose of this work was to develop a technique for the generation of multi-directional random waves, which could be used for driving wave-makers in a wave basin or for the simulation of elongated structures in a real sea.

The method used was an extension of the idea of digital filtration of white noise, which has been used successfully for the generation of waves in unidirectional tanks.

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1. Introduction

There are several existing methods for the generation of 3-dimensional random waves. The Wallingford Method [1] consists of an arc of flaps all generating one dimensional spectra along different axes focussing to produce a required directional spread within a central working area of a square basin. This is not suitable for work involving structures which are longer than the small central region of the tank.

The snake [2] superimposes discrete wave fronts, each having a random starting phase. This method is not continuous in either direction or frequency but does produce flexible control of the types of spectra used. The diffraction technique [3] utilising large independently driven wavemakers which produce spreading by the natural diffraction of the waves from the paddles. This produces waves whose directionality is continuous, rather uncontrolled but known.

There is therefore a need for a method of generation which gives controllable continuous spectra over a large area of the wave basin. The technique presented here is an extension of a method already used to generate waves in 2-d wave flumes [4]. A random Boolean series is passed through a filter corresponding to the desired spectrum to produce the signal record.

2. Theory

An array of wave-makers can be thought of as a structure, the power spectrum of whose motion matches the spectrum of the required sea.

The response of an infinitely long regular linear system, lying along the x-axis, to a driving force $A(\tau,X)$ can be expressed as

$$R(t,x) = \int_{\tau=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(t-\tau,x-X)A(\tau,X)d\tau dX \qquad (1)$$

where $h(\tau,X)$ is the unit impulse response function. The power spectrum, $S_O(\omega,c)$ of the response of the system to a driving spectrum $S_I(\omega,c)$ is given by [5]

$$S_O(\omega,c) = |H(\omega,c)|^2 S_I(\omega,c) \qquad (2)$$

ω = angular frequency

C = X-component of the wave vector

$H(\omega,c)$ is the system function, or inverse fourier transform of the response function.

If $S_I(\omega,c)$ = const.

Then assuming an anti-symmetric phase distribution:

$$h(\tau,X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(\omega,c) \sin(\omega\tau + cX)d\omega dc \qquad (3)$$

The full integral (1) is not required as any real sea has only a finite coherence time and correlation distance so:

$$R(t,x) = \int_{\tau=(t-T)}^{(t+T)} \int_{X=(x-L)}^{(x+L)} h(t-\tau,\ x-X)\ A(\tau,X)d\tau dX \tag{4}$$

T > coherence time.

L > correlation length.

We are only interested in the response of the wavemakers at discrete times and places i.e. $t=a\Delta t$ and $x=b\Delta x$

hence $T = M\Delta t$ and $L = N\Delta x$

so that

$$R(a,b) = \int_{\tau=(a-M)\Delta t}^{(M+a)\Delta t} \int_{X=(b-N)\Delta x}^{(b+N)\Delta x} h(a\Delta t-\tau,\ b\Delta x-X)A(\tau,X)d\tau dX \tag{5}$$

and if the driving force is also discrete

i.e. $A(\tau,X) = A'(p,q)\ \delta(\tau-p\Delta t,\ X-q\Delta x)$

$$A'(p,q) = 0 \text{ or } 1$$

$$\text{and } p,q = 0,1,2,3,\ldots..$$

(5) becomes

$$R(a,b) = \int_{\tau=(a-M)\Delta t}^{(M+a)\Delta t} \int_{X=(b-N)\Delta x}^{(b+n)\Delta x} h(a\Delta t-\tau,\ b\Delta x-X)\ A'(p,q)\delta(\tau-p\Delta t,\ X-q\Delta x)\ d\tau dX \tag{6}$$

$$R(a,b) = \sum_{p=(a-M)}^{(M+a)} \sum_{q-(b-N)}^{(b+N)} h(a-p,\ b-q)\ A'(p,q) \tag{7}$$

where $h'(m,n) = h(m\Delta t,\ n\Delta x)$

3. Method

The programme used to generate the waves was written in P.A.S.C.A.L. on an ICL 2972 mainframe. An array of shift registers containing 1's and 0's was convolved with the digital filter to produce the wave records. The input to the shift registers was produced using a linear congruential random number generator.

Figure 1 shows an example of a 2-dimensional spectrum which has a Gaussian form in the frequency domain and $\cos^2$ directional spreading. Notice that the drawing is drawn in C space rather than Θ space. The corresponding digital filter is shown in figure 2. To simplify the diagram only the positive time part of the filter is plotted.

4. Results

The results were tested in two ways.

The total power spectrum of the wave signals was compared with the model power spectrum. Then cross-correlations of the time records at various x-positions were compared with the predicted cross-correlation.

Figure (3) shows the superimposition of the signal spectrum on the desired Gaussian form. These show very good agreement and the conclusion is that this method provides excellent spectral reproduction at the wavemakers. Only one tank test has been run to date; the resulting power spectrum is shown in figure (4). The high frequency shift of the spectrum is due to the transfer function of the tank which had not been accurately determined.

Figure (5) is a 3 dimensional plot of the wave signals on which the analysis was carried out. The wave direction is indicated. The diagram displays, admirably, the short crested nature of the wave field.

The correlation of the wave signals is plotted on figure 6. The separation of the wave-makers was assumed to be 0.3 m. This is to be compared with the correlations obtained between wave gauges positioned at a separation distance of 0.35 m. The inaccuracies in the latter may be accounted for by diffraction effects at the edges of the tank (note that only 7 wavemakers were used in the preliminary test).

5. Conclusions

The method of 2 dimensional filtering of random noise has been successful in producing the desired sea states. However several tests still need to be performed before full implementation. The method lends itself for use on small tanks by direct signal production from a micro-computer. Larger tanks may require a somewhat faster machine although the possibility of calculating the signals before running the sea state should be borne in mind.

Acknowledgements

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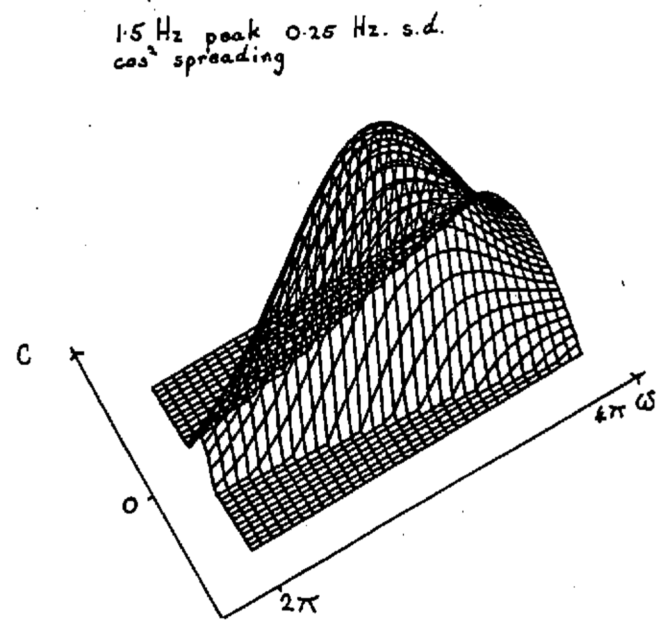

Figure 1

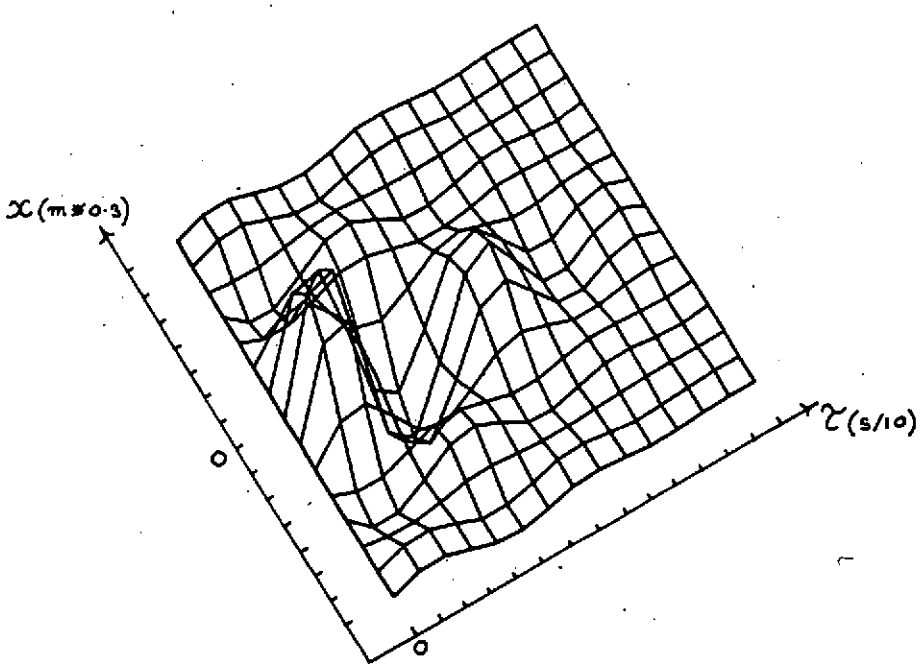

Figure 2

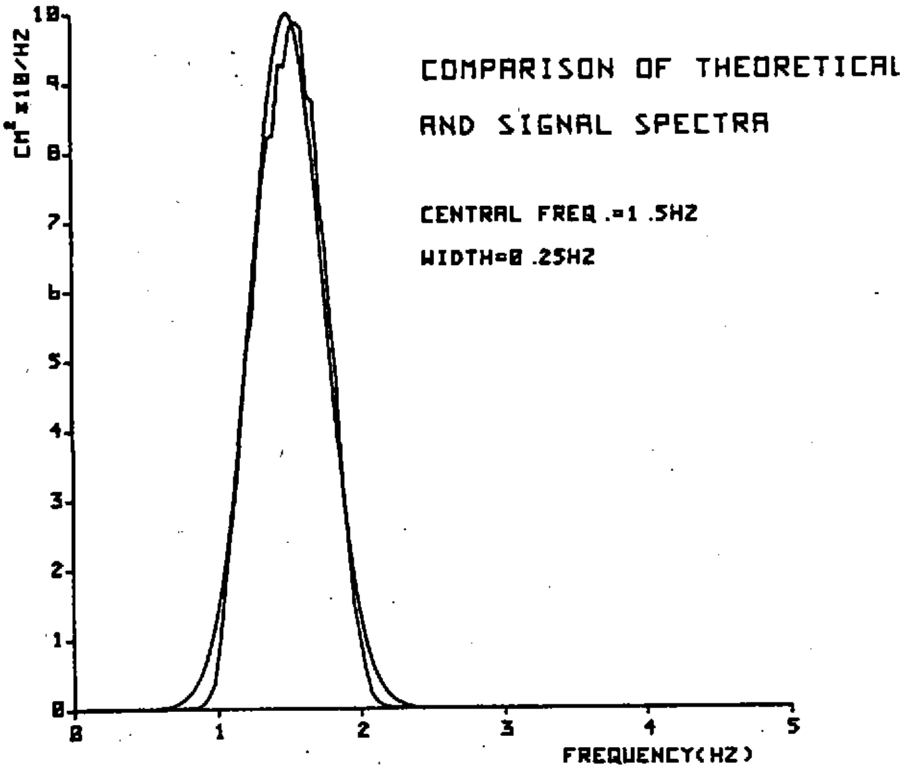

Figure 3

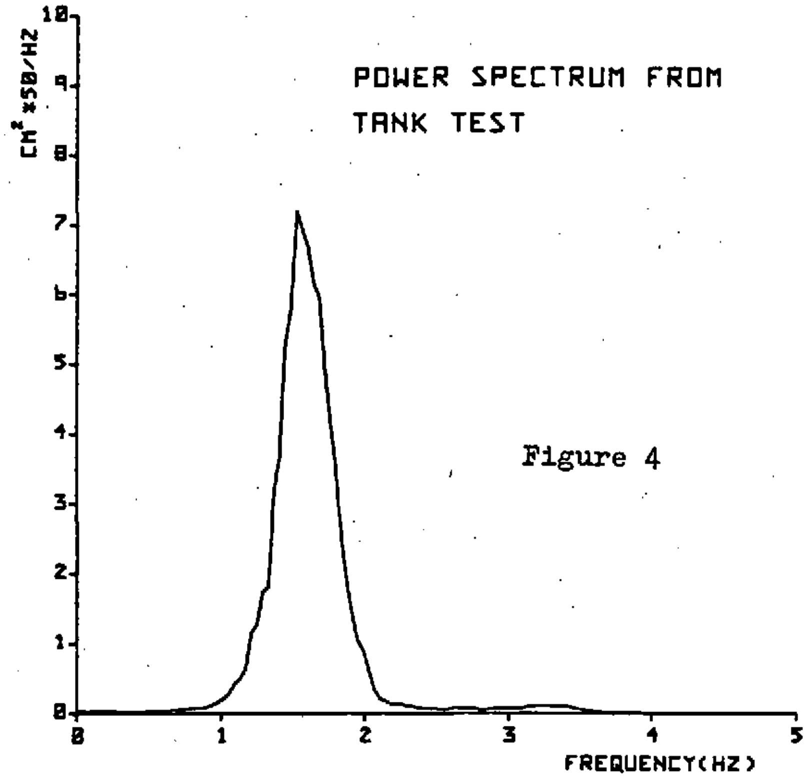

Figure 4

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Wave Signals Using Digit 0315 Filter

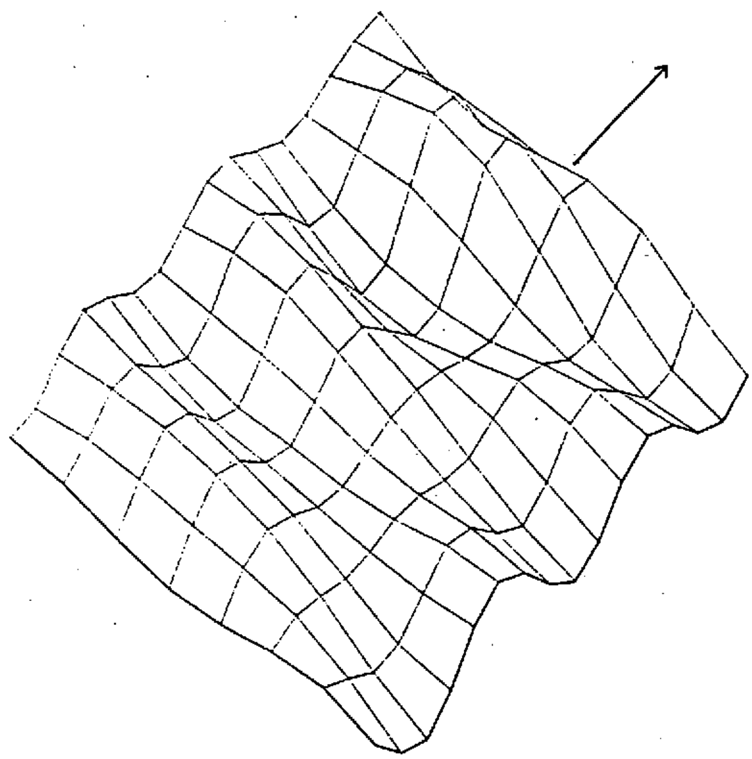

Figure 5

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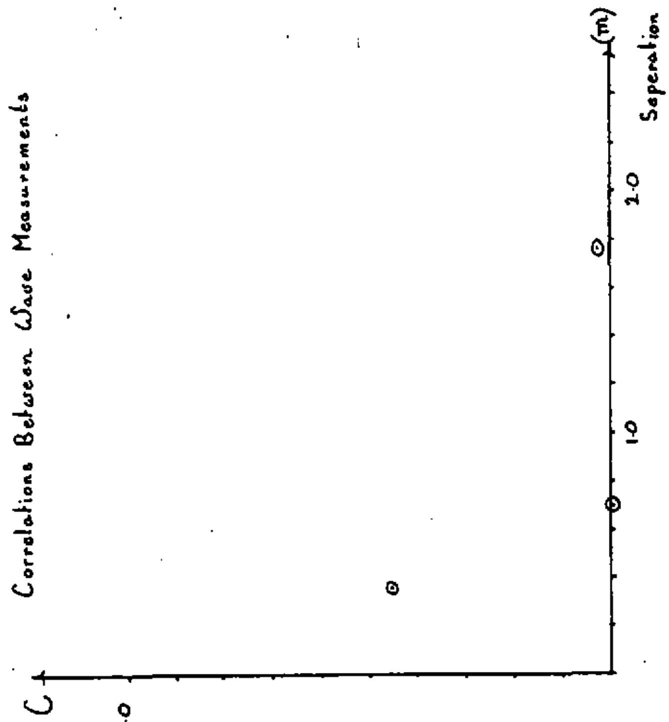

Figure 7

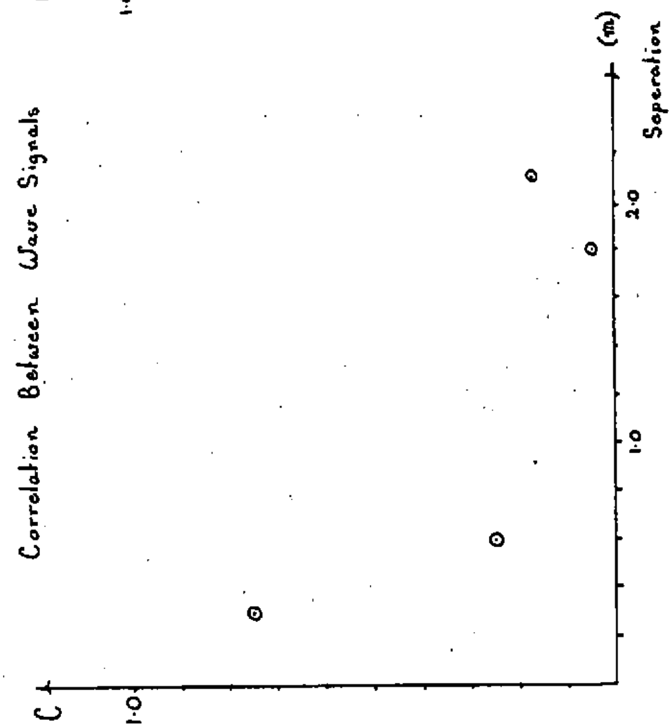

Figure 6