PULSE COMPRESSION OF STEP FM PULSES

J. Cibis, S. Ries and M.-Th. Roeckerath-Ries

Krupp Atlas Elektronik GmbH, Bremen

1. INTRODUCTION

Pulse compression in radar or sonar is accomplished by employing frequency or phase modulation to widen the signal bandwith. Frequently, a linear frequency modulated (LFM) pulse is used. The instantaneous frequency $f_i(t)$ increases linearly over the duration of the pulse. In practice, the LFM pulse is often approximated by a step-like frequency modulated (SFM) pulse with step-like increasing instantaneous frequency. In order to get good pulse compression results, phase correction of the SFM pulse at the discontinuities of $f_i(t)$ is most important. Phase discontinuities produce high frequencies and lead to fading in the magnitude spectrum. In the time domain, this influences the autocorrelation function and results in high side-lobe levels. For sonar practice, this yields a deterioration in range resolution and signal-to-noise ratio. The theoretical results this problem are confirmed by computer simulations.

2. PULSE COMPRESSION OF FREQUENCY MODULATED PULSES

2.1 Pulse compression of an LFM pulse

An LFM pulse of duration T is given by

$$s(t) = \cos(2\pi(f_u+\alpha t)\, t)\ \mathrm{rect}((t-T/2)/T)\,. \tag{1}$$

The instantaneous frequency $f_i(t) = f_u+2\alpha t$ is defined as the derivative of the phase; see [1] and [2]. For $\alpha = w/(2T)$ and a sufficiently large time-bandwith product wT, the magnitude spectrum of (1) is given by

$$|S(f)| \approx \mathrm{rect}((f-f_m)/w), \tag{2}$$

where $f_m = f_u + w/2$ is the centre frequency of the pulse. Processing s(t) in a matched filter, the output is the autocorrelation function $\varphi_s(t)$ of the modulated pulse,

$$\varphi_s(t) \approx (\sin(\pi wt)/(\pi wt))\, \cos(2\pi f_m t). \tag{3}$$

The long pulse of duration T is compressed to a duration 1/w and the peak power of the pulse is increased by the signal duration T; see [3] and[4]. The pulse compression ratio is a measure of the degree to which the pulse is compressed and might be as small as 10 and as large as 10^5. In Figs 1 and 2 the power spectrum (PS) and the autocorrelation function (ACF) of an LFM pulse can be seen. The PS and the ACF are given as $20\log|S(f)|$ and $20\log|\varphi_s(t)|$, respectively. The ACF is normalized to 0 dB.

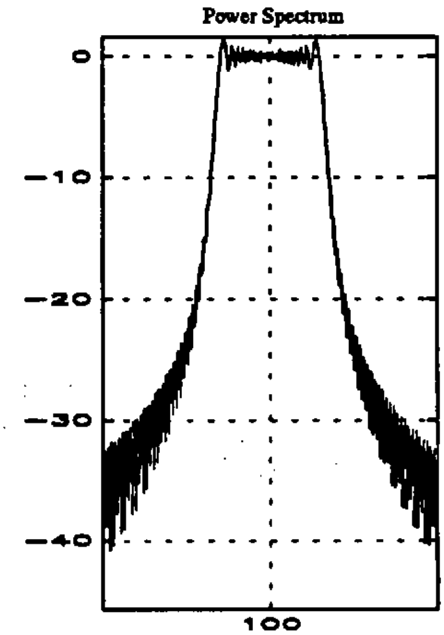

Fig. 1 T = 10ms; w = 10kHz (kHz)

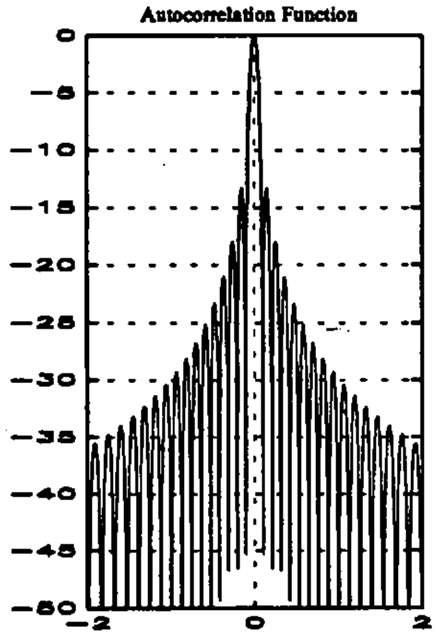

Fig. 2 T = 10ms; w = 10kHz (ms)

2.2 Pulse compression of an SFM pulse

In practice, instead of the LFM pulse of duration T, an approximating discrete frequency- shift waveform is frequently implemented. This waveform is generated by a sequence of n contiguous subpulses of duration $T_n = T/n$ with shifted carrier frequencies f_k, $k=1,\dots,n$, from one subpulse to the next. The theory of discrete frequency modulation has been dealt with in detail by Rihaczek [3]. Denoting an additional phase-term of the k.th subpulse as ϕ_k, the resulting waveform is given by

$$s(t) = \sum_{k=1}^{n} \sin(\,2\pi f_k(t-(t_k-T_n/2))+\phi_k)\ \ \mathrm{rect}(\,(t-t_k)/T_n\,)\,. \qquad (4)$$

The instantaneous frequency

$$f_i(t) = \sum_{k=1}^{n} f_k\, \mathrm{rect}(\,(t-t_k)/T_n\,) \qquad (5)$$

of (4) is a step-like function with discontinuities at the frequency shift times $t_k - T_n/2$. Fig. 3 shows the instantaneous frequency of an LFM pulse and an SFM pulse, respectively. The carrier frequencies are given by $f_k = f_u + (k-1/2)w/n$, where n is the number of subpulses, $f_u = f_m - w/2$ the lower band limitation and w the available bandwidth.

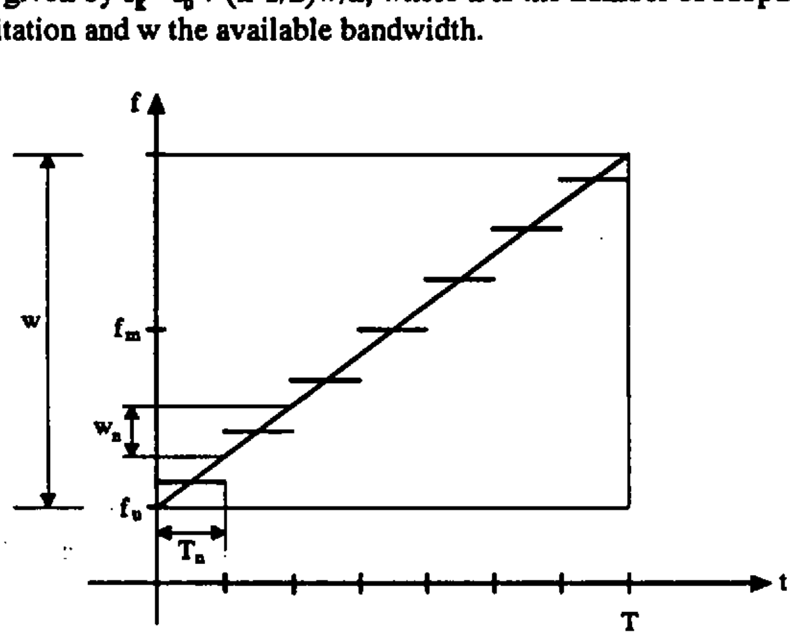

Fig. 3

The spectrum of this step-like frequency modulated (SFM) pulse depends strongly on the number of subpulses n and on the phase terms ϕ_k, which have to be chosen properly to avoid discontinuities in the phase function. This is done by defining

$$\phi_1 = 0, \ \phi_{k+1} = \phi_k + 2\pi f_k T_n, \tag{6}$$

and will be called phase correction (PC) of the SFM pulse in the following. Missing phase correction (abbreviated to MPC in the following), $\phi_k = 0$, means that whenever the frequency is shifted, the phase of the subpulse starts at zero. Figs 4 and 5 show a pulse train with and without phase correction.

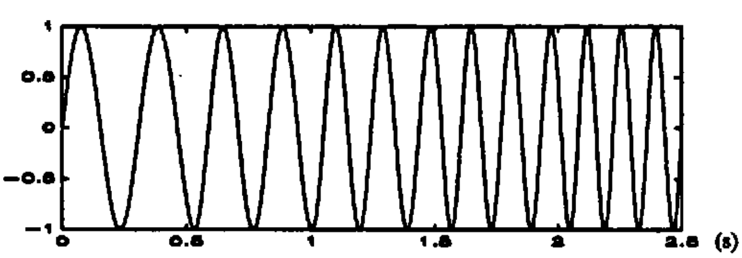

Fig. 4

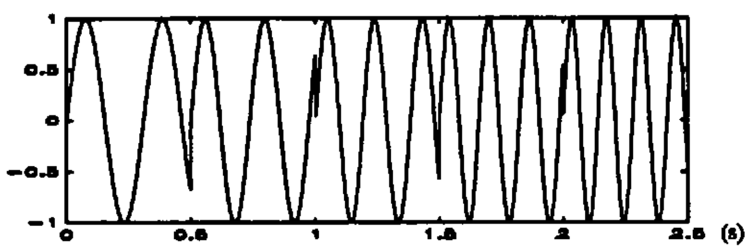

Fig. 5

For $n < \sqrt{wT}$, the spectra of the individual pulses do not result in a contiguous power spectrum of the entire pulse train. The gaps that appear do not lead to a good approach to the rectangular spectrum of the LFM pulse. This leads to undesired side-lobes in the autocorrelation function, and the pulse compression ratio is degraded relative to wT. In Figs 6 and 7 the PS and the ACF for a number $n < \sqrt{wT}$ are given.

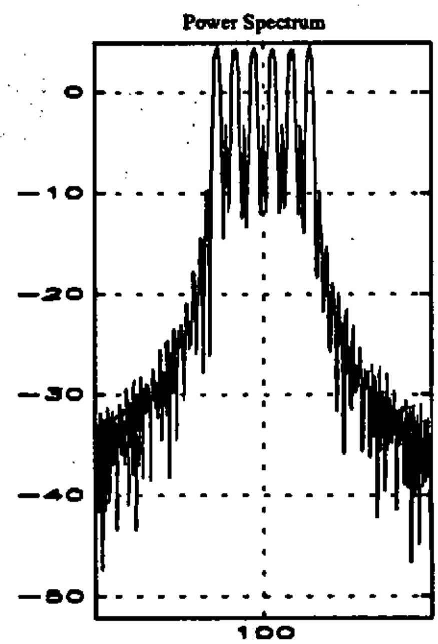

Fig. 6 PC n = 6; T = 10ms; w = 10kHz (kHz)

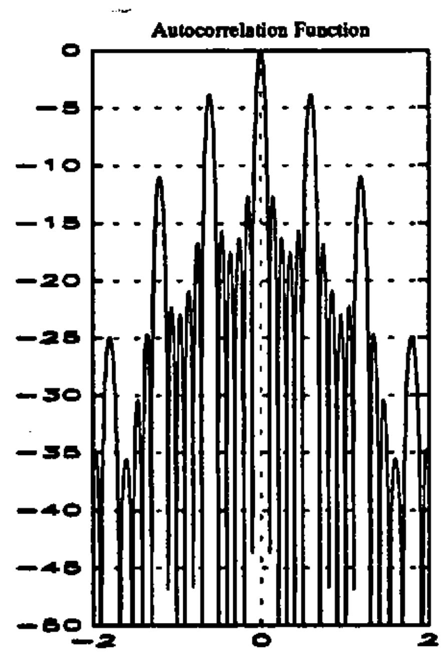

Fig. 7 PC n = 6; T = 10ms; w = 10kHz (ms)

The desired pulse compression ratio wT can be reached by a number $n \geq \sqrt{wT}$ of subpulses under the assumption of phase correction.

Figs 8 and 9 show the PS and ACF of a phase corrected SFM pulse for $n = \sqrt{wT}$. Figs 10 and 11 demonstrate the results for the same pulse but missing phase correction. For $n \leq \sqrt{wT}$, the PS of the subpulses are more or less non-overlapping, so that the phase discontinuities at $t_k - T_n/2$ do not essentially affect the ACF .

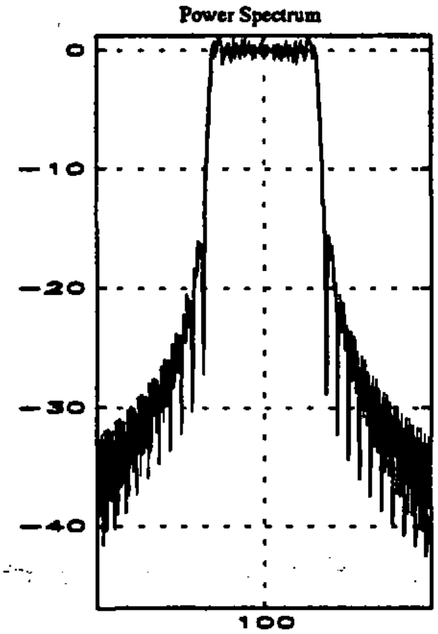

Fig. 8 PC n = 10; T = 10ms; w = 10kHz (kHz)

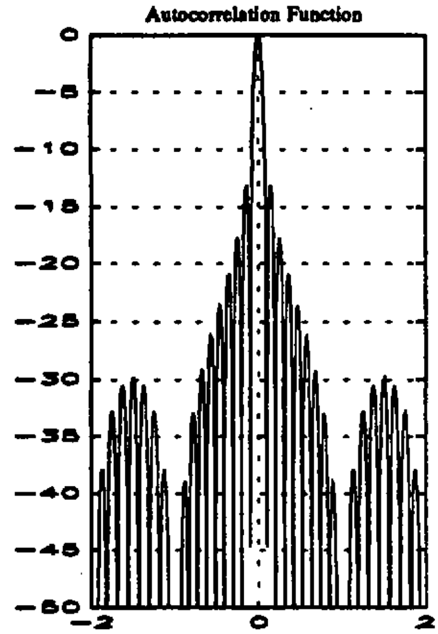

Fig. 9 PC n = 10; T = 10ms; w = 10kHz (ms)

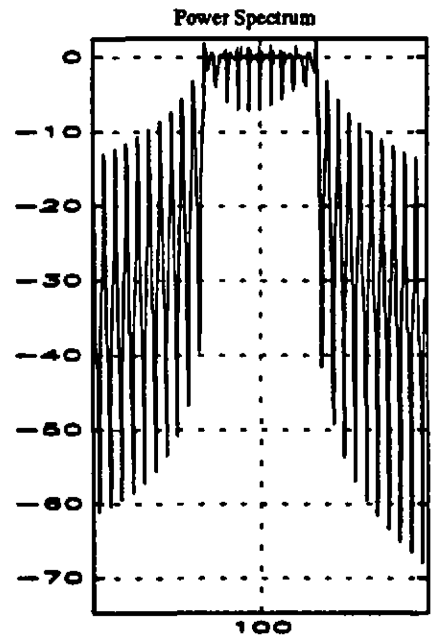

Fig. 10 MPC n = 10; T = 10ms; w = 10kHz (kHz)

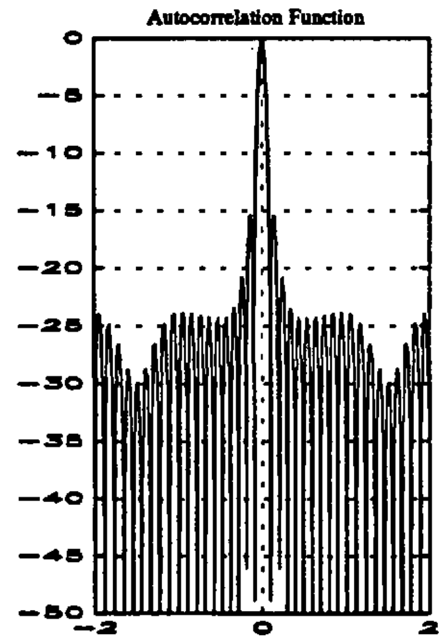

Fig. 11 MPC n = 10; T = 10ms; w = 10kHz (ms)

Comparing Figs 8 - 11 to Figs 1 and 2 it can be said that a number of $n = \sqrt{wT}$ subpulses leads to satisfactory pulse compression results.

On the other hand the ACF only demonstrates the results for the Doppler zero case. Because of its Doppler-intolerance, the SFM pulse with $n = \sqrt{wT}$ is not suitable for the Doppler-shifted case. Then, a Doppler-tolerant LFM pulse or an SFM pulse (approaching an LFM pulse) with a large number $n > \sqrt{wT}$ is more suitable. In this case a missing phase correction can lead to considerably degraded pulse compression results even for very large n. With phase correction the phase function of an SFM pulse is continuous and converges to the phase function of the LFM, such that the autocorrelation process results in a good pulse compression.

Figs 12 - 15 give the PS and ACF of a SFM for $n > \sqrt{wT}$ with and without phase correction.

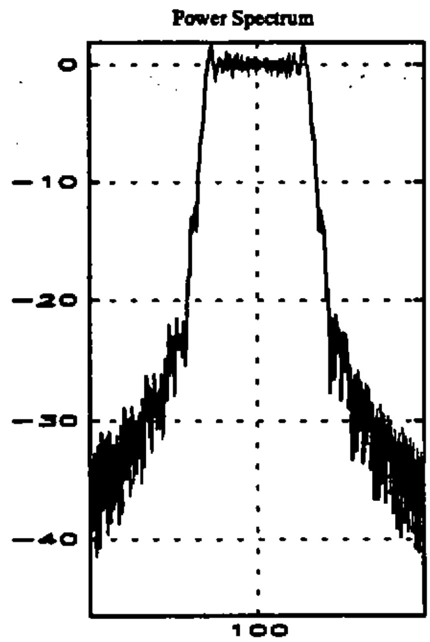

Fig. 12 PC $n = 20; T = 10ms; w = 10kHz$ (kHz)

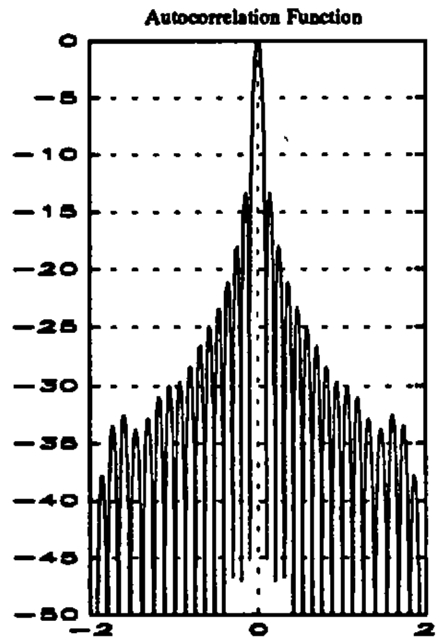

Fig. 13 PC $n = 20; T = 10ms; w = 10kHz$ (ms)

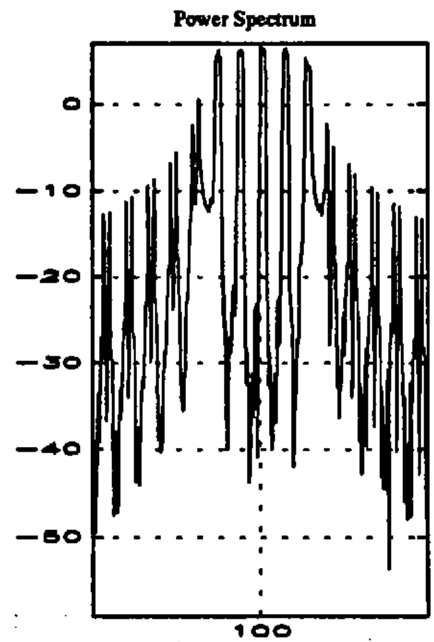

Fig. 14 MPC n = 20; T = 10ms; w = 10kHz (kHz)

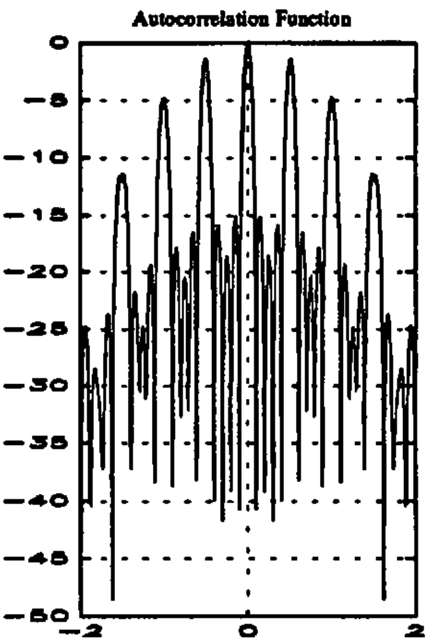

Fig. 15 MPC n = 20; T = 10ms; w = 10kHz (ms)

The discontinuities in the phase function can also be avoided without use of the phase terms ϕ_k by choosing variable subpulse lengths $T_k \approx T/n$.

3. CONCLUSIONS

Pulse compression in sonar is mostly accomplished by the use of linear frequency modulated pulses. When in practice an LFM pulse is to be approximated by a step-like frequency modulated pulse, it is very important to avoid discontinuities of the phase whenever the frequency is shifted. Therefore, either the pulse length of the individual pulse has to be chosen differently or the pulse train has to be corrected in phase.

4. REFERENCES

[1] W. S. BURDIC, 'Radar Signal Analysis', Prentice-Hall, Inc., Englewood Cliffs, N.J., 1968

[2] A. PAPOULIS, 'Signal Analysis', McGraw-Hill Book Company, New York, 1984

[3] A.W. RIHACZEK, 'Principles of High-Resolution Radar', McGraw-Hill Book Company, New York, 1969

[4] M. SKOLNIK, 'Introduction to Radar Systems', McGraw-Hill Book Company, New York, 1980