

## ACTIVE ATTENUATION OF BENDING WAVES IN BEAMS

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### 1. INTRODUCTION

The attenuation of bending waves may be achieved by different means. Like other wave types, especially guided waves in fluids, their propagation can be affected by obstacles (e.g. blocking masses or elastic interlayers), changes in material and cross-section or corners and branches.

But the control of wave propagation can be equally realized by active means, that is by the application of additional exciters. This may be easily illustrated by the fact that any changes in the amplitudes of propagating waves may be viewed as the superposition of additional waves, whose amplitudes correspond to the differences related with these changes.

Active control of bending waves may be tackled from two different points of view which represent well established areas of application: active control of mechanical vibration fields and of hydrodynamic (acoustic) wave fields. In spite of the fact that these differ only conceptually, they have developed almost independently from each other and hardly made reference to each other.

Starting from airborne acoustical wave fields, the control of bending wave propagation may be formulated as a generalization to other media, which in general are dispersive. On the other side, having in mind the vibrational behaviour of structures, the control of wave propagation is aimed to suppress modes of vibration by breaking off the internal mechanical feedback which is a necessary condition for the occurrence of any modes.

### 2. CONCEPTS FOR ACTIVE CONTROL

Any bending wave or vibration field may be equally interpreted as a superposition of waves with different amplitudes propagating in different directions as well as a superposition of different modes of vibration. It was the latter approach which presumably was undertaken first in a systematic way. In control theory, theoretical investigations on the control of distributed parameter systems mostly favoured systems whose dynamical behaviour could be described by Kirchhoff's theory of thin plates or - in the one-dimensional case - by the Euler-Bernoulli equations. Early references may be found in (/1/).

These first theoretical studies as well as early experimental realizations (/2/) were based on a modal approach. But since 1981 approximately, growing emphasis was laid on the wave propagation approach (/3/-/11/). Starting from comparable studies in airborne sound applications (/12/), the theoretical possibilities of active control of bending waves, in particular their reflection and absorption, and the related power flows were investigated. If applied at the end of finite beams (/8/, /10/), secondary forces may serve as modal boundary controllers as theoretically described in (/1/) and (/13/).

## ACTIVE ATTENUATION OF BENDING WAVES IN BEAMS

The modes of finite structures may be suppressed by absorbing elements anywhere in the propagation path, which forms a mechanically closed loop and as such is responsible for the existence of free modes. Powerful modal control thus may be achieved as a byproduct of wave propagation control. Impressive simulation results for the vibration of strings based on optimal controllers may be found in (/7/) while the corresponding generalization for bending waves is described in (/14/).

Besides the concept of modal control, which can be realized by wave propagation control, other concepts were established to achieve active vibration control: direct control of the power flow (/15/) and impedance control (/16/). The latter usually is aimed to realize by active means prescribed impedances which act on the structure and thus influence the propagation of waves. In a special case, the maximization of the absorbed power leads to the realization of a matched impedance.

The implications of this approach are - in the case of bending waves - hardly to determine. Due to the fact that the application of impedances not only redistributes the power flow but also may influence the power introduced by primary sources, local considerations based upon local impedances do not necessarily allow conclusions on the global behaviour. As an example it may be stated that the power absorbed by secondary sources may be exceeded by the amount of power which is additionally introduced by the primary forces. A thorough treatment of such power considerations is given in (/17/) for typical examples.

For bending waves, this general statement is supported by the fact that energy may also be transported by the superposition of two near fields which, on the other hand, may be of no interest at all in certain applications. In addition, direct realizations usually only deal with forces and their related impedances. This disturbs the equilibrium between the two coupled pairs of field quantities and finally leads to a conversion of power related with the bending moment to power related with transversal forces.

The great advantage of impedance control lies in the fact that its realization overcomes any need to model the structure under consideration. But this advantage has to be seen together with its shortcomings which essentially can be summarized in the difficulty to get local criteria for the global vibrational behaviour.

An approach more directly related to this global behaviour is given by the control of power flow. Under certain limitations, the intensities in beams and plates can be measured and thus serve as a basis to determine the signals needed to drive secondary sources. But again, as power flow is a consequence of the relations between different amplitudes, its evaluation may leave relevant details out of consideration. On the other hand, as the direct control of the field amplitudes inherently includes the control of power flow, it incorporates a very general concept of active control of wave propagation which is closely related to other formulations of the vibration control problem.

In the following, this paper therefore focuses on the concept of influencing bending wave propagation in beams by controlling certain amplitudes of these waves. As stated before, active control is equivalent to the selective superposition of partial waves with controlled amplitudes. In this context, the term partial wave shall include near fields although these are not really propagating.

# ACTIVE ATTENUATION OF BENDING WAVES IN BEAMS

## 3. SELECTIVE CONTROL OF BENDING WAVE AMPLITUDES

Selective superposition requires selective measurements of the amplitudes to be controlled. Thus, the application of multipoint measurements as well as the multipoint excitations are necessary prerequisites. It can be shown (/17/), (/18/) that the measurement of acceleration in at least four points enables the evaluation of all amplitudes within a limited frequency range. This frequency range is a function of the geometrical arrangement of the measurement points and can be enlarged by augmenting their number. This similarly holds for selective excitations by different point forces. Detailed information on theoretical possibilities including aspects of their realization as well as selective measurements and their relation to intensity or power flow measurements may be found in (/17/) and (/18/).

This paper finally will restrict itself to the presentation of one example of active attenuation of a propagating bending wave by one force. Fig. 1 shows the overall arrangement aimed to realize the active reflection of an incoming bending wave without nearfield.

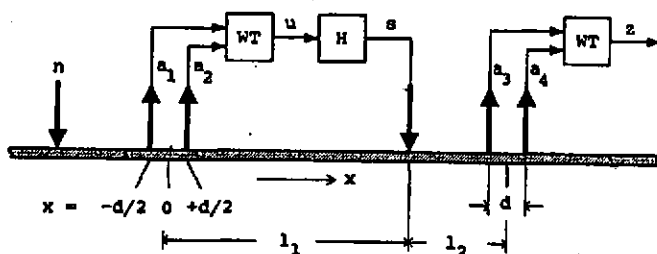


Fig. 1

A primary source "n" excites a beam and generates a wave propagating in positive x-direction together with a nearfield. At  $x = 0$  two accelerometers with distance  $d$  give two signals,  $a_1$  and  $a_2$ , which are fed to the first stage WT of the signal processing unit. The task of WT is to determine selectively the relevant amplitude of the forgoing wave. The dimensions of the experimental setup could be chosen such that above 100 Hz any nearfields were negligible at  $x = 0$ . Thus, WT has to separate the two waves propagating in positive and negative x-direction.

As  $u$  should represent the amplitude of the forward going wave, this separation should be carried out according to the formula

$$A_+(x) = \frac{a_1 - a_2 e^{-jkd}}{1 - e^{-j2kd}} e^{-jkd} \left( \frac{1}{2} + \frac{x}{d} \right) \quad (1)$$

Given the distance  $d$ , the only unknown in equation (1) is the wavenumber  $k$ . Thus, the system identification of WT reduces to the experimental determination of the frequency dependence of this quantity. It

# ACTIVE ATTENUATION OF BENDING WAVES IN BEAMS

could be shown under most variable field conditions that the wavenumber accurately could be determined from three-point measurements (/17/). Given  $k$ , the real time realization of equation (1) can be viewed as a filter design problem.

The second stage  $H$  of the signal processing unit has to utilize the information on the forgoing wave's amplitude as contained in  $u(t)$  to supply a signal  $s$  which is able to control the secondary source in the intended way. The intention in the experiment described here was to reflect the incoming wave by the secondary source at  $x = l_1$ .

The quantitative registration of this reflection was achieved by another two-point measurement at a distance  $l_2$  behind the secondary force. The system identification process for  $H$  has to make sure, that the amplitude of the positive going wave at  $x = l_2$  vanishes. It turns out, that  $H$  thus can be composed by a set of five transfer function measurements. Suitably combined, they give the relation between  $u$  and  $s$  such that the secondary field at  $x = l_1 + l_2$  negatively equals the primary field at the same location.

Again, after being qualified by these measurements, the realtime implementation reduces to a filter design problem. Figure 2 shows the resulting structure of the signal processing unit, which was realized on a TMS 32010 signal processor.

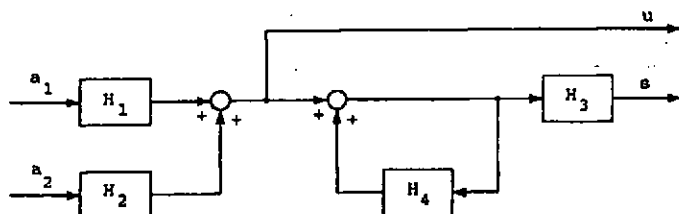


Fig. 2

## 4. EXPERIMENTAL REALIZATION OF AN ACTIVE REFLECTOR

The appropriate filter design was complicated by two facts. Essentially, the transfer functions involved have to compensate for the transfer characteristics of the electromechanical transducers and to model the transmission of waves within the beam. For bending waves, the latter results in great difficulties because of the arbitrarily high propagation time at low frequencies. It was shown in (/17/) and (/19/) how the mid-frequency behaviour of transfer elements may be optimized by allowing - under certain restrictions - arbitrary behaviour at low and high frequencies. Based upon this design procedure, it was possible to achieve a good performance of the active reflector. Fig. 3 shows the measured transmission losses with (a) and without (b) active reflector for an experimental setup with the following specifications.

$$l_1 = 165 \text{ cm}, \quad l_2 = 65 \text{ cm}, \quad d = 10 \text{ cm}$$

# ACTIVE ATTENUATION OF BENDING WAVES IN BEAMS

The frequency band of interest was limited by

$$f_1 = 0.1 f_T, \quad f_2 = 0.4 f_T$$

where  $f_T$  represents the sampling frequency which - in this experiment - was chosen to a value of 3.2 kHz. The cross-section of the beam, which was suspended on light cables, was given by a rectangle whose edges had a length of 1 cm and 5 cm respectively.

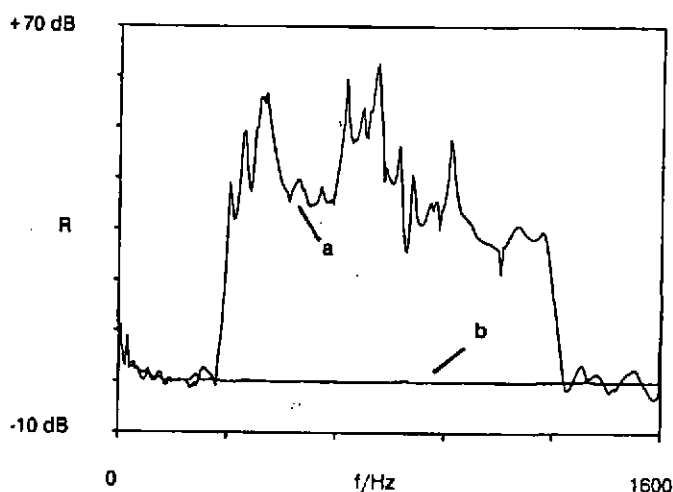


Fig. 3 : transmission loss  $R$  with (a) and without (b) active reflector

The transmission loss was evaluated according to the formula (/17/)

$$\tau = \frac{1 - H_{43} e^{-jkd}}{1 - H_{21} e^{-jkd}} H_{31} \quad (2)$$

$$R = -20 \log \tau, \quad (3)$$

## ACTIVE ATTENUATION OF BENDING WAVES IN BEAMS

where  $H_{jk}$  represents the measured transfer function between  $a_k$  as input and  $a_j$  as output signal. In the mean, the transmission loss has a value of 35 dB which means that approximately 99.97 percent of the incoming power is rejected by the secondary force.

Fig. 4 again shows the measured transmission loss with active reflector together with the predicted curve on the basis of the filter performance. The differences between measurement and prediction can be

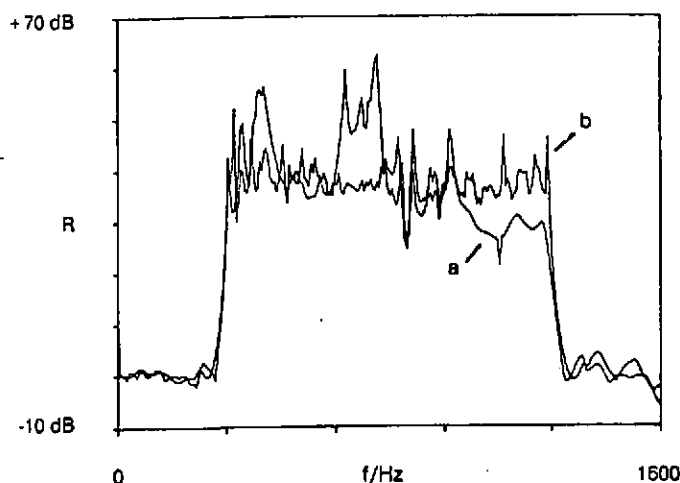


Fig. 4 : measured (a) and predicted (b) transmission loss  $R$  with active reflector

explained by different phase errors related with the two point measurements before and behind the secondary force.

### 5. CONCLUSION

It can be seen that the procedure described here is able to come up with good practical results for the active control of bending wave propagation. Its shortcoming not to cope with low frequencies may be overcome by deliberately choosing low sample frequencies. The good agreement between prediction and measurement, which equally was obtained for active absorption at the free end of beams (/10/), gives rise to the assumption that according to appropriate simulation results (/17/) the simultaneous active control of different bending wave amplitudes could be realized with comparable success.

