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SPRUCE FOR SOUNDBOARDS: ELASTIC CONSTANTS AND MICROSTRUCTURE

J. Woodhouse

Cambridge University Engineering Department,
Trumpington St., Cambridge CB2 1PZ

INTRODUCTION

For many problems in musical-instrument acoustics, a good understanding of the elastic and damping behaviour of wood, with and without surface treatments, is needed. As has been explained elsewhere [1,2], the simplest model of wood (as an orthotropic solid) requires measurement of nine elastic and nine damping constants. This is a difficult task, and some approaches to it are reviewed here.

As a starting point, we consider the simplest problem of interest, that of a flat plate cut from the tree in such a way as to contain one of the principal axes (called L(ongitudinal), R(adial) and T(ransverse)). Such a plate needs only four elastic (and four damping) constants, for which we shall follow the notational conventions of refs. [1,2] and call the elastic constants D_1, D_2, D_3 and D_4. Some data is published for all nine elastic constants for certain woods [3,4], and we first show the values of the constants D_1-D_4 deduced from those data, for the important cases of plates which are not precisely quarter-cut. Specifically, we consider plates which depart from exact quartering <u>either</u> by having tilted annual rings <u>or</u> by having tilted grain lines, but not both. In Figures 1(a) and (b) we show the four elastic constants plotted as a function of ring tilt angle (measured in the TR plane) and grain tilt angle (measured in the LT plane) respectively, using a set of nine elastic constants taken from Table 2 of ref. [3], which we take to be a typical set of spruce values. Because of the large spread of values of these constants, a logarithmic vertical scale is used in both cases. The values for zero angle are the same in each case, and correspond to the ideal quarter-cut plate, with D_1 in the L direction and D_3 in the R direction.

These graphs show the sensitivity of some of the plate constants to grain and ring angles. Most important are the very strong variations of D_2 and D_3 with ring angle - one reason why makers pay such careful attention to ring angle [5]. Much of the wood sold for instrument making is not precisely quarter-cut, and we see from Fig. 1(a), for example, that D_3 can change by a factor of two or so with a ring angle of only 10°.

A BRIEF SURVEY OF MEASUREMENT METHODS

We now examine the various methods of measuring the elastic and damping constants of wood which are to be found in the literature. These can be divided into three broad categories, based respectively on static deformations, low-frequency vibrations and ultrasonics.

<u>Static tests</u>
Static methods are perhaps the first to spring to mind for measuring elastic constants. After all, the textbook definitions of Young's modulus, Poisson's ratio and shear modulus are given in terms of the deformation of the material

in response to static stresses of various kinds. Thus static compression or shear of suitably-shaped specimens could be used to make direct measurements of the constants from their definitions. However, while this is true in principle, in practice it is not easy to carry out such measurements on wood with sufficient accuracy because the magnitudes of the moduli make the elastic strains very small.

This problem is exacerbated by the fact that in the case of wood, we need to work with rather small specimens, a constraint which affects all measurement methods. There are two reasons. First is the spatial inhomogeneity of the material (heartwood and sapwood, and the multifarious growth anomalies that wood is heir to). Second, and more serious, is the fact that the simple orthotropic theory we are using ignores the curvature of the annual rings. To cut samples for testing purposes in the T direction without significant problems arising from curvature requires very small dimensions (recall the extreme sensitivity of the constants to ring angle demonstrated in Fig. 1(a)).

On the other hand, specimens cannot be too small. We are using a continuum theory which ignores the ring structure of the wood. The conditions for this to be a reasonable approximation are not easy to specify succinctly, but the main requirement is that samples should not get so small that they have any radial dimension comparable to the ring spacing. Otherwise large fluctuations in results are to be expected, depending on precisely how much spring wood compared with summer wood is included. This constraint is again unavoidable, for any method, and together with the first one poses a serious dilemma regarding sample sizes, for any measurement method.

A more useful form of static testing than simple compression or shear involves bending or torsion of rods or plates. For a given level of elastic strain, much larger measurable displacements are obtainable this way, with a corresponding improvement in accuracy. Such methods have been used extensively in the past. Hearmon [3] reviews several such approaches, and compares the results with those of other approaches.

For our purposes, we should note two serious difficulties with all static measurements which make it not worth our while to pursue the method in much detail. First, static tests cannot be used to give any of the damping constants. Even if energy loss in the static test were measured, there is no very good reason to expect this to relate closely to the vibration damping. The reason points up the second difficulty with static tests. It has been extensively reported in the literature [e.g. 3,4,6,7] that any static test involving shear strains relative to the principal axes of the wood is likely to exhibit creep. The deformation will continue to increase with time after the load is applied, and the specimen will not in general recover its original dimensions fully when the load is removed. This is the reason for the familiar permanent sagging or yielding of wood under conditions of sustained load. One result of this is that static measurements of shear moduli tend to give consistently lower values than do vibration tests, the precise value depending on the time taken to perform the measurement.

Low-frequency vibration tests
The second category of measurement methods is the broadest of the three.

"Low-frequency vibration tests" means low audio frequencies, and includes the loaded cantilever and torsion-bar vibrations discussed by Hearmon [3], bending-strip vibration tests [e.g. 5], measurements on plates such as our own [2], and so on. In all of these cases, the procedure is essentially the same. One or more normal modes of vibration of the wood sample, perhaps with some added masses or constraints, are studied, to deduce the frequency, damping or Q factor, and perhaps the mode shape. To be useful, such measurements must be made on vibration modes which are well understood theoretically. One can then relate the measured frequencies and Q-factors to the desired elastic and damping constants of the material.

To determine the complete set of elastic and damping constants by studying low-frequency vibration modes, one obviously needs to measure at least nine different modes. For cross-checking one needs more. Ideally, one would like to think of sample geometries such that each constant is deduced directly from one measurement. In practice, however, this simplicity cannot be achieved for all the constants. In most cases, the frequency measured depends on a combination of the elastic constants (and similarly the Q-factor depends on a combination of the damping constants [8]). One then has to solve an "inverse problem" to deduce the separate constants from the measurements.

Certain commonly-used sample configurations avoid the difficulties of tricky inverse problems to a considerable extent. The Young's moduli can be measured directly (using strip bending tests or loaded cantilever tests, for example) because one-dimensional thin-beam bending depends only on Young's modulus. Similarly, the three shear moduli can be deduced almost independently from torsional vibration tests. A rod of some uniform cross-section is cut parallel to one of the principal axes of the material (subject of course to the sample-size constraints discussed above). One end is clamped in some suitable way, and a mass having a large moment of inertia is attached to the other end. The low frequency torsional oscillations of this system are then observed. The frequencies are governed by a combination of two of the shear moduli alone (in two planes, each perpendicular to the rod's cross-section). The precise combination depends on the cross-sectional shape of the rod.

However, for our full problem of determining all the constants, two difficulties are still evident. First, bending and torsion tests about principal axes still leave us with three of the nine constants to be measured. These are, for example, three of the Poisson's ratios. We have already noted that static methods are not desirable for the elastic constants, and are unlikely to be possible for the damping constants. Second, it would be better for experimental efficiency and economy of sample material if some way could be found to measure all the constants from a minimal number of separate samples, rather than having to cut a range of different samples for measurements of the different constants. As noted above, we also want in particular to avoid having to cut any shapes which are large in the T direction. For example, conventional strip-tests for E_T would be error-prone for this reason.

It would perhaps be useful to pause here and summarise our "shopping list" of features which the elusive ideal measurement method should offer. It should involve the cutting of a few - say no more than two or three - specimens of wood. These will have all dimensions small compared with the radius of curvature of the annual rings, but large compared with the ring spacing; in other words of the order of a centimetre or two. The shapes should not present great technical difficulties in the cutting process (such as, for example, a requirement for exact spheres or hollow shells would pose). The specimens would then be used in such a way that they can be tuned to produce low vibration modes at a wide range of audio frequencies, involving all nine of the independent elastic constants in such a way that, at each frequency separately, we can solve the inverse problem to give good accuracy on all the elastic and damping constants. These should, of course, be in agreement with results obtained by other reputable methods such as those discussed above. All of this makes a tall order, but it is as well to have the ideal clearly in mind when assessing practical possibilities.

None of the low-frequency resonance approaches known to us from the existing literature come close to satisfying this demanding shopping list. We now offer some speculations about a method which might possibly come close to doing so. These are untested ideas, and are intended more as a spur to further thought and experiment than a proven recipe for solving the problem. The hardest part of the problem as stated is perhaps the requirement for tunability of the frequencies of resonances over a significant part of the audio range without violating the stringent conditions on sample size. One solution to this might be make measurements off resonance, using forced vibration. A machine is apparently made by DuPont to do this for simple bar tests, but it is well beyond the means of the violin-making community! Resonance methods do not require high technology, and for such a method the only simple but practical way to achieve tunability seems to involve adding masses to the wooden specimen. The simplest configuration involves a cubic or rectangular block of wood with blocks of metal firmly attached to a pair of opposite faces - in other words a metal/wood/metal sandwich. Varying the masses of the metal blocks (which need to be effectively rigid in comparison with the wood for a tractable theory) could allow the lowest few resonances of the composite body to be tuned over a useful range.

These low resonances will fall into four distinct types - compressional modes, two orientations of shear modes, and twisting modes. Thus measurements on one configuration will yield information about several combinations of the elastic and damping constants of the solid wood. The hope would be that by using "meat" for the sandwich cut in more than one orientation relative to the principal axes of the tree, one might be able to deduce all the constants by solving the appropriate inverse problem. In terms of economy of specimens, cubes of wood have a lot to recommend them. Provided the metal blocks could be fixed in a way which can be undone without damaging the wood, a single cube could be used in all three orientations with the same set of metal weights. There is thus a faint possibility that one cube in principal axes might be sufficient to yield the full set of constants, elastic and damping. A stronger possibility is that two cubes, one principal and one skew, could do the job. Some careful thought about both theory and experimental technique is needed before it becomes clear whether this hope is justified. Experimentation with computer simulations is needed to assess the various possibilities, and indeed to evaluate the feasibility of the suggested method as a whole.

Ultrasonics

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The use of ultrasonic testing to determine the elastic constants of wood is a relatively recent development - see e.g. [9,10]. The method is simple in concept, and shares some advantages with the speculative method outlined above. Small cubes of wood are cut at various orientations to the principal axes, and transmitting and receiving transducers are attached to a pair of opposite faces. A short pulse of high-frequency vibration is then transmitted through the sample, and its travel time measured.

By varying the type of transducers used, both compressional and shear wave speeds can be measured on the specimen. Provided the wavelength is short compared with the sample dimensions, the waves can be assumed to be plane. With that assumption, the measured wave speed can be used to deduce an appropriate stiffness of the material. If enough orientations are measured to give the complete 6×6 stiffness matrix (see Hearmon [3,4]), this can be inverted to give the compliance matrix [op. cit.] from which the Young's moduli etc. may be deduced.

The method just described is attractive in many ways, and is the natural extension to wood of a method which works extremely well for homogeneous materials like metals. However, there are two likely drawbacks in applying it to wood. First, in common with static tests described earlier, it cannot be used to measure damping constants since it involves frequencies well outside the audio range. It would be surprising if the damping "constants" were in fact constant over this wide frequency range.

The second problem relates to the measurement of elastic constants, and is a new manifestation of the sample-size constraints mentioned earlier. It is most clearly illustrated for the case of compressional waves travelling in the R direction, perpendicular to the annual rings. We are using a continuum theory which ignores the ring structure of wood, but if one looks in more detail, of course the elastic behaviour and density of wood varies roughly periodically with radial distance, because of the rings. The continuum theory can be used safely only if the wavelength is long compared with the ring spacing.

To be safe, the wavelength should be at least four times the ring spacing. Published data [9,11] give typical values of the sound speed in the R direction in spruce of 1400m/s. Thus the highest frequency one could use reliably is about 350kHz for fairly close rings of 1mm spacing, dropping to about 100kHz for ring spacing of 3mm common in spruce used for cello top plates. This is on the low side for conventional ultrasonic testing, where other problems (associated with longer pulse lengths) are beginning to appear which affect the measurement accuracy. Bucur does present some results measured at 100kHz [9], which perhaps represent the most promising line to follow.

MICROSTRUCTURE MODELLING.

We have now surveyed briefly the range of methods available for direct measurement of the elastic and damping constants of wood. However, direct measurement is not the only way of gaining useful insight into the problem, and in this final section we look at what may prove to be a powerful alternative approach. The theory we have been using up to now deduces the existence of nine independent elastic constants from very general arguments about the mirror symmetry of the material in three mutually orthogonal planes [4]. However, these constants are "independent" only to the extent that we use no further knowledge of the structure of wood in building our theory.

If we understood the microscopic mechanics of wood better, we might find useful inter-relations between the nine constants. This could have two important benefits. First, it would reduce the number of degrees of freedom in the problem, so that fewer measurements would suffice. In view of the difficulty of finding reliable ways of measuring all the constants, this would obviously be very useful. However, it would require the inter-relations to be known rather accurately.

The second advantage does not rely so much on accuracy. Even approximate inter-relations backed by physical understanding can provide some valuable prejudices about the results of direct measurements. Since we have repeatedly stressed the difficulties of such measurements, any well-founded expectations we may have about the relative magnitudes of the various constants are likely to be valuable in helping to detect errors in experiments or their interpretation.

The microscopic structure of wood is described in many standard texts, for example Bodig and Jayne [6]. For our present purposes it is fortunate that we are particularly interested in Norway spruce, since this has a rather simple structure (at least in a first approximation to what appears under the microscope). It consists of hollow, rather thin-walled cells which are very long compared with their cross-sectional dimensions. Most of these cells are aligned parallel to the axis of the tree - these are the grain fibres, and are known botanically as tracheids. A small proportion of the cells form medullary rays. For the present purpose ray cells are similar to the main fibres, except that the ray cells are aligned in the radial direction. No cells run in the transverse direction. The walls of all these cells have anisotropic mechanical properties, because they contain "microfibrils" whose orientations vary between cells of different types.

As long ago as 1928, this structure suggested a simple theoretical model to A. T. Price, who analysed it in a paper [7] which is highly significant for our purposes. He ignored the finite length of the cells, and considered them as indefinitely long tubes of constant cross-section. His model thus consisted of close-packed tubes aligned axially, interpenetrated by a small proportion of similar (but not necessarily mechanically identical) tubes running radially, to model the medullary rays. He analyses quantitatively the mechanics of the close-packed axial tubes, and comments qualitatively on the effect of the rays. This second stage of the calculation was later made more quantitative by Barkas [12]. Price devoted the rest of his paper to consideration of the effect of the radial modulation in cell size constituting the annual rings. We will not review all his work in detail, but we will sketch the clear physical understanding he gives of the relative magnitudes of the elastic constants. It seems that this simple model goes a long way toward explaining the measured values, and thus appears to capture some of the essential properties of wood.

A qualitative explanation of the anisotropy of Young's modulus along and across the grain is the most immediate deduction from this model. For stretching along the grain, the individual tubes must be stretched. For stretching across the grain, on the other hand, the tube walls need only bend, approximately inextensionally, giving a much lower modulus. Price suggests that the smaller degree of anisotropy between the two cross-grain directions (T and R) is explained by the medullary rays. E_T is always smaller than E_R, usually by about a factor of two. The idea is that E_T represents just cell-wall bending as just described, whereas E_R involves extra stiffness from stretching of the small proportion of ray cells.

To make these ideas quantitative, Price analysed the simplest case of tube geometry. He imagined the tubes to have circular cross-sections, and used results from Love [13] for inextensional (bending) deformation of the cross-sectional shape. This model, combined with geometric information about cell configuration from microscopic examination, gave first estimates of the ratio $E_L:E_T$ which were rather larger than those observed [7, p9], but close enough to suggest that this theory was indeed modelling the dominant mechanism for L-T anisotropy. Effects of finite wall thickness and departures of geometry from that assumed could plausibly be believed to account for the deviation from observation.

In a similar way, the "tube model" can give information about the relative magnitude of the other elastic constants. Interesting light is shed on the method by recent work of Gibson et al. [14]. They examined a family of two-dimensional cellular materials formed from various kinds of hexagons, and they give formulae for all four in-plane elastic constants. By using a different geometry from Price, they reveal which features are special to circular cylinders and which are more general.

As a final note on microstructure modelling, we draw attention to the interesting paper on balsa wood by Easterling et al. [15]. They use arguments similar to those of Price, and combine them with more up-to-date direct measurements of cell-wall elastic properties [16] to produce (among other things) predictions of the three principal Young's moduli as functions of density of the wood. Using their own measurements on balsa specimens having a wide range of densities together with published measurements for other woods, they plot an interesting graph showing that the predicted correlations and values are encouragingly well supported by measurements ([15], Fig. 10). There is surely much more to be learnt about wood properties by developing this line of thinking.

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Figure 1. The four elastic constants D_1 to D_4 for a flat plate of spruce, calculated from data in ref. [3]. In (a), the L axis (the 'grain') lies in the plate, while the angle (measured in the TR plane) of the annual rings varies linearly from zero (the ideal quarter-cut plate) to 90° (plate in the LT plane). In (b), the R axis lies in the plate, while the grain angle (measured in the LT plane) is varied.