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MODELISATION OF THE RAIN NOISE IN UNDERSEA ACOUSTICS.

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SUMMARY

____Sometimes the rain noise is clearly detected behind a sonar antenna. It is caracterized by a dissymetricalspectral repartition of the energy aroundf a main frequency. Actually we carry on researchs on mathematical mocelization of the different causes of the undersea ambiant noise to separate and identify them. This is the reason why we are interested by an explanation of the rain noise. We put the hypothesis that this one is due to the collapsus of the vacuum under the surface generated by the shock of a raindrop on this surface. Generally a random number of random phenomena generates a so-called lorentzian noise of which the power spectral density is symmetrical.In fact the dissymetrical is simply due to the upper limit of the diameter of raindrops of which the jet frequency is driven.

I) INTRODUCTION:

Carying on a study for modelizing the undersea acoustic noise we are interested by the study of the rain noise. This one is indeed disturbing because it is very well heard in an area of convergence . For detecting it and rejecting it it is necessary to know its spectral properties.

II) SURVEY OF THE RAINFALL PHENOMENUM:

The rainfall phenomenum is driven by clouds that can be clustered in two main species: The cumulus and status types. The latest raindrops are due to the cumulus clouds during the strong rainfalls. We showed that the weak rainfalls due to the stratus clouds are statistically distributed according to a log-normal law instead of a gamma distribution for the strong rainfalls due to the cumulus clouds (Cf. fig. 1). For a given rain rate the diameter of raindrops is statistically distributed (Cf. fig.2). Ipso facto the general statistical law of the diameter of the raindrops is driven by the rate of rainfalls. We note that the value of the maximum likelihood is approximatively 1.5 mm. The speed of fall of the raindrops is approximatively constant and near of 10m/s.

III) EXPERIMENT RESULTS

If we look at experiment results we observe immediately a very particular caracterization of the shape of the radiated spectrum. We note a peak frequency at 15 kHz. and a dissymetrical level around it: The decreasing below it is stronger than this one above it. (Cf. fig. 3) This work tries to explain this observation.

IV) FIRST COMPARISON WITH ACTUAL THEORETICAL RESULTS

Suppose that a raindrop with 1 mm. radius falling with a 10 m/s. speed. Its volume is $\frac{4\pi}{3}10^{-9}m^3$ and its weight is $\frac{4\pi}{3}10^{-9}kg$. Because of the fluid damping we asuppose that the raindrop is break on a lenghtequal to its radius after the shock. We obtain a decceleration equal to : $\gamma = \frac{v^2}{r} = 10^5 m/s^2$

The stroke pressure is equal to the weight of the raindrop time the acceleration divided by its section .And we obtain: $P = \frac{F}{s} = \frac{m\gamma}{s} = \frac{4}{3}10^{2}\,\mathrm{Pa}$

Following Donald ROSS we give the idealized spectrum of an theoretical cavitation in the figure 4 and we note that it is very near of the spectrum of the rain noise. If we apply the formula given by Ross for obtaining the peak frequency : $f_m = \frac{1}{r}\sqrt{\frac{P}{\rho_0}}$; ρ_0 : Density of the fluid.

we obtain this value: $f_m = \frac{1}{r}\sqrt{\frac{P}{\rho_0}} = 10^{3}\sqrt{\frac{\frac{4}{3}10^{2}}{1}} \approx 12\,\mathrm{kHz}$

We can conclude that the physical phenomenum of the rain noise generation and the cavitation generation are similar phenomena.

As examples we show several curves of growth and collapsus of a bubble of cavitation. We note that the collapsus can take very different shapes without any characteristic features. As far as we are concerned with the spectrum of the rain noise we can modelize it a priori with the following power spectral density:

$$|S(\omega)|^{2} = \frac{A^{2}\omega^{2N}}{B^{2} + \omega^{2N+2}}$$

How is it possible?

VI) PHYSICAL-STATISTICAL MODELIZATION OF THE RAIN NOISE FOR A LITTLE SURFACE.

The physical-statistical process of the supply of the rain noise is:
Raindrops bit the sea surface according on a random process. Each of them triggs an excitation of the fluid medium on the surface and under the surface. The surface pezrturbation is due to the surface wave generated by the variation of pressure. The undersea perturbation is due to the formation of a gas vacuum generated by the penetration of the raindrop under the sea surface. If we consider a little surface we can suppose a poissonian characterization of the counting process. So the generated acoustical noise can be written so:

$$I_j(t) = G_j(t) * e(t) = \frac{1}{R}\int_0^t e^{-\frac{\lambda_j}{RC}(t-\tau)} e(\tau)\,d\tau; \text{ Where: } G_j(t) = \frac{e^{-\frac{\lambda_j t}{RC}}}{R}$$

$$B(t) = \sum_{n=0}^{N(t,dS(P))} s_n(t-\tau_n) \quad \text{With: } s_n(t) = \alpha e^{-\beta t}\sin\theta$$

Where theta is the site angle. (We suppose that the image source is with a - Pi phase related to the original source of noise.

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— — — — — Sea surface.

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So we admitt a dipolar radiation. For a given unit surface we admitt a poissonian process with a $\lambda(dt,dS)$

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rate. The characteristic function of B(t) is equal to:

$$E(e^{iuB(t)}) = \sum_{n=0}^{\infty}\left(E\left(e^{ius(t-\tau)}\right)\right)^n P(N(t,dS(P))=n)$$

With hypotheses of independance and equidistribution:

$$P(N(t,dS(P))=n) = e^{-\int_0^t \lambda(d\tau,dS(P))}\frac{\left(\int_0^t \lambda(d\tau,dS(P))\right)^n}{n!} \quad ; \quad E_\tau(E(e^{ius(t-\tau)})) = \frac{\int_0^t E(e^{ius(t-\tau)})\lambda(d\tau,dS(P))}{\int_0^t \lambda(d\tau,dS(P))}$$

According to the rate of rainfalls the radiation is different because the rate is depending on the point of measurement. If s(t) is depending itself on the point of measurement :

$$\varphi_{B(t)}(u) = e^{\int_s \left[E\left(e^{ius(\tau,P)}\right)-1\right]\lambda(d\tau,dS(P))}$$

In the same way if we admit a space decorrelation between the elementaries signals sn we obtain the space -time of the received noise B(t):

$$C_B(t-\tau,h,S) = \iint_S \int_0^t C_s(t-\tau,h,P)\frac{\partial^2\lambda}{\partial\tau\partial S}(\tau,P)d\tau dS(P)$$

$$; P=(x,y) ; dS(P) = dxdy$$

So:

$$C_s(t-\tau,h,x,y) = \int_0^t \alpha^2 e^{-\beta(t-\tau)}e^{-\beta(t-\tau+h)}\lambda(d\tau,dS).\sin^2\theta = \alpha^2 e^{-\beta h}\int_0^t e^{-2\beta(t-\tau)}\frac{\partial\lambda(\tau,dS)}{\partial\tau}d\tau.\sin^2\theta = \alpha^2 e^{-\beta h}\wedge(dS)\frac{1-e^{-2\beta t}}{2\beta}.\sin^2\theta$$

And we have:

$$\lim_{t\to\infty} C_s = \alpha^2 e^{-\beta h}\wedge(dS)\frac{1}{2\beta}.\sin^2\theta$$

VII) PHYSICAL-STATISTICAL MODELIZATION OF THE RAIN NOISE FOR A GREAT SURFACE.

For a great surface we cannot admitt that the counting process is a simply poisson process. We will suppose that it is a compound poisson process defined by:

$$N_s(t) = \sum_{n=0}^{N(t)} Y_n(S)$$

Where the Yn random variables represent the number of raindrops that are simultaneously present at the t instant..N(t) is the poisson process counting the arrivals of the clusters So the B(t) process can be written as:

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$$B(t)=\sum_{n=0}^{N(t)}\sum_{m=1}^{Y_n(S)} s_m(t-\tau_n)$$

The characteristic function of B(t) is:

$$E\left(e^{iuB(t)}\right)=\sum_{N=0}^{\infty} E\left(e^{iu\sum_{n=0}^{N}\sum_{m=1}^{Y_n(S)} s_m(t-\tau_n)}\right)P(N(t)=N)=\sum_{N=0}^{\infty}\prod_{n=1}^{N} E\left(e^{iu\sum_{m=1}^{Y_n(S)} s_m(t-\tau_n)}\right)P(N(t)=N)$$

Where:

$$E\left(e^{iu\sum_{m=1}^{Y_n(S)} s_m(t-\tau_n)}\right)=\sum_{M_n=1}^{\infty} E_\tau\left[e^{iu\sum_{m=1}^{M_n} s_m(t-\tau_n)}\right]P(Y_n(S)=M_n)=\sum_{M_n=1}^{\infty}\left(E_\tau\left[e^{ius_m(t-\tau_n)}\right]\right)^{M_n}P(Y_n(S)=M_n)$$

$$E\left(e^{iuB(t)}\right)=\varphi_{B(t)}(u)=\sum_{N=0}^{\infty}\left(E\left[e^{ius_m(t-\tau)}\right]\right)^{N}P(N(t)=N)$$

Hence:

Where:

$$P(N(t)=N)=e^{-\int_0^t\lambda(\tau)d\tau}\frac{\left(\int_0^t\lambda(\tau)d\tau\right)^N}{N!}\ ;\ E_\tau\left\{E\left(e^{\sum_{m=1}^{Y(S)} s_m(t-\tau)}\right)\right\}=\frac{\int_0^t E\left(e^{iu\sum_{m=1}^{Y(S)} s_m(t-\tau)}\right)\lambda(\tau)d\tau}{\int_0^t\lambda(\tau)d\tau}$$

The characteristic function of B(t) becomes: $\varphi_{B(t)}(u)=e^{\int_0^t\left(E\left(e^{iu\sum_{m=1}^{Y(S)} s_m(t-\tau)}\right)-1\right)\lambda(\tau)d\tau}$

In the same way we can show that its correlation function can be written under this relationship:

$$C_B(t,t-h)=E(B(t)B(t-h))=\int_0^t E\left(\left(\sum_{m=1}^{Y(S)} s_m(t-\tau)\right)\left(\sum_{m=1}^{Y(S)} s_m(t-\tau-h)\right)\right)\lambda(\tau)d\tau$$

$$C_B(t,t-h)=E(B(t)B(t-h))=\int_0^t E\left(\sum_{M=1}^{\infty}\sum_{m=1}^{M}\left(s_m(t-\tau)\right)\left(s_m(t-\tau-h)\right)\right)P(Y(S)=M)\,\lambda(\tau)d\tau$$

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$$= \sum_{M=1}^{\infty} \sum_{m=1}^{M} \int_0^t E(s_m(t-\tau)s_m(t-\tau-h))\hat{\lambda}(\tau)d\tau \; P(Y(S)=M)$$

With: $s_m(t-\tau) = \alpha_m \sin\theta \, e^{-\beta_m(t-\tau)}$

Admitting the homogeneous character of the Poisson process we have:

$$\int_0^t E(s_m(t-\tau)s_m(t-\tau+h))\lambda d\tau = \sin^2\theta\,\alpha_m^2 e^{-\beta_m h}\int_0^t e^{-\beta_m(\tau-\tau)}d\tau$$

$$\lim_{t\to\infty}\int_0^t E\{s_m(t-\tau)s_m(t-\tau+h)\}\lambda\, d\tau = \frac{\lambda \sin^2\theta\,\alpha_m^2 e^{-\beta_m h}}{2\beta_m}$$

$$S_M(\omega) = \sum_{m=1}^{M} \lambda \sin^2\theta \frac{\alpha_m^2}{2\beta_m}\cdot\frac{1}{\beta_m + i\omega}$$

Put: $P(Y(S) = M+1) = \left(1-e^{\frac{S}{S_0}}\right)^M . e^{\frac{S}{S_0}}$; $M \geq 0$

$$S_M(\omega) = S_{M0}\sum_{m=1}^{M}\frac{\alpha_m}{\beta_m + i\omega} = S_{M0}\frac{\sum_{m_0=1}^{M}\alpha_{m_0}\prod_{j\neq j_0}(\beta_j + i\omega)}{\prod_{m=1}^{M}(\beta_m + i\omega)}$$

The $S_M(\omega)$ spectra can be written so:

$$S_M(\omega) = \frac{(i\omega)^{M-1}\sum_m \alpha_m + (i\omega)^{M-2}\sum_{m\neq m_0}\alpha_{m_0}\sum_m \beta_m + (i\omega)^{M-3}\sum_{\substack{m_0\neq m\\ m_0\neq n}}\sum_m\sum_n \alpha_{m_0}\beta_m\beta_n + \ldots}{(i\omega)^M + (i\omega)^{M-1}\sum_m \beta_m + (i\omega)^{M-2}\sum_m\sum_n \beta_m\beta_n + (i\omega)^{M-3}\sum_m\sum_n\sum_p \beta_m\beta_n\beta_p + \ldots}$$

Developping it is obtained:

We can admitt that the relative phase are becoming random by multiple products of severall terms So we can suppose that the sum of multiple terms tend to zero. for the great values of M If we keep only the single

$$S_M(\omega) \cong \frac{(i\omega)^{M-1}\sum_m \alpha_m + \ldots}{\prod_m \beta_m + (i\omega)^M + (i\omega)^{M-1}\sum_m \beta_m + \ldots}$$

sums we obtain: $M >> 1$

The square of the modulus is equal to:

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$$|S_M(\omega)|^2 \cong \frac{(\omega)^{2(M-1)}\left|\sum_m \alpha_m\right|^2}{\left|\prod_m \beta_m - \mathrm{Im}(\sum_m \beta_m)\right|^2 + \left|(\omega)^M + (\omega)^{M-1}\mathrm{Re}(\sum_m \beta_m)\right|^2}$$

$$M \gg 1$$

Hence all P.S..D.. have an asymptotic behavior as $\omega^{\frac{1}{2}}$ when the frequency tends to infinity and a quick decreasing to zero when the frquency tends to zero. So the p.s.d. admitts a peak value. For the rain noise it is well known that its one is equal to 15 kHz.

VIII.) PRACTICAL FORESEEING OF THE LEVEL OF THE RAIN NOISE.

From meteorological data we can know the forecasting of rainfalls and the sea surface covered by the rain clouds. So we can use for the studied area the probability function of the rainfall.If we call R the rate of rainfall and D the diameter of raindrops we have: $P(R \geq R_0)$; (Cf. Fig. 1) $P(D \geq D_0 / R_0)$; (Cf. Fig.2)

$$P(D \geq D_0) = \int P(D \geq D_0 / R_0)\, p(R_0)\, dR_0$$

So we can deduce the probability: $$p(R_0) = -\frac{d}{dR_0} P(R \geq R_0)$$

We admitt an homogeneous spreading of raindrops on a little area:

$$S_m(\omega, D_0) = \Lambda(dS)\sin^2\theta \,\frac{\alpha_m^2}{2\beta_m}\,\frac{1}{\beta_m + i\omega}$$

$\alpha_m = \alpha_m(D_0)$; $\beta_m = \beta_m(D_0)$; (i.i.d.r.v.)

We suppose the distribution of raindrops is independant of the dS element.

$$E|S_M(\omega, D_0)|^2 = E\left|\sum_{m=1}^{M} S_m(\omega, D_0)\right|^2 \cong E\frac{\omega^{2(M-1)}\left|\sum_1^M \alpha_m\right|^2}{\prod_1^M \beta_m^2 + \left|\omega^M + \omega^{M-1}\sum_1^M \beta_m\right|^2} \cong E\frac{\omega^{2(M-1)}|M\bar{\alpha}|^2}{\bar{\beta}_0^{2M} + \left|\omega^M + \omega^{M-1}M\bar{\beta}\right|^2}$$

putting:

$$\frac{1}{M}\sum_{m=1}^{M}\alpha_m \to \bar{\alpha}(D_0)\;\frac{1}{M}\sum_{m=1}^{M}\beta_m \to \bar{\beta}(D_0) \;;\; \text{(Arithmetical average)}$$

$$\left(\prod_{m=1}^{M}\beta_m\right)^{\frac{1}{M}} \to \overline{\beta_0}(D_0) \;;\; \text{(Geometrical average)}$$

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So we obtain:

$$E\left|S_M(\omega)\right|^2 = \int \frac{\omega^{2(M-1)}\left|M\alpha\right|^2}{\beta_0^{-2M} + \left|\omega^M + \omega^{M-1} M\beta\right|^2}(D_0)\, p(D_0)\, dD_0$$

Where: $$p(D_0) = -\frac{dP(D \geq D_0)}{dD_0}$$

So the final Power spectral density is equal to:

$$\left|S(\omega)\right|^2 = \sum_{M=1}^{\infty} E\left|S_M(\omega)\right|^2 \left(1-e^{-\frac{S}{S_0}}\right)^M e^{-\frac{S}{S_0}}$$

IX) BIBLIOGRAPHY:

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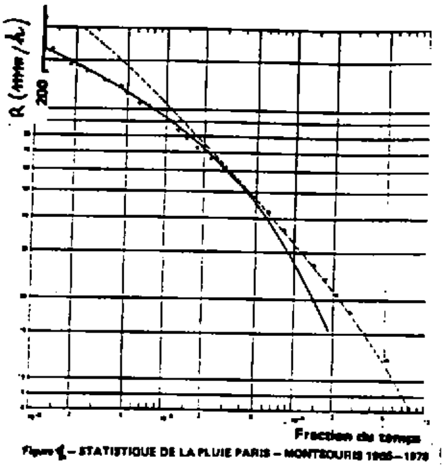


Figure 1 – STATISTIQUE DE LA PLUIE PARIS – MONTSOURIS

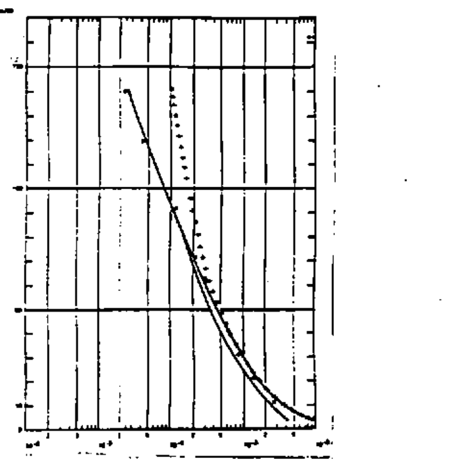
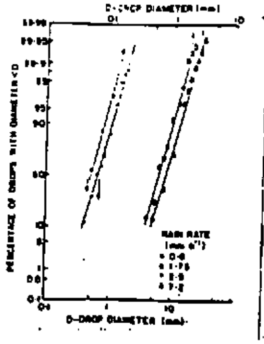
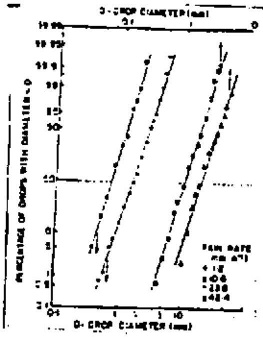


Figure 5 STATION DE BRAZZAVILLE – PERIODE DE MI AVRIL 80 A FIN AVRIL 81





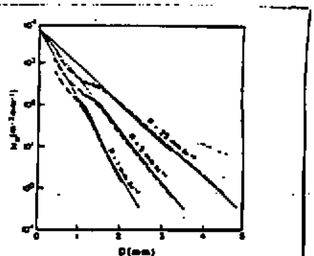


FIG. 1

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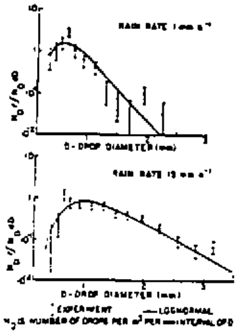
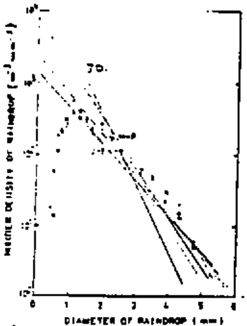
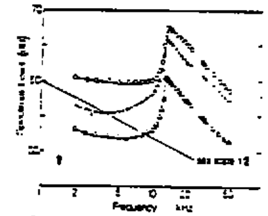






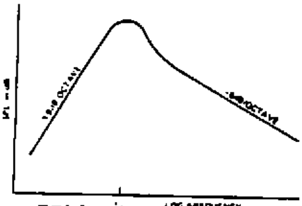
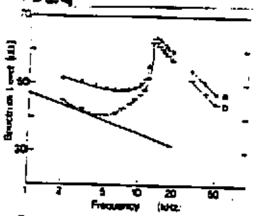
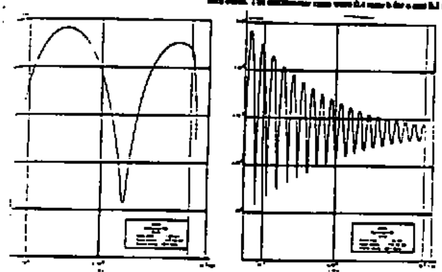


FIG.4





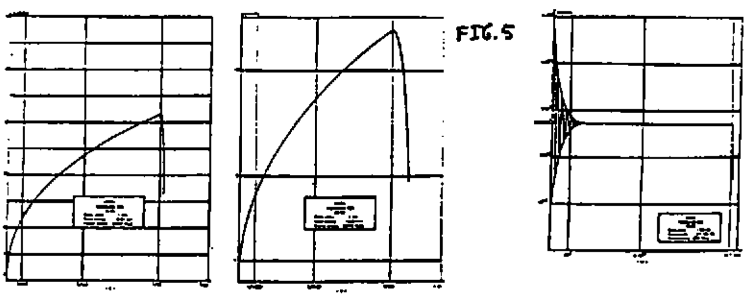


FIG.5

FIG.5