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A STUDY OF THE COMPUTATIONAL PRINCIPLES OF THE ACOUSTIC TELESCOPE AND POLAR CORRELATION

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Introduction

The two source location techniques of the Acoustic Telescope and Polar Correlation have for some time been regarded as competing rivals. Much has been made of the differences between these two methods; this paper has the aim of bringing out their similarities, and of finding common ground for improving both techniques.

The analysis starts from the point of view of the Acoustic Telescope, and continues to show that the computations implied by the two algorithms have very much in common.

General

In each method, data is taken from an array of microphones in the form of digital samples either retained in computer memory or recorded onto backing store.

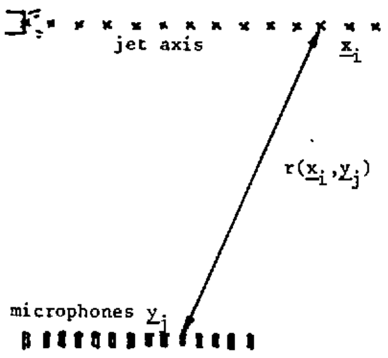

Fig. 1(a) Telescope

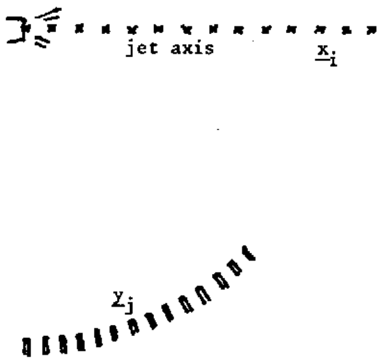

Fig. 1(b) Polar Correlation

These samples may be taken 'live', or from tapes recorded with analog signals. The signal from the j^{th} microphone at time t will be denoted by

$$s_j(t)$$

lower case letters being used for a function of time and upper case for the corresponding function of frequency.

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The Telescope Algorithm

Microphones are arranged in a linear array as shown in Figure 1(a). The signal from source position $\underline{x}_i$ will reach the j^{th} microphone after a delay $t_{ij} = \frac{1}{c} \times r(\underline{x}_i, \underline{y}_j)$, where $r(\underline{x}_i, \underline{y}_j)$ is the distance from the i^{th} source focus to the j^{th} microphone position. Now if sources and microphones are respectively equally spaced we may write

$$\underline{x}_i = \underline{x}_o + i\delta\underline{x} \quad (1)$$

and

$$\underline{y}_j = \underline{y}_o + j\delta\underline{y}, \quad (2)$$

and so $r(\underline{x}_i, \underline{y}_j) = |\underline{x}_i - \underline{y}_j|$ is approximated by

$$r(\underline{x}_o, \underline{y}_o) + i\delta\underline{x}.\ \nabla_x\ (j\delta\underline{y}\nabla_y r)$$

$$= r(\underline{x}_o, \underline{y}_o) + (\underline{y}_o - \underline{x}_o).\ \delta\underline{y}_i \nabla_x\ (\frac{1}{r}).\delta\underline{x} - ij\ \frac{\delta\underline{x}.\ \delta\underline{y}}{r}$$

In this expression, the first term is constant. The second term is zero if $\delta\underline{y}$ is orthogonal to $(\underline{y}_o - \underline{x}_o)$, and so it is the third term which gives the variable part of the delay (as far as this approximation is concerned).

The telescope algorithm employs a table of delays, in practice computed from the actual distances. For the purpose of this comparison we approximate the delay-table entries by

$$t_{ij} = r(\underline{x}_o, \underline{y}_o)/c - ij\ \frac{\delta\underline{x}.\ \delta\underline{y}}{rc} \quad (3)$$

The (simplified) algorithm is as follows, assuming an array of 14 microphones. For focus point $\underline{x}_i$

1. Construct a time series $\sum_{j=o}^{13} s_j(t+t_{ij}) = s(t,i)$ say.
2. Take the FFT of this time series, giving $S(\omega,i)$
3. Multiply S by its conjugate to give the power $S(\omega,i)\ S^*(\omega,i)$.

Now note that this expression is the transform of the autocorrelation function of $s(t,i)$; this can be represented by

$$\phi\ [s,s](\tau) = \overline{s(t,i)\ s(t+\tau,i)}$$

$$= \overline{\sum_{j=o}^{13} s_j(t+t_{ij})\ \sum_{k=o}^{13} s_k(t+t_{ik} + \tau)}$$

$$= \sum_{j=o}^{13} \sum_{k=o}^{13} \overline{s_j(t) s_k(t+\{t_{ik} - t_{ij}\} + \tau)}$$

for stationary sources

$$= \sum_{j=o}^{13} \sum_{k=o}^{13} \phi[s_j, s_k](\tau + \{t_{ik} - t_{ij}\}) \quad (4)$$

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If we substitute the apprixomate values of t_{ik}, t_{ij} from (3):

$$\phi[s,s](\tau) = \sum_{j=o}^{13} \sum_{k=o}^{13} \phi[s_j, s_k](\tau - i(k-j)\, \frac{\delta \underline{x}.\ \delta \underline{y}}{rc}) \qquad (5)$$

This can be grouped into terms, writing τ_o for $\frac{\delta \underline{x}\ \delta \underline{y}}{rc}$

$\sum_{o}^{13} \phi[s_j, s_j](\tau)$ — 14 terms of auto-correlation.

$+ \sum_{o}^{12} \phi[s_j, s_{j+1}](\tau - i\ \tau_o)$
$+ \sum_{o}^{12} \phi[s_{j+1}, s_j](\tau + i\ \tau_o)$ — 13 + 13 terms of cross corr. for microphone separation of one unit

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$+ \phi[s_o, s_{13}](\tau - 13\ i\ \tau_o)$
$+ \phi[s_{13}, s_o](\tau + 13\ i\ \tau_o)$ — 1 + 1 term separation 13 units (6)

The Polar Correlation Algorithm

The microphones are now arranged in a circular arc as shown in figure 1(b). Their separation is such that the component resolved along the jet axis is constant, $\delta \underline{y}$.

The algorithm is now as follows:

1. Take the FFT of each microphone time series to give $S_j(\omega)$
2. Take the product of the conjugate of each transform with the transform for microphone O, $S_o(\omega)\ S_j^*(\omega)$ $j = 1, 13$
3. Normalise by dividing by the square root of the product of the power spectra, store away as 13 arrays (plus the power spectrum for microphone O to relate levels between frequencies).

Note that if we neglect the normalisation, these arrays will be the transforms of the cross correlation functions $s_o(t)s_j(t + \tau)$.

For each selected frequency:

4. Fish the entry for that frequency from each array, f_j and enter it (possibly weighted) into the (j+1)th position of a new array of length N.
5. Fill the rest of the new array with zeroes and take an inverse FFT.

The result can be related as source intensity (at that frequency) versus position. The sum of the (i+1)th entry in the array with its conjugate gives the source strength at position $\underline{x}_o + \delta \underline{x}$, where $\delta \underline{x} = \frac{2\pi}{N\omega_o} \frac{rc}{\delta y}$.

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Now let us inspect the (i+1)th term of the inverse FFT. It is derived from the file entries f_j by the sum

$$\sum_{j=o}^{N-1} w_j f_j e^{-ij\sqrt{-1}\,\frac{2\pi}{N}},\quad w_j \text{ being the weighting factor}$$

$$= \sum_{1}^{13} w_j f_j e^{-ij\sqrt{-1}\,w_o\,\frac{\delta x\,\delta y}{rc}}$$ (since the entries for j=o and from 14 to N-1 are filled with zeroes)

Now to within a constant, f_j is the transform of $s_o(t)s_j(t + \tau)$, so we have a result which is the transform of

$$\sum_{j=1}^{13} w_j s_o(t) s_j(t + \tau - ij\,\frac{\delta x\,\delta y}{rc})$$

i.e. $$\sum_{j=1}^{13} w_j \phi[s_o, s_j](\tau - ij\,\frac{\delta x\,\delta y}{rc})$$

For Bartlett weighting w_j has the value $14 - j$ $(0 \leq j \leq 13)$.

We may now write τ_o for $\frac{\delta x\,\delta y}{rc}$, substitute for w_j and expand the sum of the (i+1)th term and its conjugate to give a result which is the transform of

$$13\,\phi[s_o,s_1](\tau - i\tau_o) + 13\,\phi[s_1,s_o](\tau + i\tau_o)$$
$$+ 12\,\phi[s_o,s_2](\tau - 2i\tau_o) + 12\,\phi[s_2,s_o](\tau + 2i\tau_o)$$
$$\ldots\ldots$$
$$+ \phi[s_o,s_{13}](\tau - 13i\tau_o) + \phi[s_{13},s_o](\tau + 13i\tau_o)$$

When we compare this with expression (6) we find a remarkable resemblance. The autocorrelation term is missing, and the sum of assorted terms for each interval is replaced by a multiple of a single term. Apart from that the expressions for source strength are almost identical.

Conclusions

There are of course differences between the two techniques compared here, and both will have their respective operational merits. In computational terms, however, their effects are almost identical over the range for which the approximations hold true. They are subject to 'aliasing' in identical ways, and techniques for countering this phenomenon must be common to both.

The limited time for presentation cut short an analysis of aliasing. Hopefully this, with suggested strategies for alias avoidance, will be published in the near future.