ANALYSIS OF GEARBOX VIBRATION IN TIME DOMAIN

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1. INTRODUCTION

Frequencies produced in a gearbox, which has all its components running in a cyclic fashion, have values, which are equal to a real multiple of shaft rotational speed. The non-integer multiples correspond to faults of rolling elements of bearing, looses in housing, or parametric vibrations, but their occurrence is relatively rare. The most common components have the frequency given by an integer multiple of the base frequency of the shaft rotation. They can usually be broken down into a combination of the following effects [1]:

a) Low harmonics of the shaft speed, which are due to additive impulses repeated once or twice per a revolution.

b) Harmonics of the base toothmeshing frequency and their sidebands, which are due to deviations from the ideal tooth profile and modulation of the otherwise uniform toothmeshing vibration. The sideband components are spaced around the toothmeshing frequency with a spacing usually equal to the appropriate gear rotational frequency. It can be noticed that sidebands are the most complicated intermodulation components in the vibration spectrum.

c) Ghost components, which frequencies are equal to integer multiples of the gear rotational frequency, like the harmonics of toothmeshing frequency but corresponding to a different number of teeth to those actually cut. These components are due to the errors in the teeth on the index wheel of the gearcutting machine. The frequency of the ghost component corresponds to the number of teeth on the index wheel.

One may conclude, that the gear tooth contact conditions can be described by spectrum components, whose frequencies are the integer multiples of the rotational frequency of this gear. This spectrum corresponds to the periodical continuous signal in time domain. The representation in the time domain gives the vibration signals produced by individual teeth. Since vibrations produce an acoustic noise, this new kind of measurement can be used for the design of quiet gears.

The analysis instrumentation used for the results presented in this paper comprises an FFT analyzer (Brüel & Kjaer Type 2034) interfaced to a computer. The vibration signal was measured by an accelerometer fixed to the surface of the gearbox housing. A photoelectric probe Brüel & Kjaer Type MM 0012 was used for triggering.

2. THEORETICAL BACKGROUND

This section discusses theoretical principles of tooth-by-tooth vibration signal measurements, using the simplest signal, which is composed of a single frequency sinusoidal component and an uncorrelated random (white) noise

$$x(t) = A.\sin(\Omega.t + \Phi) + e(t) ,$$

where t is time, A is an amplitude, Ω is a constant angular frequency, Φ is the initial phase angle at time zero, and e(t) is the white noise. Let this time signal be sampled with the frequency f_s (the reciprocal of the time interval T_s between samples). Further it is supposed that the set of the samples is divided into time records with the same length of N samples. As the record length is a fixed number and a power of two (N = 2048 for B&K Type 2034), the sample frequency is chosen in such a way that the samples are taken into one record from more than one revolution of the gear, that a toothmeshing response is chosen to be measured. It is supposed as well that the capture of the j^{th} time record is started at time $t_j = 2\pi.h.j/\Omega_o$, where Ω_o is a constant angular frequency of the shaft rotation and h is an integer. The previous equation shows that time records begin at the same position of the gear or the shaft, on which this gear is fixed. It means that the time records are triggered synchronously with the shaft rotation. The measurement and processing of one record take the time interval, which is h times longer than the time needed for one revolution. Thus, the continuos time signal is sampled at time $t_{ij} = t_j+i.T_s$, i = 0,1,...,N-1, j = 1,2,... A signal enhancement is an average of the time records (1), i.e.

$$x_i = \frac{1}{M}\sum_{j=1}^{M} x(t_{ij}) \quad , \quad i = 0,1,\ldots,N-1 ,$$

where M is the averaging number. According to the previous formula, the following equation for the average of the time signal can be formulated

$$x_i = \frac{1}{M}\sum_j (A.\sin(2\pi \frac{\Omega}{\Omega_o}.h.j+i.T_s+\Phi) + e(\frac{2\pi}{\Omega_o}.h.j+i.T_s)) .$$

The root mean square (RMS) of the average of the uncorrelated random values is proportional to the reciprocal of the square root of the averaging number. It is sufficient to suppress the random noise by averaging if the number M is about 150. The sum of sinusoidal components depends on the ratio between Ω and Ω_o. If Ω is the integer multiple of Ω_o, including equality, then the following equation can be obtained

$$\frac{1}{M}\sum_j A.\sin(2\pi.k.h.j+i.T_s+\Phi) = A.\sin(i.T_s+\Phi),\ k = 1,2,...$$

It should be noted that all the harmonics of the base angular frequency Ω_o are kept in the spectrum after averaging. In this case the trigger signal is taken from the shaft, which is rotating at the same angular frequency Ω_o. If Ω is the angular frequency of another gear, and Ω is different from Ω_o, then the previous sum depends on the gear ratio between the mesh gears and the value of the integer h. The gear ratio can be expressed as a result of dividing one integer by another, i.e. $\Omega = \Omega_o.m/n$. If the product of m and h is not any integer multiple of n then after the substitution of Ω the limit of the sinusoidal component sum is zero for $M \rightarrow \infty$ in general sense. The zero sum can also be reached if the averaging number M is equal to the integer multiple of n. On the other hand, if the product of m and h is the integer multiple of n, then the averaging cannot remove the sinusoidal component with the frequency Ωo and its harmonics. It can happen for m = n, m = 2n,.., or n = h. The case mentioned above deals with the toothmeshing frequency in the vibration spectrum. The conclusion can be extended to the real ratio between Ω and Ω_o. The signal enhancement reduces not only the noise but also the periodical component with the base frequency, which is not any integer multiple of the frequency of the trigger signal.

3. TIME SIGNAL FILTRATION

The signal enhancement, using the average of the time records, makes it possible to isolate vibrations caused by meshing gears mounted on a specific shaft from vibrations of gears, which are mounted on another shaft, rotating at the different speeds. The spectrum of this signal contains only the harmonics of the base frequency of the trigger signal. If there is only one engaged gear on the shaft, then the synchronously averaged time record can be considered as a response of the toothmesh of this gear. If there are two or more engaged gears on the same shaft and at least two of them are under mechanical load, then the averaged record is the result of the toothmeshes of all these gears and cannot be attributed to any specific gear. An example of the trace of acceleration versus time produced by two gears with different numbers of teeth (29 and 34) under torque 900 Nm, rotating at the same speed (1500 RPM), is in Fig. 1. The time signal is synchronously averaged over 150 cycles. An amplitude modulation effect is caused by the beats of two sinusoidal components with close frequencies, as it is evident from the frequency spectrum. Information about the toothmesh of both gears in this enhanced time signal is distorted.

On condition that gears have different numbers of teeth and the interference of sidebands from adjacent toothmeshing frequencies can be ignored, it is possible to separate the tooth-by-tooth vibration patterns using a very selective band-stop filter. The problem is that the toothmeshing frequencies are very close, and it is necessary to remove not only the base frequency from the spectrum but its harmonics as well. It is not possible to carry out this filtration in the time domain. An efficient way of performing the filtration of some components of the frequency spectrum is to transform the time signal into the frequency domain, and after removing these components from the spectrum, the "frequency" signal is transformed back into the time domain. For this reason, the enhanced discret-time signal is considered as one period of the unlimited periodical signal. Hence it is possible to use the Fast Fourier Transformation (FFT) and the Inverse FFT (IFFT). Both the FFT and IFFT operators transform a vector of limited length into a vector of the same length. As noted above the record takes the time interval, which is longer than one complete revolution of the gear, because the frequency spans of the analyzer are usually equal to a finite number of values

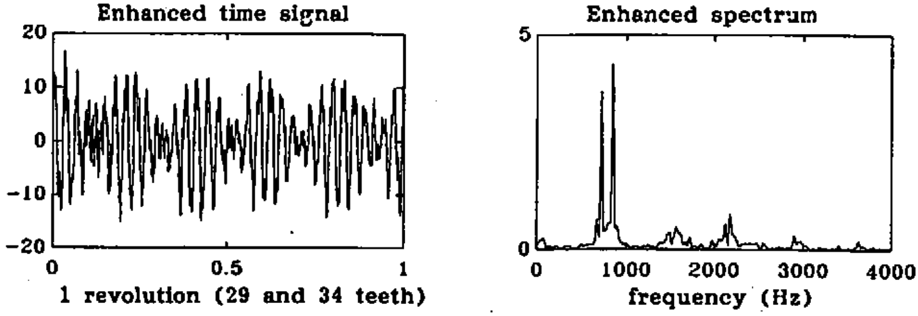

Figure 1 - Typical enhanced time signal of acceleration

creating a geometric sequence. The components of the vector FFT(x) belong to frequencies, which are spaced by the reciprocal of the time period. Thus, the time record must be shortened to the time interval of one revolution of the gear and resampled to reach the same number of samples as is contained in the original time record. The components of the vector FFT(x) are denoted by X_i, i = 0,1,...,N-1. The first component X_0 is the main value of the time record. Because the values of the time record x are real, the vector FFT(x) is conjugate symmetric [3], i.e. $X_i = X_{N-i}^*$, i = 1,2,...,N-1. Therefore both the i^{th} and $(N-i)^{th}$ components of FFT(x) correspond to the i^{th} harmonic of the base frequency of the rotational speed. The real and imaginary part of these components must be set to zero (X_i = 0, and X_{N-i} = 0) so that the i^{th} harmonic can be removed from the spectrum. The Inverse FFT of this vector FFT(x) gives the time record without the i^{th} harmonic of the base frequency in the spectrum. There are two methods for isolating the selected components of the spectrum, involving the stop or pass-band filtration, respectively, carried out by a computer. An example of the stop-band filtration is in Fig. 2. The harmonic components of the toothmeshing frequency of the 34-tooth wheel with their one pair of the sideband components are removed from the spectrum, because the toothmeshing frequency of the 29-tooth gear is analysed. Another example in Fig. 3 shows the effect of the pass-band filtration. The harmonic components of the toothmeshing frequency of the 29-tooth gear with their three pairs of sideband components are filtered out and the other components are removed from the spectrum. Both the filtered time signals give the valuable diagnostic information about the condition of toothmesh of the 29-tooth gear.

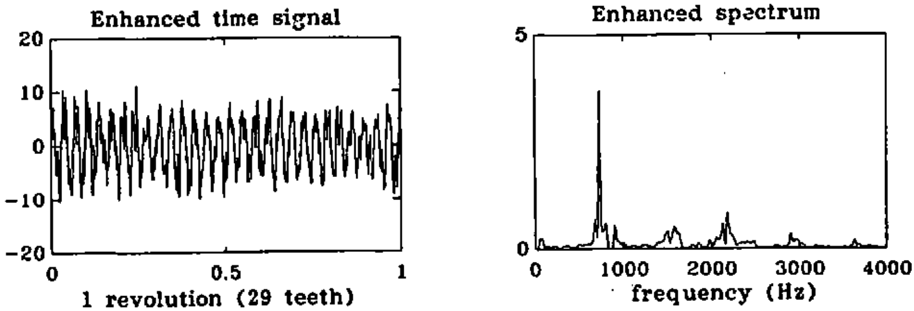

Figure 2 - Enhanced time signal after band-stop filtration

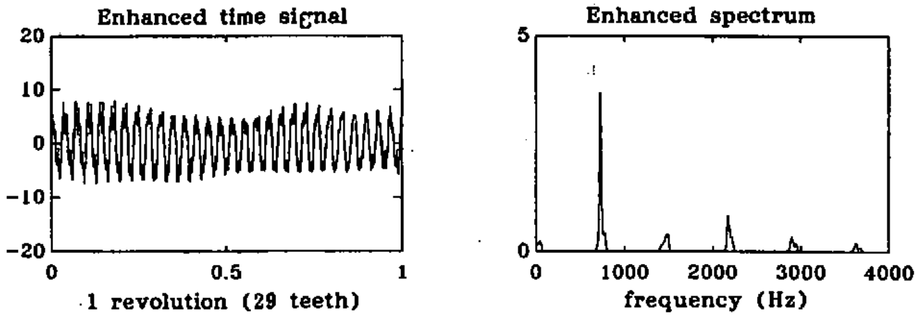

Figure 3 - Enhanced time signal after band-pass filtration

4. GEARBOX VIBRATIONS

This section presents some measurements of the gearbox which were made to reduce the transmission noise level. The toothmesh of gears under load was studied on a back-to-back test ring. Some measurements were aimed at toothmeshing of an input-shaft helical gear with 29 teeth to test tooth surface modification. An accelerometer was fixed on the surface of the gearbox housing near the input-shaft bearing which is the closest to the 29-tooth gear. As the input shaft of the gearbox is driven by the output shaft of an auxiliary gearbox with the 34-tooth gear, the component, with a frequency corresponding to this gear, was found in the acceleration spectrum. Therefore this component had to be removed using the same procedure as it was described above. Fig. 4 shows tooth-by-tooth acceleration signals with only 6 teeth in 29 of the input-shaft gear rotating at 1900 RPM. The signals on the left hand side (Gear A) in Fig. 4 differ from ones on the right hand side (Gear B) in the way of the tooth profile modification. In Fig. 5 it is shown how the RMS of acceleration depends on the load. It can be noticed that the design load is the torque of 1200 Nm. As it is evident, the RMS of the tooth-by-tooth vibration pattern
corresponding to the gear A is greater than the RMS corresponding to the gear B at the designed load. It means that the noise which is produced by the gear A is less than the noise produced by the gear B. Of course, the tooth profile of the gear A is modified while the profile of the gear B is not. The surfaces of both gears were improved by grinding but using different types of machines.

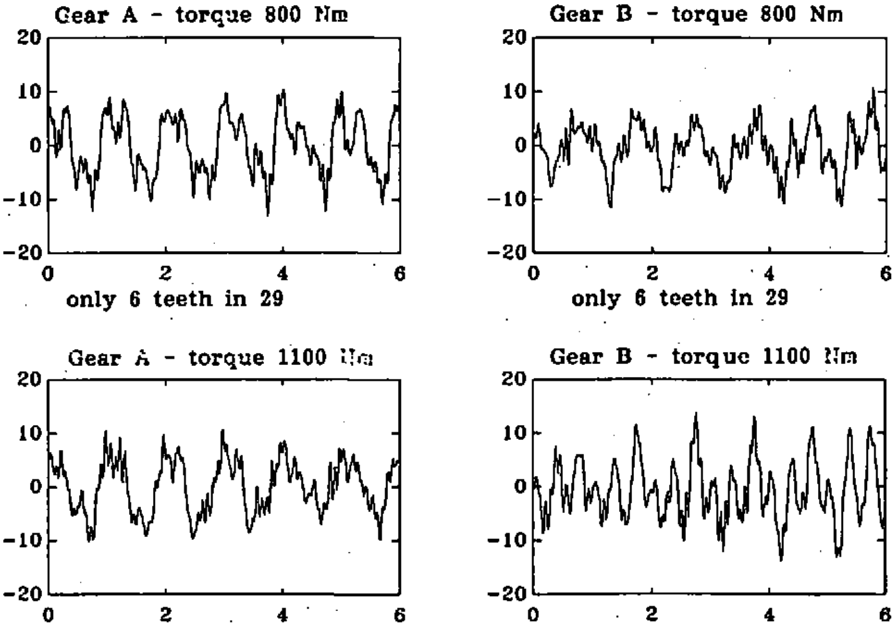

Figure 4 - Acceleration signals corresponding to different levels of load and types of gears

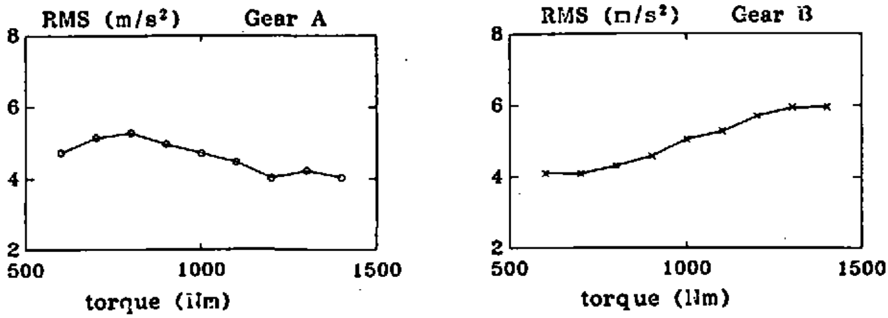

Figure 5 - Effect of load on RMS of acceleration

6. CONCLUSION

It has been shown that it is possible to measure and analyse tooth-by-tooth vibration signals in time domain, involving signal enhancement in an FFT analyzer and filtration in a computer, on condition that the toothmeshing frequency of this gear is different from that one of the other gears. The signal enhancement suppresses not only the noise but also the components, which are, due to the toothmeshing of gears, rotating at a different speed. The filtration, using the FFT and IFFT, supplements the signal enhancement to suppress the components, which are caused by gears having different numbers of teeth and rotating at the same speed as the gear, which was chosen to be analysed. The tooth-by-tooth vibration signals give a more clear indication of the gear tooth contact than the spectrum or cepstrum. This new kind of measurement can be used to measure gear quality in the gearboxes and to improve the designs of the gearboxes and especially gears.

7. REFERENCES

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