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THE ACOUSTICS OF PERFORATED LININGS USED TO SUPPRESS SCREECHING COMBUSTION

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1. Introduction

In the 1940's, the quest for greater thrust from jet engines led to the introduction of afterburners. In these devices, fuel is injected upstream of a flame-stabilising bluff-body in the jet-pipe. The reheat flame which is produced is a powerful source of sound, and it was soon discovered that a variety of acoustically driven combustion instabilities occurred. One such instability is referred to by the onomatopoeic term 'screech', in which the reheat flame excites a transverse mode of oscillation. Combustion instabilities are potentially hazardous, and there have been many attempts at suppressing them; Markstein [1] reviewed much of the early work in this field.

Research workers suspected that by inserting a sound-absorbent lining in the jet-pipe, near the reheat flame, screeching combustion could be suppressed. The idea was simply that if the sound produced by the flame at the screech frequency could be absorbed, then the sound level should remain acceptably low. The wall of the jet-pipe was protected from the intense heat by a heat shield. This cylindrical lining has a pressure drop across it, and supplies cool air to the afterburner through a series of cooling rings. Experiments showed that an engine was less inclined to screech when additional holes were drilled in the heat shield. The mean pressure drop across the liner ensures that there is a mean bias flow through the screen. There is a need for a theory to determine the optimal combination of the perforation geometry and the bias flow.

Crude theoretical models have been developed in which the cylindrical lining was modelled as a plane collection of Helmholtz resonators without a bias flow; see, for example, Lewis & Garrison [2]. Each component 'cell' in such models is assumed to act independently, as if there were a honeycomb structure behind the perforated screen. In a real engine a honeycomb structure is impractical. Sound waves at oblique incidence must cause fluid to travel between the imaginary cells, and so these models are unrealistic.

Leppington & Levine [3] presented a detailed theory for the reflection of sound by a plane rigid screen perforated with a regular array of circular or elliptical apertures and backed by a plane rigid wall. However, no mechanism by which sound energy could be absorbed was included. Their analysis shows that, unlike the simple Helmholtz resonator theory, the resonance frequency depends upon the angle at which sound is incident. For sound at normal incidence the resonance frequency coincides with the Helmholtz resonance frequency of an individual cell.

Theoretical models of the mechanism by which sound energy is absorbed by perforates, such as those presented in references [2, 4-8] usually have no mean flow through the apertures. Without a mean flow, the absorption mechanism is complicated; above a certain sound level, around 100 dB, non-linear viscous effects predominate [9].

The effect of blowing cooling air through a screech liner has received little attention. Some experimental work on the effects of a mean flow through the apertures in a small plane-section of a typical liner was reported by Garrison et al [10]. Howe [11] showed that a bias flow greatly affects the acoustic properties of a perforated screen. The sound and the mean flow interact at the aperture rims to produce vorticity. This process provides a mechanism for the conversion of sound into vortical motions which radiate sound ineffectively; sound energy is then effectively absorbed. We show that the interaction between sound and a steady low Mach number, high Reynolds number bias flow, such as the flow of cooling air, is of prime importance in determining the acoustic properties of screech liners.

We examine the sound absorption properties of a backed perforated screen with circular apertures. When the backing plate is sufficiently far from the perforated screen for the local incompressible flow in the mouths of the apertures to be unaffected by its presence, then we can model the perforated screen as a homogeneous compliant plate [12,13]. The rigid backing plate merely passively reflects the sound which is transmitted through the perforated screen.

In §2 we develop a relatively general theory of the scattering of sound by a plane perforated screen in front of a backing plane. Some experimental results are presented in §3, and encouraging agreement between theory and experiment is noted.

2. The Scattering of Sound by a Perforated Screen with an Infinite Rigid Backing Plane

An infinite rigid wall occupies the plane $x_1 = -\ell$ of a Cartesian coordinate system (x_1,x_2,x_3). Parallel to this plane, at $x_1 = 0$, there is a thin rigid plate which contains a uniform, acoustically homogeneous, array of apertures. A steady bias flow, of high Reynolds number and low Mach number, is maintained, outwards, through the apertures. The mean value of the velocity in the mouths of the apertures is U. The geometrical arrangement is illustrated in figure 1.

We consider the reflection of plane sound waves by the backed screen. The sound field in far from the screen of the scale of the apertures may be written as

$$p = e^{i[k_2x_2 + k_3x_3 - \omega t]}\left[e^{-ik_1x_1} + R\, e^{ik_1x_1}\right], \tag{1}$$

where k_i is the component of the wavenumber $k_0 = \omega/c$ in the i-direction, and R is the reflection coefficient of the backed screen. Between the perforated screen and the hard wall, the sound field may be written as

$$p = e^{i[k_2x_2 + k_3x_3 - \omega t]}\left[A\, e^{-ik_1x_1} + B\, e^{ik_1x_1}\right], \tag{2}$$

where A and B are constants.

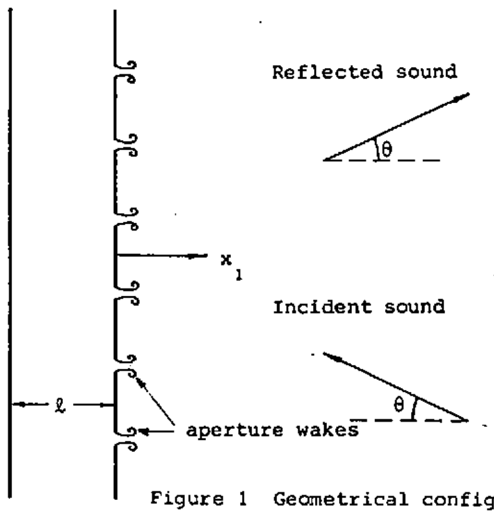

Figure 1 Geometrical configuration for the reflection of plane sound waves by a backed screen in the presence of a bias flow.

When the rigid plane is sufficiently far from the perforated screen for the local details of the flow to be unaffected by its presence, the acoustic properties of the screen can described by the boundary condition

$$\frac{\partial p}{\partial x_1} = \eta \left[p\right]_{x_1=-0}^{x_1=+0}, \text{ on } x_1 = 0, \tag{3}$$

where η is the effective compliance of the perforated screen. It can be shown tht the compliance is simply related to the Rayleigh conductivity K [14] of an aperture: $\eta = NK$, where N is the number of apertures per unit area. Essentially, this 'smoothed boundary condition' results from smearing the perturbation volume flux through each aperture over the surface; when the aperture spacing is acoustically compact the surface appears acoustically homogeneous.

Howe [11] determined the appropriate conductivity for circular apertures. For a square array of circles, spaced a distance d apart, the effective compliance is

$$\eta = 2a\chi/d^2, \tag{4}$$

where χ is a function of the Strouhal number $\kappa a = \omega a/U$;

$$\chi = \gamma - i\delta. \tag{5}$$

The form of the functions γ and δ is given in [11]; both are real, positive and less than unity.

The boundary conditions of vanishing normal velocity at $x_1 = -\ell$ and continuity of normal velocity at $x_1 = 0$, together with the linearised momentum equation, specify the sound field. We find that the reflection coefficient is given by

$$R = \frac{(ik_0 d^2 \cos\theta/2a\chi) + 1 - [i/\tan(k_0 \ell \cos\theta)]}{(ik_0 d^2 \cos\theta/2a\chi) - 1 - [i/\tan(k_0 \ell \cos\theta)]}, \tag{6}$$

where θ is the angle between the direction of propagation for the plane waves and the x_1 axis. When there is no bias flow, $\chi = 1$. With the additional constraint $k_0 \ell \cos\theta << 1$, this expression reduces to that determined by Leppington & Levine [3]; the compactness condition was used in Appendix A of their paper. Leppington and Levine noted that the magnitude of their reflection coefficient is always unity: no sound energy is absorbed. With a bias flow through the holes, χ is complex. Then $|R| \leq 1$ and acoustic energy is absorbed; the absorbed energy appears as virtually incompressible vortical motions produced by the interaction of the sound and the mean flow at the rims of the apertures.

In elementary examinations of screech liners, where the backed screen is modelled as a bank of discrete Helmholtz resonators, it is assumed that a lining absorbs sound well at the Helmholtz resonance frequency. We can now examine the absorption characteristics of a lining more clearly.

Leppington & Levine pointed out that their backed screen reflects sound like a perfectly soft screen, that is the reflection coefficient is minus one, when the wavenumber $k_0 \cos\theta$ satisfies the resonance condition

$$k_0 \cos\theta = \surd(2a/\ell d^2). \tag{7}$$

Rayleigh [14] showed that the resonance frequency of a Helmholtz resonator, which consists of an acoustically compact rigid container of volume V_0 in which there is an aperture with a Rayleigh conductivity K, is determined by the relationship

$$k_0 = \surd(K/V_0). \tag{8}$$

The Rayleigh conductivity of a circular aperture of radius a is 2a when there is no mean flow through the neck, and other dissipative mechanisms are ignored. The volume of each, imaginary, cell which makes up the backed screen is ℓd^2, and, for normally incident sound, $\cos\theta = 1$. Therefore the resonance condition for the backed screen is identical to the Helmholtz resonance condition as expected. Equation (7) shows that the dependence of the resonance frequency on the angle of incidence of the sound takes a rather simple form. However, for large angles of incidence there is a significant difference between the resonance frequency predicted by assuming a cellular structure for the screen and that predicted by Leppington & Levine's theory.

In order to discuss our expression for the reflection coefficient, equation (6), which includes the effects of a mean flow through the apertures, it is

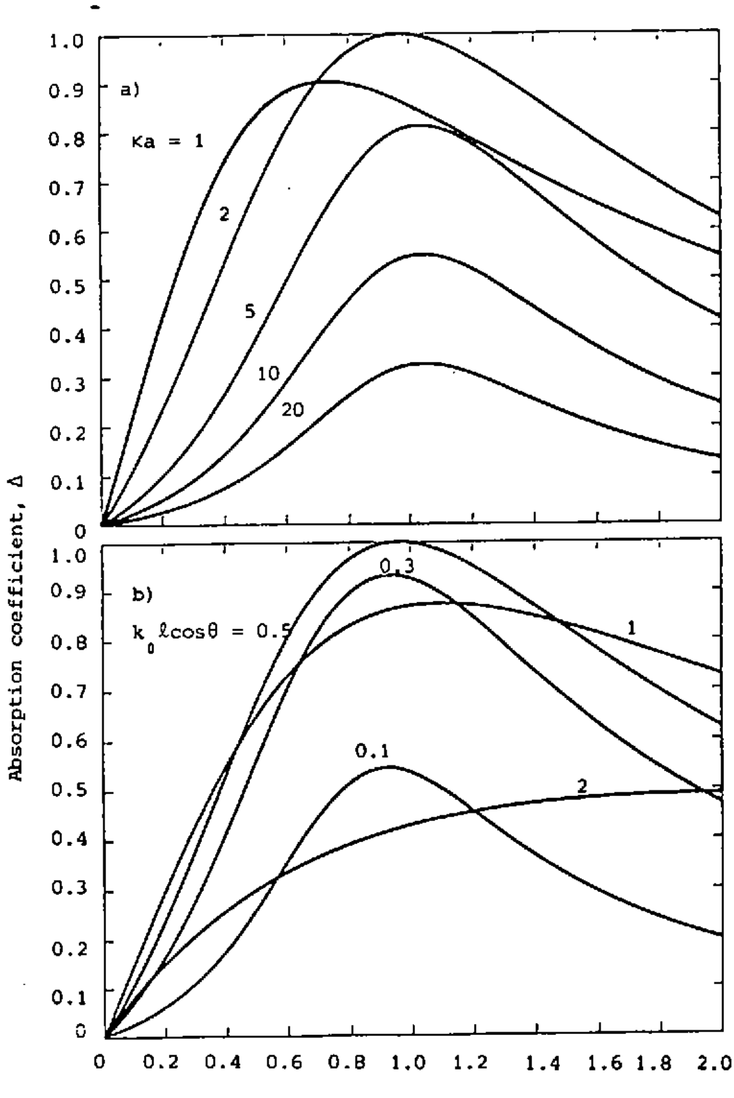

Figure 2 The absorption coefficient for a plane liner

a) $k_0 \ell\cos\theta = 0.5$

b) $\kappa a = 2$

useful to introduce a 'resonance parameter'

$$Q = (k_0 d \cos\theta)^2 \ell/2a, \tag{9}$$

which is proportional to the square of the frequency. Q is a convenient single-valued function of frequency which is unity at the modified Helmholtz resonance frequency given in equation (7). We can then rewrite equation (6) as

$$R = \frac{(Q/\chi) - ik_0\ell\cos\theta - [k_0\ell\cos\theta/\tan(k_0\ell\cos\theta)]}{(Q/\chi) + ik_0\ell\cos\theta - [k_0\ell\cos\theta/\tan(k_0\ell\cos\theta)]} \tag{10}$$

This result shows that the reflection coefficient, and consequently the amount of sound energy absorbed by the screen, is a function of three non-dimensional variables : the resonance parameter Q, the Helmholtz number $k_0\ell\cos\theta$, and the Strouhal number κa. One may expect, from elementary Helmholtz resonator theory, that the maximum sound absorption will occur near the modified Helmholtz resonance frequency, $Q = 1$. Any departure of the position of the maximum from $Q = 1$ arises as a consequence of the bias flow and cavity non-compactness.

The absorption coefficient $\Delta = 1 - |R|^2$ is plotted as a function of Q in figure 2(b), with the Helmholtz number $k_0\ell\cos\theta = 0.5$, and in figure 2(a), with the Strouhal number $\kappa a = 2$. Perhaps the most striking feature of these plots is the efficiency of the backed screen as a sound absorber. For certain combinations of the dependent variables all the incident sound can be absorbed. However, the plots also show that designing a liner by merely choosing the resonance frequency, without consideration of the bias flow through the holes, does not guarantee a large absorption coefficient.

Figure 2 reveals that, for small values of the Helmholtz number $k_0\ell\cos\theta$, the peak absorption, at a particular Strouhal number, occurs close to $Q = 1$. As the Helmholtz number is increased, so the resonance parameter becomes less useful as an indicator of the position at which the peak absorption will occur; we noted that the Helmholtz resonance theory relies on the cavity volume being acoustically compact. The relevance of the resonance parameter also decreases as the Strouhal number tends to zero, since the bias flow is then large, χ is significantly different from unity, and the resonance frequency shifts away from where $Q = 1$. If a sound absorbent lining may be treated as planar, then the theory which we have presented in this chapter may be used to design one which is highly absorptive.

3. Experimental investigation of the acoustical properties of a plane backed perforated screen

In theory, a backed perforated screen can absorb all of the sound which is incident upon it at a particular frequency when there is a mean bias flow through the screen. In this section we present some typical results of the experimental investigation which we have carried out to test the theory. Garrison et al [10] did some experiments on the effect of a mean flow of air through a small plane section of a backed screen, However, that examination was limited to studying the way a mean flow could degrade the non-linear absorptive properties of a liner. The absorptive properties screen in the presence of a bias flow of the type considered in this chapter has not previously been fully investigated experimentally.

We used the version of the popular 'impedance-tube' technique developed by Seybert & Ross [15]. The experimental apparatus is illustrated schematically in figure 3.

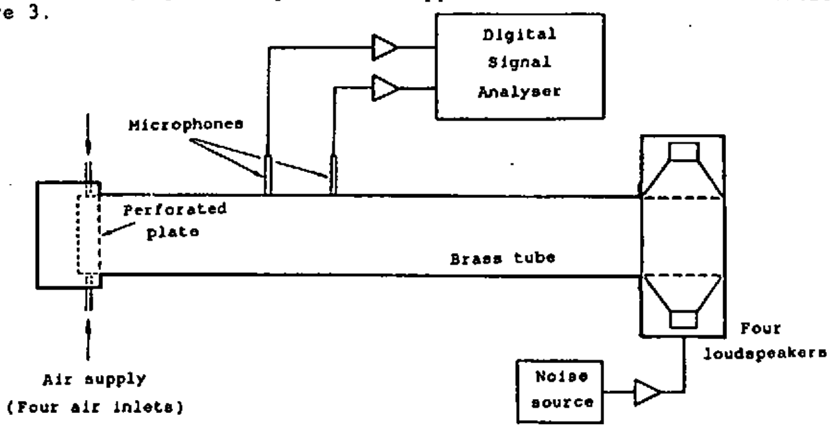

Figure 3 Schematic of the apparatus used for the experimental examination of perforated liners with a bias flow.

Seybert & Ross showed that by measuring the auto- and cross-spectral densities at the two microphone positions when a sample is irradiated with white noise, in the frequency range appropriate for plane wave propagation, the acoustic properties of a simple impedance surface can be calculated. However, when Howe determined the Rayleigh conductivity of a circular aperture with a bias flow, the linear contributions to the fluctuating flow in the apertures from the turbulence in the wake of the screen were neglected because they are of a different frequency to the incident harmonic sound. With white noise excitation, this assumption is invalid. We therefore evaluated the response of the backed screen using discrete frequency excitation. From the measurements, we calculated the absorption coefficient Δ.

We first checked that the measured quantities were independent of the sound pressure level over the range 85 dB to 140 dB. The sound pressure level was not kept constant for the measurements and typically varied over this range. It is convenient to present the experimental results using the following four non-dimensional parameters: the resonance parameter Q is equal to $k_0^2 \ell d^2/2a$ for normally incident sound, the Mach number M, based on the mean flow velocity in the mouths of the apertures, and two parameters, $a\ell/d^2$ and a/ℓ, which depend upon the geometry alone; only Q is frequency dependent. The mean velocity in the mouths of the apertures is approximately equal to the vorticity convection velocity [11] because the vorticity convects at about half the uniform fluid velocity at the vena contracta whose cross-sectional area is approximately half that of an aperture. The Mach number M is therefore simply related to the Strouhal number κa which is based on the vorticity convection velocity, given this approximation.

A representative set of experimental results is presented in figure 4. The theoretical properties are plotted for comparison. The agreement between the theoretical and experimental acoustical properties is very encouraging. We see from this figure that the theoretical prediction of total sound absorption, at a particular frequency, can be attained in practice. We also see that when the parameters are altered to produce a less efficient sound absorber, the correlation between the theory and experiment is also rather good.

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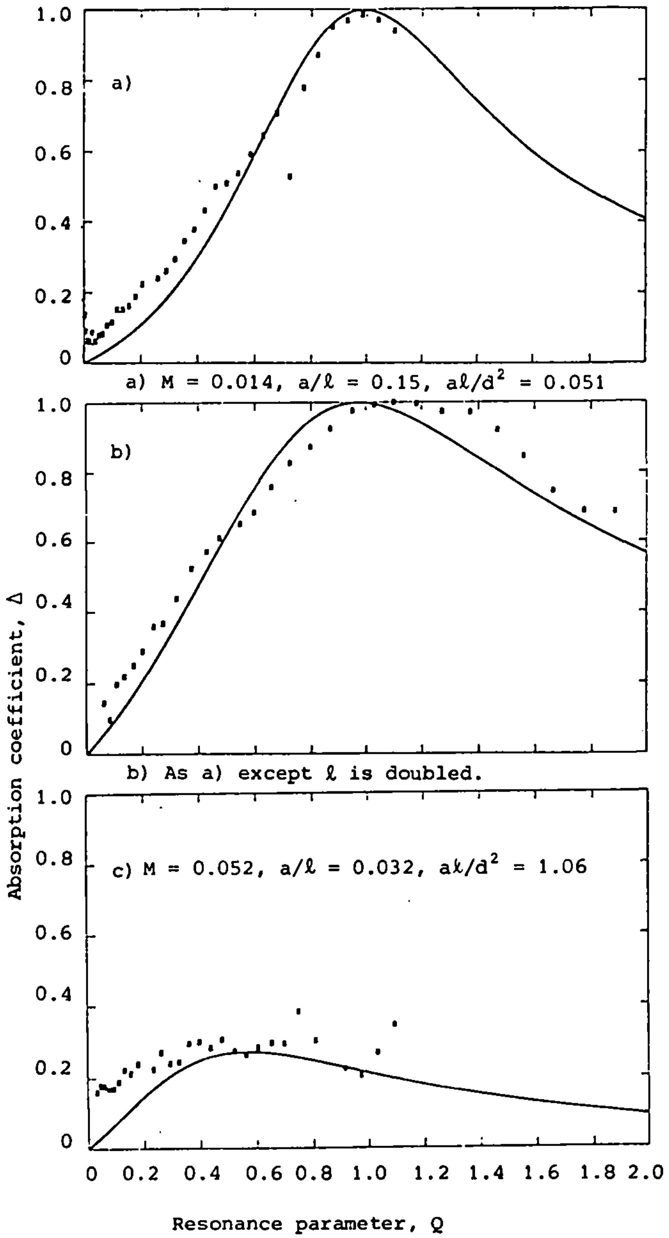

Figure 4 The experimentally determined absorption coefficient. Theory: solid line. Experiment: filled squares.