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COHERENT SCATTERING MEASUREMENT IN PENETRABLE INHOMOGENEOUS MEDIA

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INTRODUCTION

A plane wave propagating in an inhomogeneous medium will, in general, as a result of the scattering inhomogeneities present, suffer fluctuations in amplitude and phase across the propagating wavefronts. The component that does travel as a plane wave (represented by the mean amplitude and phase in a plane parallel to the initial wavefronts) includes the effect of absorption in the matrix and the coherent part of the (forward) scattering. The remainder, i.e. the fluctuations about the mean value represent the incoherent part of the scattering [1].

The use of finite piezo-electric transducers has the potential disadvantage of causing systematic (but unquantifiable) errors due to phase cancellation artefacts[2] when the measurement of velocity or attenuation is attempted using conventional methods on inhomogeneous materials. The tendency is to over-estimate the attenuation and to produce small random errors in the velocity. The use of phase insensitive detectors[3] has the disadvantage of not providing velocity measurements. In addition, it has recently been shown[4] that phase sensitive receivers in general measure a fundamentally different attenuation from phase insensitive receivers - a difference which can be interpreted in terms of the coherent and incoherent scattering contributions. However the limitations introduced by the finite size of the receivers in discriminating between the coherent and incoherent parts have yet to be defined in rigorous quantitative terms.

An alternative approach to the measurement distinction of the coherent and incoherent contributions involves the use of a scanned point receiver (hydrophone). Provided the diameter of the probe is small compared to the wavelength and the measurement sampling is adequate, the phase and amplitude distributions of the waves emergent from a parallel-sided specimen, in a plane parallel to that of the incident wavefronts can be recorded. Considerable care needs to be taken in the phase measurements with regard to thermal and mechanical stability[5]. Nevertheless from such a set of measurements the coherent and incoherent components of the scattering may in principle be determined. The practical limitations of the finite receiver and finite sampling aperture also still require careful evaluation.

If the matrix in which the scattering inhomogeneities are embedded is itself highly absorbing, then this may have a further adverse influence in significantly limiting the signal to noise ratio available. Analytical formulations of this problem are still in the process of development[6,7].

For materials into which a hydrophone may be inserted without structural damage it appears that there is an alternative approach to measurement of the coherent component of the scattering and it is this procedure which is described in the present paper. The method is based upon the approximate analytical formulation of the axial field from a focussed transducer. It is most easily interpreted for homogeneous fluids whereby the absorption coefficient of an unknown fluid

can be determined relative to that of a second fluid. Thus if the second fluid has known absorption, that of the first fluid can be determined. At megahertz frequencies, the obvious choice of the second fluid is water with effectively negligible absorption.

Inclusion into the first liquid of a significant population of scatterers then permits the method to determine the difference between the attenuation of the new mixture and the absorption of the original fluid. The difference may, in the first instance be seen to be the coherent part of the scattering produced by the inhomogeneities. It is not improbable that the method may be extended to give the incoherent part of the scattering also, but this has not been achieved so far by the present authors.

ANALYTICAL BASIS

The underlying principle of the method is based on the field distribution from a finite (weakly) focussed pistonlike bowl transducer analysed by O'Neill in 1949[8]. The expression for the axial acoustic pressure radiated into a fluid is given by

$$p = \rho c u_o P \, e^{i(\omega t - km)} \tag{1}$$

where c, ρ are the velocity and density of the fluid, u_o is the surface normal velocity of the pistonlike source, $k = \frac{2\pi}{\lambda}$, λ is the wavelength and P and m are defined by:

$$P = \frac{2}{1-\frac{z}{A}} \sin k\delta \tag{2}$$

where $2\delta = [(z-h)^2+a^2]^{1/2} - z$ (3)

and $2m = [(z-h)^2+a^2]^{1/2} + z$ (4)

Here z is the axial distance, A the radius of curvature of the bowl, h its depth, and a its circumferential radius.

Loss in the fluid can be incorporated by writing the wave number k in complex form:

$$k = k - i\alpha \tag{5}$$

Equation (1) then becomes:

$$p = \frac{2\rho c u_o}{1-\frac{z}{A}} i \sin(k-i\alpha)\delta \; e^{i(\omega t - km + i\alpha m)} \tag{6}$$

Putting the sine function into exponential form and rearranging we have

$$p = \frac{\rho c u_o}{1-\frac{z}{A}} \left[e^{ik\delta} e^{-\alpha(m-\delta)} - e^{-ik\delta} e^{-\alpha(m+\delta)}\right] e^{i(\omega t - km)} \tag{7}$$

$$= \frac{\rho c u_o}{1-\frac{z}{A}} \left[e^{ik\delta} e^{-\alpha z} - e^{-ik\delta} e^{-\alpha(2\delta+z)}\right] e^{i(\omega t - km)} \tag{8}$$

since $m - \delta = z$ and $m + \delta = 2\delta + z$ (9)

For $\delta << z$ we have

$$p = \left(\frac{\rho c u_o}{1-\frac{z}{A}}\right) e^{-\alpha z} \left[e^{ik\delta} - e^{-ik\delta}\right] e^{i(\omega t - km)} \tag{10}$$

$$= \left(\frac{\rho c u_o}{1-\frac{z}{A}}\right) e^{-\alpha z} 2i \sin k\delta \, e^{i(\omega t - km)} \tag{11}$$

Thus the amplitude of the axial pressure is given approximately by

$$|p| = \left(\frac{2\rho c u_o}{1-\frac{z}{A}}\right) e^{-\alpha z} \sin k\delta \tag{12}$$

If we write the pressure amplitude at axial position z in medium 1 as $p_1(z)$ and in another medium, 2, as $p_2(z)$, where the media have, respectively, densities of ρ_1, ρ_2, speeds of sound of c_1, c_2 and attenuation coefficients of α_1 and α_2, then from equation (12) we have:

$$p_1(z) = 2u_o \left(\frac{\sin k_1\delta}{1-\frac{z}{A}}\right) \rho_1 c_1 e^{-\alpha_1 z} \tag{13}$$

and $$p_2(z) = 2u_o \left(\frac{\sin k_2\delta}{1-\frac{z}{A}}\right) \rho_2 c_2 e^{-\alpha_2 z} \tag{14}$$

Thus $$\frac{p_1(z)}{p_2(z)} = \frac{\sin k_1\delta}{\sin k_2\delta} \cdot \frac{\rho_1 c_1}{\rho_2 c_2} \cdot e^{-(\alpha_2 - \alpha_1)z} \tag{15}$$

Whence $$\ln \frac{p_1}{p_2} = \ln \left(\frac{\sin k_1\delta}{\sin k_2\delta} \cdot \frac{\rho_1 c_1}{\rho_2 c_2}\right) - (\alpha_2 - \alpha_1)z \tag{16}$$

Thus subject to the constraint $2\delta << z$, it can be seen that the natural logarithm of the ratio p_1 to p_2 as a function of z is a straight line of gradient $(\alpha_1 - \alpha_2)$ and intercept $\ln\left(\frac{\sin k_1\delta}{\sin k_2\delta} \frac{\rho_1 c_1}{\rho_2 c_2}\right)$, provided this last term is sensibly independent of z over the range which measurements are made.

The condition $2\delta << z$ can be interpreted in the following way. From equation (3), 2δ can be seen to be equal to the difference in distance from an axial point to a point on the periphery of the bowl and to the centre of the bowl, O. The range of validity for this condition will need to be checked in a given experimental situation. However some general guidelines can be given for medium and weakly focussed bowls. The occurrence of axial minima in the field implies that in the regions close to these extrema, the ratio $\frac{p_1}{p_2}$ would become indeterminate in value, and that such regions should be avoided in the ranges of z over which measurements are taken. Provided $h < \lambda$ there are no extrema at values of z greater than A, the radius of curvature of the bowl[8]. Thus, the criterion of having h slightly less than λ appears to present a good

compromise in having the greatest degree of focussing, while still leaving the region beyond the focal zone free for measurement.

The benefit of having a reasonable range of z over which to take measurements suggests that the focal length be reasonably long to maintain signal levels. Typically the focal length should be comparable with the distance over which it is hoped to take measurements. This permits the introduction of the approximation in equation (3).

$$\frac{2\delta}{z} \simeq -\frac{h}{z} + \frac{h^2}{2z^2} + \frac{a^2}{2z^2} + \ldots.$$

Since, by the intersecting chord theorem, $h \simeq \frac{a^2}{2z^2}$ (for $a^2 << z^2$) and $h << z$,

$$\frac{2\delta}{z} \simeq \frac{1}{2}\ \frac{a^2}{z^2} \quad \ldots. \qquad (17)$$

Thus the condition for 2δ to be small compared to z is primarily related to the relationship between the aperture a and the axial distance z. If this is small, the analysis is valid to a good approximation.

The major constraint in practice is the condition mentioned after equation (16) above that $\ell n \left(\frac{\sin k_1\delta}{\sin k_2\delta} \frac{\rho_1 c_1}{\rho_2 c_2} \right)$ should be sensibly independent of z. Since the ratio of the characteristic acoustic impedances may vary without any practical constraint, it can be seen that since δ is in general a function of z (equation (3)), the first term introduces limitations on the speeds in the two media in relation to the positions at which measurements are made, through the relationship:

$$k_1\delta = n\pi + (-1)^n k_2\delta \qquad (18)$$

Solutions for $n \neq 0$ of this have limited practical value. Thus we find that we need to have $k_1 = k_2$, i.e. the speed of sound must be (almost) the same in the two fluids. If the technique is to be used to measure the absorption of one homogeneous fluid by comparing it with that of another whose absorption coefficient is known, the latter needs to be selected with care in view of this constraint. If the technique is to be used to measure the coherent scattering contribution to attenuation in a fluid of known absorption coefficient, it will be necessary to estimate the velocity changes that the scatterers may produce using either the method of mixtures (e.g. [9]) or Foldy's analysis[1], in order to ascertain that equation (18) is still valid.

Given that $\frac{\sin k_1\delta}{\sin k_2\delta}$ can be made very close to unity, by virtue of the fact that c_1 can often be found such that $c_1 = c_2$, it can be seen that the intercept of $\frac{\rho_1}{\rho_2}$ against z give an internal check on the consistency of the method by giving the ratio of the densities of the two fluids. This may either be known a priori or may, for suspensions, be calculated with little error using the mixture theory[9].

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EXPERIMENTAL METHOD

The determination of the attenuation or absorption coefficients required can be approached in two main ways. The first is to fix the frequency and to make a large number of measurements at different axial distances. Whereas this gives improved precision in the fitting of a straight line to the data (and thus in the determination of the coefficients) it clearly requires repetition at a number of frequencies. The alternative is to make measurements at a few values of z (in the limit only two are needed) and to sweep the frequency at each position.

The former is clearly the better method for the initial validation of the analysis, although the latter turns out to be far more valuable in interpretation of the identity of the scattering contribution to the attenuation.

The experimental tanks have been described elsewhere[10]. An outer tank bears a three dimensional rectilinear scanning mechanism that is microprocessor controlled. It has a resolution of 5μm along each axis. The outer tank can serve as a thermal reservoir for the inner tank in which the transmitting transducer and liquids or suspensions under investigation were placed. The main virtue of the dual tank system is the ease of removing potentially obnoxious materials from the scanning mechanism.

The transmitting transducers used had piezoelectric ceramic focussed elements of aperture radius 1.5cm (nominally), a focal length of 10cm and a nominal resonant frequency of 2MHz. The beam pattern of the transducer had been extensively investigated before its use in the present investigations. The hydrophone receiver was of the needle type with a circular ceramic element (PZT) of diameter 1.0mm. The transmitter was aligned (acoustic axis) with the scanning mechanism on the outer tank and the hydrophone was adjusted until it travelled along the axis.

The electronic system consisted of a transmitter generator which excited the source with approximately 20μs long tonebursts. A boxcar system sampled the pulse received by the hydrophone in the middle of the pulse, where the amplitude was essentially constant. The peak amplitude was stored by the computer (a Motorola 6809) which then moved both the scan delay generator gate of the boxcar and the hydrophone along the axis for the next measurement. Measurements were made in 250μm intervals between 1cm and 13cm from the transmitter surface.

RESULTS

As mentioned above, water is an obvious reference medium because of its very low ultrasonic attenuation in the low megahertz frequency region. The choice of absorbing liquids was limited by the need to match the sound speeds, i.e. to make equation (18) a good approximation. In addition it was necessary to select a liquid in which it would be possible to suspend the scatterers. The availability of scatterers of appropriate size and density was a severe constraint on the materials that could be used. The compromise adopted was to use castor oil as the liquid and 500μm diameter polystyrene particles in suspension. This combination satisfies most of the requirements[11] although the temperature sensitivity of the density of castor oil implies the need to incorporate a stirring mechanism. The suspension was stirred for at least an hour before the measurements were taken.

Axial distributions for caster oil showed the true focus to be inside that with water as may be expected from the absorption in the oil. Confirmation that the velocity was close to that of water at the same temperature, i.e. that the condition of equation (18) is satisfied, was achieved by reference to data in the literature, although it proved relatively easy to check it directly. The ratio of the water and oil curves plotted on a logarithmic scale (see equation (16)) provided excellent linearity over a central region from 4 to 9cm. It was excellent confirmation of the theory presented in section 2 and the validity, for these two media under these conditions, of the assumptions involved - particularly the critical one of closely similar sound speeds. The limits to the linear fit are; at the lower axial distances, the sensitivity of the ratio to the exact shape of the peak at the true focus, and at the longer axial distances, the absorption reducing the signal in castor oil almost to the noise level. A least squares fit to the data in the linear region was used to obtain the slope of the data. The value of the absorption coefficient obtained from the slope, namely 23.9 nepers m^{-1} compared well with that from the literature (e.g. 25.3 nepers m^{-1}[12]). Similar results were found at five discrete frequencies in the range 1.8 to 2.0 MHz. There is some variation in the data found in the literature, and castor oil appears to exhibit ageing effects (Lele, private communication) in its ultrasonic properties.

For the measurements on the suspensions a preliminary set of data was obtained in castor oil at frequencies between 1.0 and 2.5 MHz in steps of 0.25 MHz. Insertion of a 1% volume concentration of 300 - 500 μm polystyrene spheres was, after thorough stirring, followed by repetition of the scans at the different frequencies. It was found that, while there was considerable fluctuation in the individual measurements, there was, again, a central section which was approximately linear. A least squares analysis of the region between 4 and 9cm gave the slope, as before, and this represented the difference between the attenuation in the suspension and the absorption in the castor oil. It was believed that this difference was the coherent part of the scattering.

In order to try to confirm that this difference was, in fact, the coherent part of the scattering, a further set of measurements was performed. Assuming that the velocity of the suspension is very similar to that of the castor oil alone and the densities of the particles are very similar to that of the castor oil, equation (16) becomes

$$\alpha_2 - \alpha_1 = -\frac{1}{2} \ln \frac{P_2}{P_1} \qquad (19)$$

Thus by making measurements at only one distance $\alpha_2 - \alpha_1$ can be measured. In fact, measurements were made at two distances, as a further check (5.0 cm and 6.0 cm), at 0.05 MHz intervals between 1.0 and 2.5 MHz. The results between the two distances were consistent and were compared with the theoretical predictions of Waterman and Truell [13]. Although greater variation was observed at the highest frequencies, the agreement was good, the coherent scattering following closely the resonance peak expected for these particles in this matrix over this frequency range. Increasing the concentration of particles to 2% in a further range of experiments produced similar agreement.

CONCLUSION

The technique described has a major limitation in the need to have the velocities of the liquids very similar if absorption is to be measured. In addition it is necessary to have sufficiently low concentrations of scatterers to affect the velocity in the suspension relatively little, and to assume that the hydrophone is effectively a point receiver. Nevertheless the technique has considerable potential for the investigation of coherent scattering in suspensions. A quantitative theoretical comparison is made more difficult at the present time by the fact that the size distribution of the particles used is significant, and by the more general difficulty of defining shear parameter data for the scatterers. The promising results of the method suggest that it may be used in the frequency sweep mode described at the end of the last section, provided some time averaging of the results is performed at each point.

The areas of potential application of the technique are many, e.g. biomedical ultrasonics, sedimentology, biotechnology. In particular it may be used potentially to identify the coherent scattering contribution to the ultrasonic attenuation in human tissues - a factor that has hitherto been explicitly or implicitly neglected.

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