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CONTROL OF TORSIONAL WAVES PROPAGATING IN A STRUCTURED BEAM

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This work presents the demonstration of the rainbow trapping effect for torsional waves propagating in an aluminium beam with rectangular cross section. The beam contains a chirped structure that is machined with notches separated with the rule $\lambda = (a_j - a_{j-1)/} a_j$, where a_j is the size of the cavity defined between notches j-1 and j, and λ is the chirp intensity parameter. We have observed the temporarily trapping of waves at positions in the beam that depend of the central frequency of the wave packet. This demonstration for torsional waves is encouraging and opens the possibility of its extension to more complex elastic waves.
Keywords: Physical acoustics, phononic crystals, sound scattering, torsional waves

1. Introduction

The control of wave propagation through materials is a long-standing field of research due to its potential applications in industry and technology. For example, vibrational energy trapping is a subject of considerable interest because of its importance in designing mechanical devices like oscillators. The phenomenon of “rainbow trapping” was characterized in optical metamaterials [1], where each frequency component of a wave packet was trapped at different positions inside a tapered left-handed heterostructure1. This phenomenon was later characterized for other type of waves, like acoustic waves [2, 3]. Thus, trapping of broadband acoustic waves was demonstrated in designed gradient subwavelength structures. The trapping positions can be predicted because of the interplay between the acoustic resonance inside individual apertures and the mutual coupling among them. Bloch oscillations are a phenomenon strongly related with rainbow trapping and has been demonstrated in acoustics using chirped structures [4]. For mechanical waves, however, the complexity associated to polarization mixing and the difficulty of selectively measure the modes has delayed the observation of rainbow trapping for elastic waves.

This work reports experiments showing that mechanical waves can be slowdown in chirped structures. Particularly, we demonstrate rainbow trapping of torsional waves propagating in a beam with rectangular cross section. Thus, wave packets with different frequencies are back-scattered at different positions inside the corrugated beam; being packets with higher frequencies the ones with larger penetration depths.

2. Numerical simulations

We have considered a finite length aluminium beam with rectangular cross section. The beam has free boundary ends and it is made of a uniform part and a chirped structure. The chirped structure consists of notches separated by the rule $\lambda = (a_j - a_{j-1})/a_j$, where a_j is the size of the cavity defined between notches j-1 and j, and λ is the chirp intensity parameter. The periodic structure is obtained for λ=0. When $\lambda \neq 0$ the chirped structure is obtained. The cells defined between notches become smaller with the j-index. First, the band structure of the chirped structure was calculated. With no chirp (λ=0) the first gap of the locally periodic structure is located between 6.5 kHz and 9.5 kHz. The last frequency defines the starting point of the second passband, which ends at around 13.5 kHz. For chirped structures ($\lambda \neq 0$) the levels in each band separate, the bands becoming wider and the gaps narrower with increasing values of λ.

The evolution of a wave packet has been numerically studied using the transfer matrix formalism [6]. The wave packet has been expanded in around 250 normal-mode wave amplitudes for both cases; the locally periodic structure and the chirped structure. After a comprehensive numerical study of the evolution of wave packets for different values, we select two structured beams with λ = 0 and 0.03, whose main results are described in what follows. The structure with λ =0 contains a periodic region consisting of 20 equal cells with lengths of 92 mm joined by rectangular cuboids (the notches) with height 18 mm, width 6 mm and length 8 mm. The structure with chirp parameter λ= 0.03 consists of 20 cells with variable length ℓ_n, determined by

$$\ell_n = \frac{\ell_0}{1+n\lambda} \tag{1}$$

with ℓ_0 = 92 mm

Figure 1 shows the predicted dynamics from the numerical simulations. For the periodic structure (λ=0) it is observed that the wave packet is completely reflected at the interface between the uniform part and the periodic structure of the beam when its central frequency lies in the gap (see Fig. 1a). However, when f_C lies within the passband, Fig. 1(b) shows that the wave packet is partially transmitted and partially reflected each time that it crosses the interface. For the chirped structure with λ=0.03, the wave packet is reflected at different positions in the structured region of the beam depending of the frequency f_C; see Figs. 1(c), 1(d), 1(e), 1(f). This behaviour represents the mechanical rainbow trapping effect.

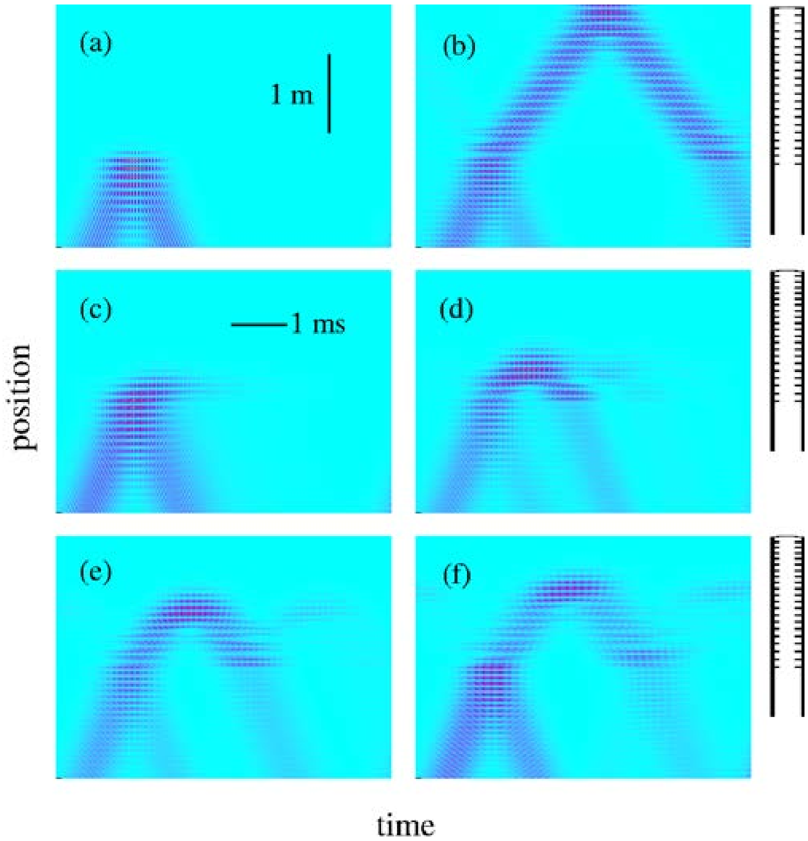

Figure 1. Calculated dynamics of the wave packet. Results are obtained using the transfer matrix method. The position of the wave packet is given as a function of time. Evolution of the wave packet within the beam with a locally periodic structure (λ=0) and central frequencies lying in (a) the gap f_C=8 kHz; (b) in the band, f_C=11.5 kHz. For the beam with a chirped structure (λ=0.03) the evolution is given for the following central frequencies: (c) f_C=9 kHz, (d) f_C=10kHz, (e) f_C=11kHz and (f) f_C=12 kHz.

3. Experimental results and discussion

The theoretical predictions have been corroborated experimentally. We have employed an experimental set up in which two beams with the same geometrical characteristics than that employed in the simulations were constructed and characterized. In addition to the chirped aluminium beam, the setup consists of signal generator, a high-fidelity audio amplifier, an electromagnetic-acoustic transducer (EMAT) and a 500V laser Doppler vibrometer. In brief, torsional waves are generated in the uniform part of the beam are sent to the chirped part. The waves traveling in the opposite direction are almost completely absorbed using a homemade passive vibration isolation system which dissipate efficiently the waves for frequencies higher than 1500 Hz. The excited Gaussian wave packet has a spatial width of 0.875 m in the uniform region of the beam. Figure 2(a) and 2(b) show the recorded dynamics of a Gaussian wave packet propagating in the periodic structure. The wave packet with an initial width of 0.5 ms has a central frequency, f_C, located at two different frequencies within the gap and inside the passband. The width used in the time domain implies than the wave packet has a spatial width of 0.875 m in the uniform part of the beam since the velocity of the torsional waves in the beam is c = 1750 m/s. When its central frequency f_C lies within the gap, it is observed how the wave packet is completely reflected at the interface between the uniform part and periodic structure of the beam [see Fig 2(a)]. However, when f_C lies within the passband [see Fig. 2(b)] the wave packet is partially transmitted and partially reflected each time that the packet crosses the interface. Notice the good agreement observed between these experimental data with the numerical simulations reported in Fig. 1(a) and 1(b).

This agreement is also extended to the appearance of the rainbow trapping effect, which is observed in the data shown in Figs. 2(c), 2(d), 2(e) and 2(f). This effect can be explained in terms of the locally periodic structure using the independent rod model [7] as follows. In this case the system can be considered as a chirped mechanical crystal in which there is a local variation of the bandgaps along the structure. Therefore, the wave traveling inside this quasi-periodic crystal is gradually slowing down, as the wave frequency which is propagating is approaching the “local" bandgaps [3].

4. Conclusions

In summary, we have reported numerical simulations and measurements showing “rainbow trapping” of torsional waves propagating in a chirped quasi-one-dimensional elastic structure. The case of no chirp, corresponding to a periodic system, has been also reported for comparison purposes. The theoretical and numerical predictions of transfer matrix and independent rod model show a good agreement with experimental data. As a potential application we can foresee the control of torsional waves propagation in metallic rods, where their penetration length can be controlled using the rainbow trapping effect. Moreover, our work leads us to conclude that other well phenomena in quantum mechanics and photonics can be attainable also for mechanical waves, opening the possibility of vibration control in automotive, building and aeronautic industries.

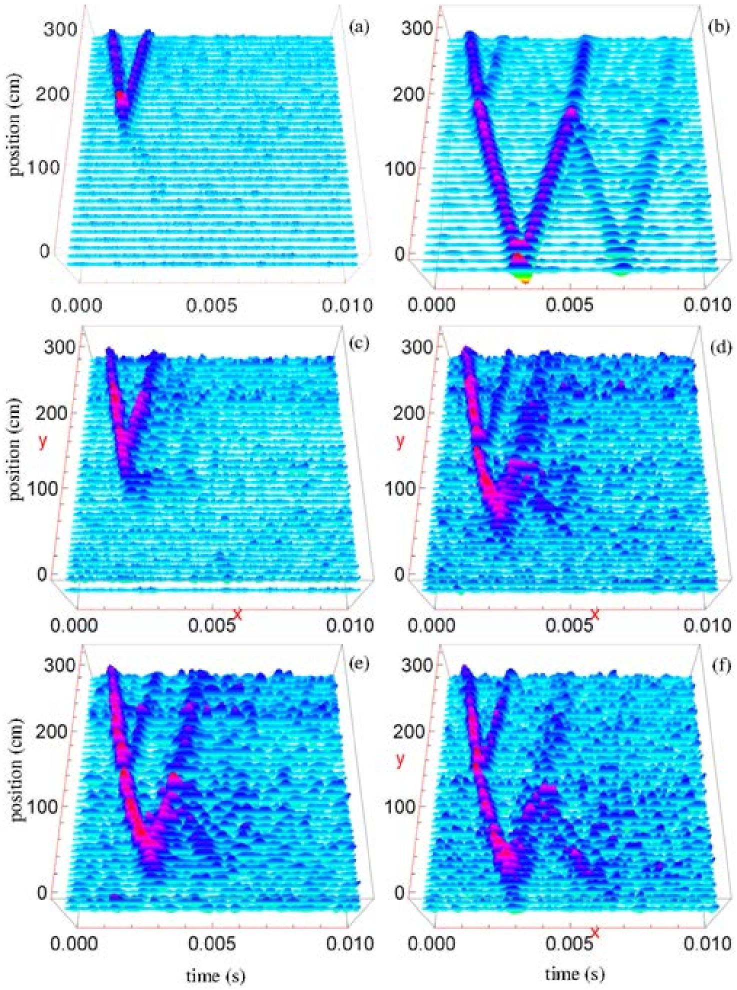

Figure 2.- Measured dynamics of the wave packet using the same parameters than in Figure 1. Notice that the beam here is oriented in the opposite direction. In other words, the structured part is in the right-hand side of the beam. Now, the propagation goes from the left (upper part in this plot) to the right (lower part in this plot).

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