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MECHANICAL SHOCK SHAPE CALCULATION FROM ESTIMATED ACCELEROMETER PARAMETERS

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Mechanical shocks are often measured by piezoelectric accelerometers. This sensor type causes shock shape changes because it is essentially an under-damped second order dynamic system, therefore it filters sharp edges of shocks and adds damped oscillations. Because of a signal change occurring on a sensor, it is hard to calculate correctly shock parameters or a shock response spectrum. Yet both are essential parameters for a device testing. Extracting a shock shape from measured data is very complicated due to signal noise and inaccuracies in sensor dynamic parameters. In this paper, we present a new method of calculation of an original shock shape and sensor dynamic parameters when the mechanical shock is measured by two accelerometers with different frequency spectrum. New method was tested on simulated data and verified during experiment which provided overall accuracy up to 5 %.

Keywords: mechanical shock, piezoelectric accelerometer, shock response spectrum, parameter estimation

1. Introduction

With increasing demands on a product reliability is necessary to test product ability to survive shocks and vibrations according to norm [1]. For a decision whether the product is able to withstand the lifetime tests is necessary to correctly measure shock parameters and a shock response spectrum (SRS). Measurement using piezoelectric accelerometers are widely used for its lightweight and wide range.

In case of vibrations there are usually no problems with their usage. However, in case of shocks, which contain high frequencies, using sensors are much more difficult and some other influences has to be taken into account. Some authors describe difficulties in mounting accelerometers to a surface [2]. Gaberson [3] shows the influence of a filtering to measured data. Doebellin [4] describes a filtration of a time signal caused by piezoelectric accelerometers. Filters there are used on a simulated shock for better matching with real data. So there is no mention about filtering measured data to suppress a sensor influence. For this suppression a new algorithm was developed because a neglection of the sensor influence to a mechanical shock shape can, in some cases, be the difference between failing and passing the shock test.

1.1 Shock response spectrum

A shock response spectrum is a way how to determine a shock severity based on the time history. The shock shape is applied to a single degree of freedom system (SDOF) and maximum of the response is one point of the SRS at a SDOF natural frequency [5]. The system response can be calculated in an acceleration or a pseudo-velocity as discussed in [6].

2. Shock shape change

Piezoelectric accelerometers can be principally modelled as an under-damped spring, damper, mass system with single degree of freedom. This model leads to a second order differential equation, which can be transformed using Laplace transform to a continuous second order dynamic system (Eq.1) [4]. For a calculation with sampled a data discrete model (Eq. 2) in Z-transform should be used. However in case where a sampling period is much shorter than system time constants the continuous model can also be used.

$$F(s) = \frac{k}{T^2 s^2 + 2T\zeta s + 1} \tag{1}$$

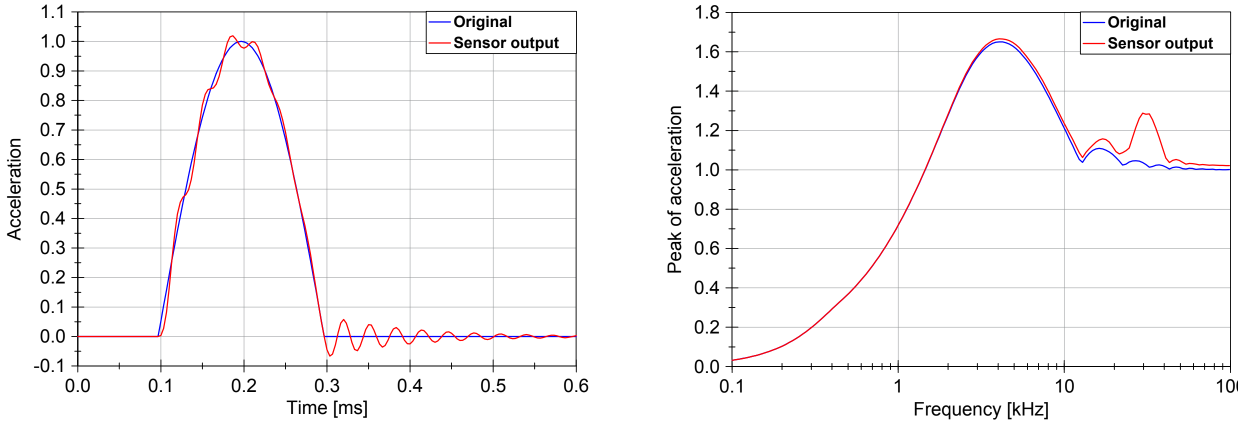
where s is Laplace operator, k is accelerometer gain, T is system time constant and ζ is damping ratio.

$$F(z) = \frac{k}{T\sqrt{1-\zeta^2}} \frac{z e^{-\zeta \frac{T_s}{T}} \sin\left(\frac{T_s}{T}\sqrt{1-\zeta^2}\right)}{z^2 - 2 z e^{-\zeta \frac{T_s}{T}} \cos\left(\frac{T_s}{T}\sqrt{1-\zeta^2}\right) + e^{-2\zeta \frac{T_s}{T}}} \tag{2}$$

where z is Z transform operator, k is accelerometer gain, T is system time constant, ζ is damping ratio and T_s is sampling period.

For shock measurements are used piezoelectric accelerometers with a small damping (usually damping ratio less than 0,05) [4]. For that reason a maximum measured frequency should be five times smaller than a sensor resonant frequency. Then the expected amplitude rise caused by the resonance is approximately 5 %. This recommendation is, in case of a short shock duration (approx. 0,2 ms), difficult to comply because frequency spectrum has to be up to 40 kHz [1].

Despite the sensor resonance error is applied only on higher frequencies of the shock it still has a significant effect to the shape (Fig. 1a) and also to the SRS (Fig. 1b). The difference between an actual and a measured shock can have significant effect to the test result. A compliant device can fail the test and vice versa. This can lead to a financial and, in worst case, to a fatal consequences.



(a) Time duration

(b) Shock response spectrum

Figure 1: Differences between original and sensor output data. Sensor resonant frequency $f_r = 200\ kHz$, damping $\zeta = 0.05$, sampling frequency $f_s = 600\ kHz$

3. Solution

The easiest way how to suppress the resonance error and avoid the wrong test result is using a different acceleration sensor. The sensor has to be lightweight (a few grams), with a high maximal measured value (several thousand g) and with a broad bandwidth (up to $40\ kHz$).

As lightweight only MEMS sensors can be considered. There are two other principles for measuring acceleration a piezoresistive and a capacitive. Both have limitation in the maximal measured frequency which is in both cases up to $20\ kHz$ [7], which is still insufficient for the shock measurement.

Other possible way is to filter the output signal using an inverse function of the sensor. It is theoretically ideal way how to find out actual pulse shape. In practice there are two major problems. First is usually inaccurate knowledge of sensor dynamic parameters, the time constant (T) and the damping (ζ). Second is noise because the sensor inverse function is derivative system so it amplifies noise. For that reasons this way can't be used on noisy data.

The sensor inverse function can be modified to filter the noise on high frequencies. Then the filtration suppress noise, but it can't reflect the difference between an ideal (which is not always known) and a real dynamic parameters of the sensor, which adds additional error. So filtered signal can have even bigger error than the unfiltered sensor output. To minimize this error system dynamic parameters identification has to be processed.

3.1 Two sensors method

Two sensors are used for measuring the mechanical shock to identify sensor's dynamic parameters and then calculate the actual shock shape. An accelerometer parameter identification using interferometer measurement is described in [8]. However, using interferometer for normal shock measurement is impractical, so interferometer was replaced with another accelerometer. It is necessary to have piezoelectric accelerometers with known gains and different frequency characteristics for the measuring. Then sensors dynamic parameters T_1, T_2 and ζ_1, ζ_2 are used to construct a sensor modified inverse transfer function (MITF) to suppress resonances and filter noise. The dynamic parameters are changed to find a minimal difference between filtered sensor outputs as described in Eq.(3). Different dynamic parameters are causing changes in an output delay. This delay has to be taken into account to calculate the difference correctly. For a calculation can be foe example used an iterative gradient minimization method.

$$
\begin{aligned}
H_1(t,T_1,\zeta_1) &= \mathscr{L}^{-1}\left\{F_1^{-1}(s,T_1,\zeta_1)\frac{1}{(T_{1_a}s+1)...(T_{1_m}s+1)}e^{-\phi_1 s}\right\}\\
\phi_1 &= \begin{cases} 2(T_2-T_1) & \text{if } T_2 > T_1\\ 0 & \text{otherwise}\end{cases}\\
H_2(t,T_2,\zeta_2) &= \mathscr{L}^{-1}\left\{F_2^{-1}(s,T_2,\zeta_2)\frac{1}{(T_{2_a}s+1)...(T_{2_m}s+1)}e^{-\phi_2 s}\right\}\\
\phi_2 &= \begin{cases} 2(T_1-T_2) & \text{if } T_1 > T_2\\ 0 & \text{otherwise}\end{cases}\\
\{T_{1_M},T_{2_M},\zeta_{1_M},\zeta_{2_M}\} &= \underset{\{T_1,T_2,\zeta_1,\zeta_2\}}{\arg\min}\sum_{t=1}^{n}\left[(S_1(t)*H_1(t,T_1,\zeta_1))-(S_2(t)*H_2(t,T_2,\zeta_2))\right]^2\\
U_1(t) &= S_1(t)*H_1\left(t,T_{1_M},\zeta_{1_M}\right)\\
U_2(t) &= S_2(t)*H_2\left(t,T_{2_M},\zeta_{2_M}\right)
\end{aligned}
\tag{3}
$$

H_1 and H_2 are impulse responses of sensor's MITF with dynamic parameters, system time constants T_1, T_2 and dampings ζ_1, ζ_2; t is time. T_{1_a} to T_{1_m} and T_{2_a} to T_{2_m} are filters constants of the MITF and s is Laplace operator. The time difference between sensor outputs are compensate using time delay ϕ_1 or ϕ_2. T_{1_M}, ζ_{1_M} and T_{2_M}, ζ_{2_M} are identified sensors dynamic parameters. U_1 and U_2 are recalculated shock shapes with suppressed sensor influence and filtered noise.

Setting the cutoff frequency is challenging because on one hand a high cutoff frequency means better reconstruction of sharp edges, however on the other hand it causes an amplification of the noise. In case of the shock measurement is necessary to suppress the sensor resonance. It is possible to obey without sharp edges if demands on the SRS are not extremely tough. The cutoff frequency should be set according to noise floor in the signal to maximize sharpness of edges and stay within desired noise floor. However, the cutoff frequency has to be always higher than the sensor resonance frequency. It is also possible to add several cutoff frequencies to suppress noise and preserve more information about sharp edges.

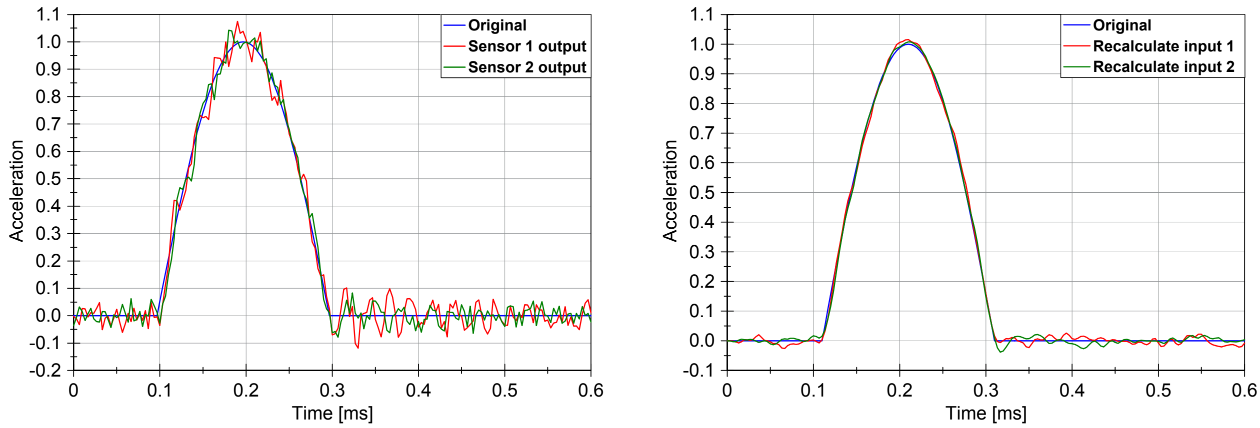
After finding actual sensor's dynamic parameters and knowing its gain is possible to calculate the shock shape with suppressed sensor influence.

4. Validation

To prove the accuracy of this method several tests were performed. The method was tested firstly on simulated data to prove functionality and check the influence of noise and the sampling frequency. Afterwards method was tested on real data to prove functionality.

4.1 Simulation

The simulation was performed on shock with normalized amplitude and duration $\tau = 0.2\ ms$. Figure (2a) shows an original shock shape (blue colour) and the outputs from sensors with added white noise ($SNR = 20\ dB$). It is visible that from sensors output data are quite complicated to measure shock parameters due to noise and sensor dynamic characteristics. After suppressing the sensor influence and lowering noise the calculated shock shape (Fig.2b) almost exactly matches the original shape. From the recalculated shock shape can be shock parameters easily measured.



(a) Sensor outputs of the shock

(b) Recalculated shock shape

Figure 2: Differences between sensors output signal and calculation of an actual shock shape from these outputs. Sensors resonant frequencies $f_{r_1} = 250\ kHz$ and $f_{r_2} = 200\ kHz$, damping $\zeta_1 = 0.02$ and $\zeta_2 = 0.05$, sampling frequency $f_s = 600\ kHz$, $SNR = 20\ dB$

After proving the algorithm functionality is necessary to check its robustness. This was done by changing the sampling frequency and the signal-to-noise ratio (SNR) while observing an error between set and detected sensor resonant frequencies. From results (Tab. 1) is visible, that the error is decreasing with increasing sampling frequency and also signal-to-noise ratio. However, the worst error is still smaller than 2 %.

Table 1: Table of algorithm error [%] in estimation of sensors resonant frequencies depending on the SNR and the sampling frequency (f_s). Sensors set resonant frequencies $f_{r_1} = 250\ kHz$ and $f_{r_2} = 200\ kHz$, damping $\zeta_1 = 0.02$ and $\zeta_2 = 0.05$.

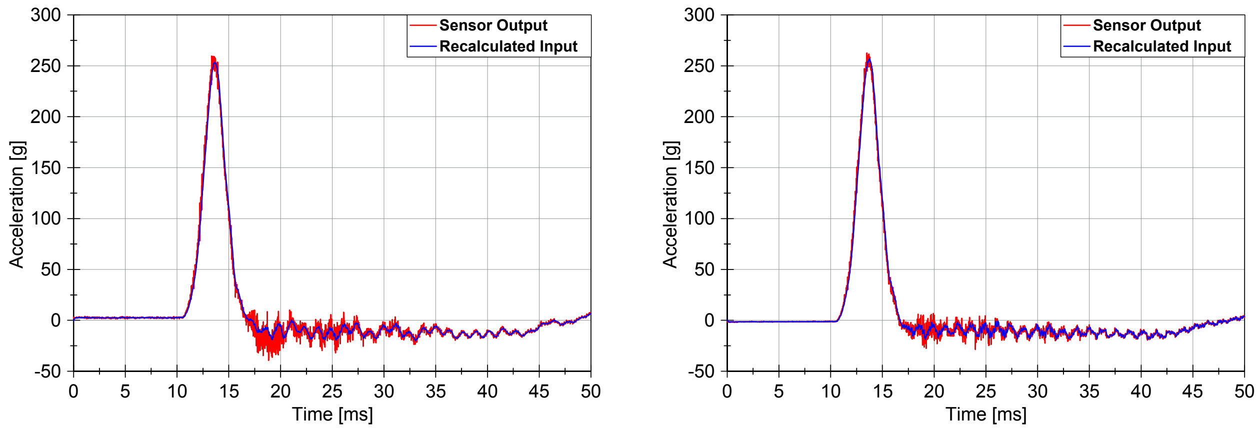
SNR [dB] \ f_s [kHz]	600	700	800	1000	2000
20	1,73	1,09	0,85	0,51	0,30
25	0,97	0,58	0,27	0,23	0,14
30	0,31	0,22	0,21	0,17	0,06
35	0,19	0,14	0,07	0,06	0,03

4.2 Real data

The real data for the algorithm validation were gathered on a shock machine AVEX SM110-MP, where parameters were set accordingly, the duration $\tau = 4\ ms$ and the peak acceleration $a_p = 250\ g$. Unfortunately the shorter duration, which imply higher frequencies, were unreachable. For measuring two piezoelectric accelerometers, a PCB M352C03 and a B&K 8309 were used. According to manufacturers the resonant frequencies of the sensors are $f_{r_{PCB}} > 50\ kHz$ for the PCB accelerometer and $f_{r_{B\&K}} = 180\ kHz$ for the B&K.

Measured data were then processed to find sensors resonant frequencies and recalculate the shock shape. Results are shown in figure (3a) for the PCB sensor and in figure (3b) for the B&K. In case of the PCB accelerometer is visible a reduction of an overshoot caused by the sensor. The B&K accelerometer has very small time constant so there is practically no influence on the shock shape. In both cases appears significant noise reduction.

On figures (3a, 3b) is also visible some other resonance on relatively small frequency (approx. $800\ Hz$). This resonance is probably caused by a shock machine [9].



(a) PCB M352C03

(b) B&K 8309

Figure 3: Differences between sensors output data and recalculated shock shapes, sampling frequency $f_s = 500\ kHz$.

The algorithm also estimates sensors dynamic parameters. In case of the PCB sensor, estimated resonant frequency was $f_{e_{PCB}} = 48,24\ kHz$. The difference between estimated and stated (according to manufacturer) frequency is $3,5\ \%$. The second sensor has the resonant frequency 18 times higher than the shock spectrum according to [1]. That's why the overshoot caused by the sensor is very small and lost in noise, so the method is not able to estimate the resonant frequency.

5. Conclusion

This study reveals the new method how to deal with resonances caused by piezoelectric accelerometers which occur when measuring mechanical shocks. The method is able to estimate sensors dynamic parameters and suppress its resonances. The method was tested on simulated and real data with following results. The resonance suppression is visible in (fig. 2) for the simulated signal and in (fig. 3) for the real signal. The accuracy of sensor parameter estimation on the real signal is 3.5 %, however the accuracy is dependent on the noise level and sampling frequency as described in (tab. 1). Further work will be focused on algorithms capable to suppress also other resonances influencing the shock measurement.

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