ON THE USE OF TIME DOMAIN SUBSTRUCTURING FOR IMPACT NOISE REDUCTION

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ABSTRACT

Transient noise and vibration exposure within residential apartments, for example caused by weight drops/impacts from rooftop gyms, can be detrimental to health and well-being. Conventional mitigation measures, typically based on empirical rules of thumb, result in overly conservative designs to safely account for the uncertainties in the underlying model. A more physically representative model would aid in reducing over-design and help minimise material usage and cost. However, a significant challenge with impact noise reduction is the occurrence of non-linear effects within isolating materials, caused by small contact areas and large material deformations. These non-linear effects cannot be accurately captured by conventional frequency-based (linear) analysis methods.

In this paper we propose a time domain-based substructuring analysis for non-linear impact noise reduction problems. Utilizing an established impulse response function (IRF) substructuring framework, we demonstrate how non-linear impact noise reduction predictions can be made, provided a material model for the isolation material is available. Example results are presented using a standard ball drop onto a standard concrete floor above an ISO-compliant reverberation chamber. Although precise material properties are unavailable for verification of the model, the results align with expected trends, showcasing the potential of this approach in accurately predicting non-linear impact noise reduction.

1 INTRODUCTION

The transmission of impact noise through floors is a persistent problem leading to annoyance and even sleep disturbance for occupants in the spaces below. Effective prediction methods for structure-borne sound are vital when attempting to meet acoustic performance standards and ensure occupant comfort. Traditionally, modelling the transmission of impact noise relies on frequency-based methods [1, 2]. While effective in many contexts, these approaches often fall short when attempting to simulate impulsive, broadband excitations, especially in systems with non-linear behaviour. This limitation motivates the exploration of time domain alternatives, particularly impulse-based substructuring (IBS).

IBS is a technique that enables the coupling of different structural components, both simulated and experimentally measured, by combining their impulse response functions (IRFs). The method operates fully in the time domain, making it well-suited to the analysis of transient and impact events, but also allows for the integration of non-linear elements. This paper investigates the development of an IBS model to predict impact noise in buildings. In order to do this the unknown impact force exerted onto the structure is first determined with the use of inverse force identification. Using the identified impact force in combination with a developed substructuring model, the sound pressure level in a receiving room can be predicted for different resilient matting properties.

2 THEORY

2.1 Response prediction

For linear time-invariant systems, the output signal $y(t)$ can be calculated using the convolution of the input force signal $f(t)$ with the system's impulse response function $Y(t)$,

$$y(t) = \int_0^t Y(t-\tau)f(t)dt. \tag{1}$$

In practice $Y(t)$, the IRF of the system, is often a time-discrete signal due to sampling in both measurement and simulation. This requires the convolution integral also be discretised in time. Using the Cauchy product, the discretised version is obtained,

$$y[k] = \sum_{i=0}^{k-1} H[k-(i+1)]F[i]\Delta t. \tag{2}$$

Here, $H[k]$ represents the discrete IRF (the discretised version of $Y(t)$), $F[i]$ the sampled input force at time i, Δt the sampling interval, and $y[k]$ the system response at time step k.

Equation 2 can be used to evaluate the performance, or impact reduction, of a resilient matting directly in the time domain. To do so, we need to: 1) determine the impact force (using inverse force identification) and 2) model the impulse response function of the coupled matting-floor-room system (using substructuring).

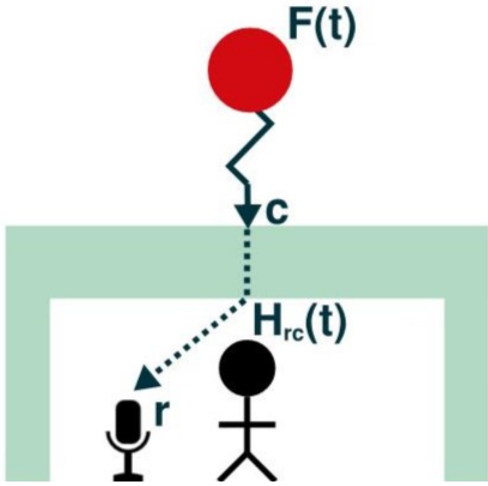


Figure 1: A diagram illustrating the measurement undertaken, with the primary structure in blue, and the transfer function measured, H_{rc}, shown with a dashed arrow

2.2 Inverse force identification

Inverse force identification (IFI) is a method that allows for the estimation of unknown input forces acting on a structure based on a set of observed responses. This is particularly useful in applications where the direct measurement of forces is impractical. In the context of this work, IFI enables the reconstruction of the impact force that generates structure-borne sound in the room below.

In structural dynamics, the so-called forward problem involves computing a system's response (displacement $u(t)$, velocity $\dot{u}(t)$, or acceleration $\ddot{u}(t)$) given the known properties of the system (mass M, damping C, and stiffness K) and the external forces acting on it $f(t)$ [3]. This can be done directly by using numerical methods to solve the governing equation, for example,

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = f(t). \tag{3}$$

Alternatively, the problem can be cast in the frequency domain and the system's Frequency Response Function (FRF, $H(\omega) = \frac{1}{M+i\omega C-K/\omega^2}$) utilised,

$$\ddot{u}(\omega) = H(\omega)f(\omega) \tag{4}$$

where $\ddot{u}(\omega)$ and $f(\omega)$ denote complex amplitudes at frequency ω. Note that for problems with multiple input forces and or responses, the above equations take on matrix/vector forms.

In contrast to the above, the *inverse problem* aims to find $f(t)$ or $f(\omega)$ given the dynamics of the system and its observed response. In an experimental context, this is most easily achieved in the frequency domain, so Equation 4 can be simplified to,

$$f(\omega) = H^{-1}(\omega)\ddot{u}(\omega) \tag{5}$$

where $H^{-1}(\omega)$ is, in general, the inverse of the system FRF matrix. Note that the force signal $f(t)$ can be obtained by $f(t) = \text{IFFT}(f(\omega))$.

2.3 Impulse-based substructuring

Dynamic substructuring is a computational technique used to reduce the complexity of large dynamic systems by decomposing them into more manageable pieces, or *substructures*. These substructures are analysed individually and recombined to understand the behaviour of the full system. Substructuring can be formulated in the physical, modal, frequency or time domains. Impulse-based substructuring (IBS) is a time domain substructuring technique that enables the coupling of different subsystems based on their impulse response functions. This method is particularly useful when components exhibit different dynamic characteristics, such as linear and non-linear behaviour, which cannot be dealt with by conventional frequency domain methods.

2.3.1 Interface conditions

When assembling substructures, two conditions at the interface between components need to be satisfied. These are:

- Compatibility - The response (displacement, velocity, acceleration) at the interface must be equal at all times ($u^1 = u^2$), preventing unrealistic gaps or overlaps between these connected DOFs [4]. This can also be expressed in the form of a signed Boolean matrix, allowing the conditions to be written as:

$$\begin{bmatrix} B^{(1)} \dots B^{(2)} \end{bmatrix} \begin{bmatrix} u^1 \\ u^2 \end{bmatrix} = Bu = 0 \tag{6}$$

- Equilibrium - The sum of forces between the two connecting DOFs should equal 0, such that there is no resultant force between the two substructures ($g^1 + g^2 = 0$) [4]. This condition can also be expressed in a similar form using a signed Boolean matrix:

$$\begin{bmatrix} g^1 \\ g^2 \end{bmatrix} = -B^T\lambda \tag{7}$$

  Here g denotes the interface forces from each of the neighbouring substructures, and λ denotes a set of interface force intensities. The compatibility condition is enforced indirectly through the interface force intensities, λ. These are the required to maintain compatibility between substructures.

2.3.2 Interface assembly

To assemble the system, a set of coupled equations is required that satisfy the above conditions at each of the substructure's interfaces. Throughout the rest of the paper, the superscripts (s) and (r) will be used to denote the IRF model (a linear subsystem representing the floor-room substructure) and an FE model (a non-linear subsystem representing the resilient matting), respectively. The uncoupled equations for each substructure is given in Equation 8.

$$\begin{cases} \ddot{u}_k^s = \frac{dt}{2}\sum_{i=0}^{k-1} H_{k-i}^{(s)}\left(f_{k-1}^{(s)} + f_i^{(s)} - g_{i-1}^{(s)} - g_i^{(s)}\right) \\ M^{(r)}\ddot{u}_k^{(r)} + p^{(r)}\left(\dot{u}_k^{(r)}, u_k^{(r)}\right) = f_k^{(r)} - g_k^{(r)} \end{cases} \tag{8}$$

In these equations, $\ddot{u}_k^{(s)}$ represents the acceleration of the IRF-based subsystem at time step k, while $u_k^{(r)}$ represents the displacement of the non-linear FE subsystem. The term $p^{(r)}(\dot{u}_k^{(r)}, u_k^{(r)})$ represents the internal forces of the non-linear subsystem, which depend on both displacement and velocity. The terms $g_k^{(s)}$ and $g_k^{(r)}$ represent the interface forces for each subsystem [5]. Using the relationships outlined in Section 2.3.1, and substituting λ for the interface forces, the coupled equations become:

$$\begin{cases} \ddot{u}_k^s = \frac{dt}{2} \sum_{i=0}^{k-1} H_{k-i}^{(s)} \left(f_{i-1}^{(s)} + f_i^{(s)} - B^{(s)T} \left(\lambda_{i-1} + \lambda_i \right) \right) \\ M^{(r)} \ddot{u}_k^{(r)} + p^{(r)} \left(\dot{u}_k^{(r)}, u_k^{(r)} \right) = f_k^{(r)} - B^{(s)T} \lambda_k \\ B u_k = 0 \end{cases} \tag{9}$$

Equation 9 enforces compatibility at the interface, ensuring that the displacements of the two subsystems match at their connection points. These coupled equations form the basis for the time-stepping IRF-FE substructuring algorithm described in great detail in [6].

2.3.3 Non-linear solution scheme

To solve Equation 9, an iterative solver combines a Newmark time integration scheme with a Newton–Raphson algorithm. The coupled system, composed of a linear IRF substructure and a non-linear numerical substructure (represented by its discretised inertial properties $M^{(r)}$ and non-linear internal forces $p^{(r)} \left(\dot{u}_k^{(r)}, u_k^{(r)} \right))$ is solved over a series of time steps, at each stage predicting displacements, velocities and accelerations, evaluating internal forces, and iteratively updating the response until convergence criteria are met. This approach enables the model to simulate transient, non-linear, behaviour of a system composed of both measured (IRF) and modelled substructures. A diagram of the process is given in Figure 2 but for a more complete explanation the reader is directed to [6].

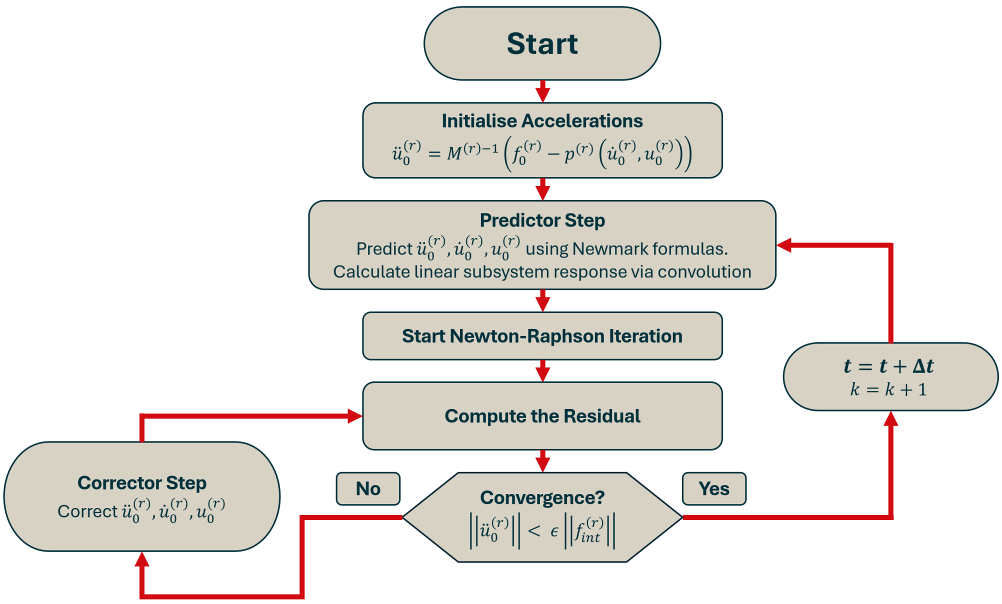


Figure 2: Flow chart showing the steps in the non-linear IBS scheme

3 Experimental case study

This study investigates the prediction of impact noise transmission through a standard concrete floor using a non-linear time domain impulse-based substructuring (IBS) approach.

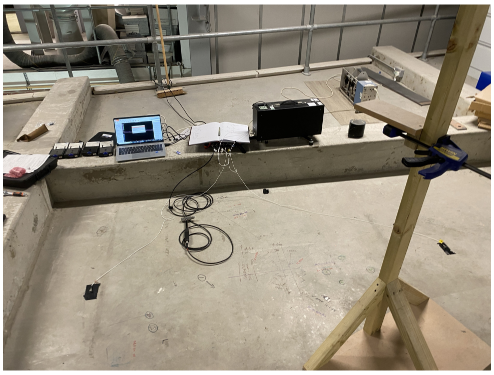
Inverse force identification was first applied to reconstruct the unknown impact forces. An impulse-based substructuring model was then implemented, combining a linear IRFs substructure to represent the floor-room system with a nonlinear mass-spring substructure representing the resilient matting. Finally, the identified forces were used as an input to the substructured model, the resulting response prediction being that of the sound pressure levels in the room below.

3.1 Experimental setup and methodology

Firstly, inverse force identification was carried out to reconstruct the unknown impact forces exciting the structure. A 2.5 kg rubber ball was repeatedly dropped at a central position on a concrete floor above the University of Salford's transmission suite. Six accelerometers were arranged in a semi-circle around the impact location to record structural responses, with an additional accelerometer placed at a separate position for validation purposes. Prior to these operational measurements, a calibrated force hammer was used at the same impact point to obtain the frequency response functions (FRFs) of the system. These FRFs, combined with the measured accelerations from the ball drops, provided the data required to estimate the unknown impact forces as per Equation 5.

The study then focused on developing and validating the substructuring model. The structural floor was represented as a linear mass-spring subsystem characterised by its simulated impulse response functions (IRFs), while the resilient matting layer was modelled as a separate mass-spring system with an embedded nonlinear stiffness. These two components were coupled using the non-linear impulse-based substructuring scheme described above, and validated by comparing the response of the substructured system with the direct numerical solution of the complete system. Note that the purpose of this numerical model was simply to verify the IBS code to be used for the experimental study.

Finally, the reconstructed impact forces were introduced into a substructuring model describing the matting-floor-room system to predict the acoustic response in the receiving room below. Microphones were positioned in the lower room to measure the structural–acoustic FRFs required for the IBS scheme. Due to a lack of material properties for the available matting, no validation was available for this prediction.



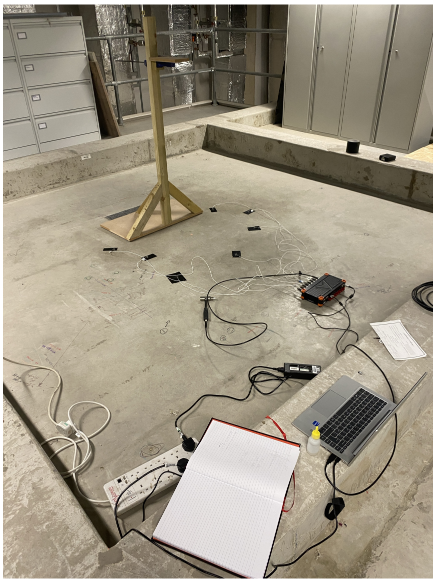
(a) Photograph showing the setup of equipment and ball dropping rig on top of the room under test.

(b) Photograph of microphone arrangement inside the room under test, with the microphones used circled in red.

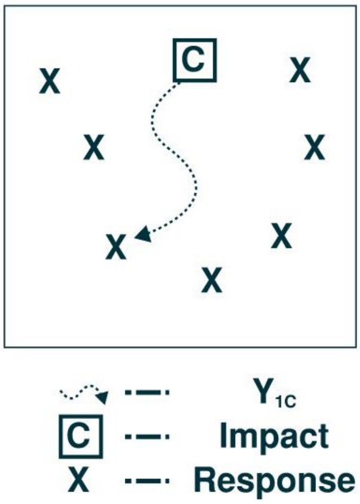
Figure 3: Photographs from the testing conducted to measure structural-acoustic FRF H_{rc} of the floor-room system seen in Figure 1, from the impact position to receiver microphones in the room below.

3.2 Force identification

Shown in Figure 4 is the experimental setup used to identify the impact force from due to a rubber ball drop from 1 m. The 7 accelerometers used can be be seen by the black tape to which they are stuck; 6 of these were used to identify the forces, with the remaining one used for validation.

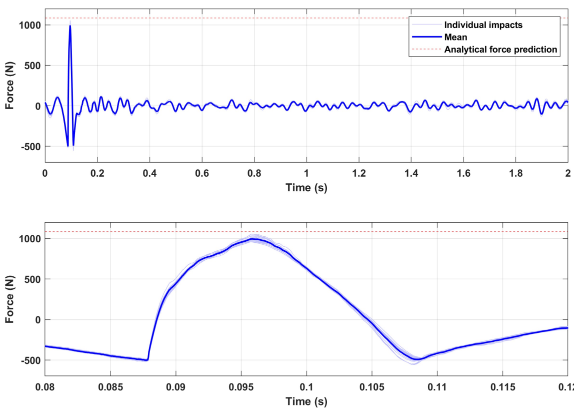


(a) Test setup on the primary structure

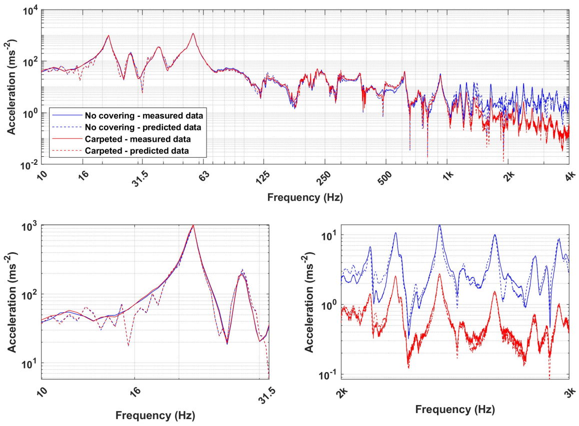


(b) Diagram of test setup

Figure 4: Transmission suite test setup on top of the primary structure and explanatory diagram



(a) Predicted impact force (bare floor) for 20 repeated ball drops.

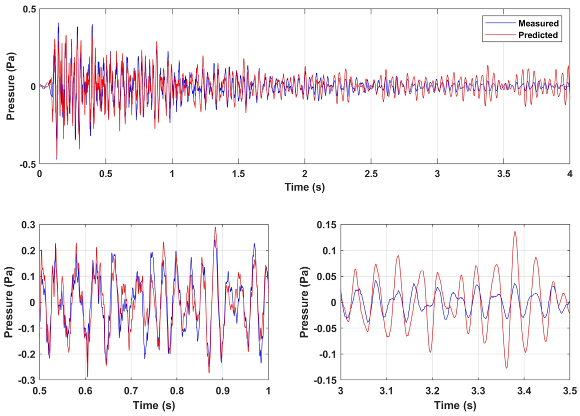


(b) Frequency domain validation of bare floor (blue) and carpet covered (red) impact forces.

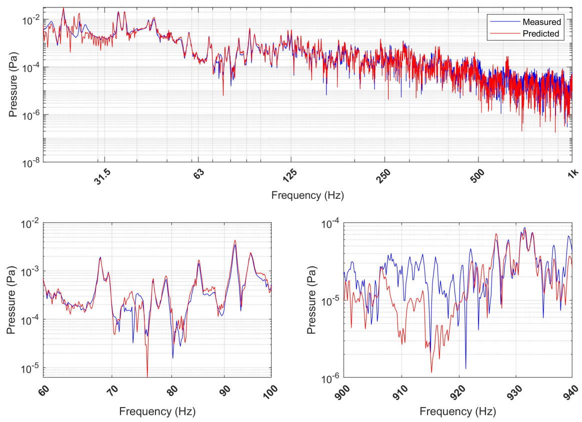
Figure 5: Results of inverse force identification and validation for rubber ball drops.

After solving Equation 5, the time domain force signal was obtained by inverse FFT. This process was repeated for 20 individual ball drops to provide a measure of repeatability. The results are presented in Figure 5a. The narrow spread illustrates the high repeatability of the repeated ball drops. To validate the identified (mean) force, it is used to predict the response at the 7th accelerometer, for which a direct measurement was also made. Results are shown in Figure 5b. For clarity and ease of comparison, we present the results in the frequency domain. Two cases are considered; a bare floor impact force, and a carpet covered impact force. In both cases the identified forces successfully predicted structural response at accelerometer 7, confirming the accuracy of the force reconstruction method.

To provide a more rigours validation, the ball drop assembly was relocated to a different part of the floor. The previously identified impact force was then convolved with measured transfer function between the relocated impact position and a microphone in the room below (H_{rc}) to estimate resulting sound pressure level. Results are compared (both in time and frequency domains) against a directly measured ball drop pressure response in Figure 6. Excellent agreement is obtained in both domains, particularly during the initial impact period.



(a) Time domain comparison.



(b) Frequency domain comparison.

Figure 6: Impact force validation using relocated impact position. Predicted and measured pressure responses inside the room.

3.3 Substructuring model

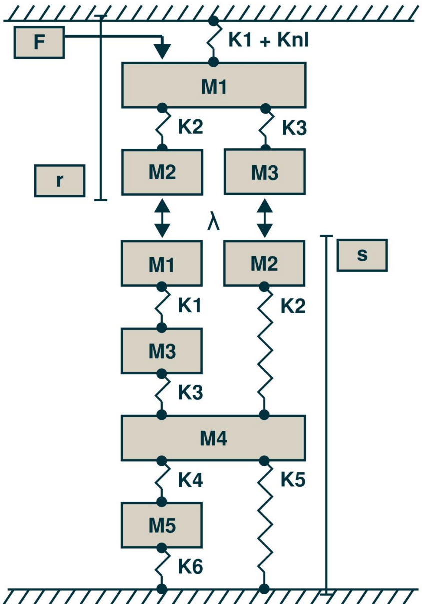
This section details the development and validation of a computational model designed to predict the response of a coupled system using non-linear IBS. The model (illustrated in Figure 7a) is designed to include the same general components as the experimental study of interest; an IRF-based linear component representing the floor-room system (denoted s) and a mass-spring-based non-linear component representing a non-linear matting (denoted r).

To verify the IBS model's accuracy, its output is compared with direct solutions calculated using MATLAB's ODE45 solver utilising an explicit Runge-Kutta scheme [7]. Figure 7b shows the displacements at a representative DOF in four cases: a linear model simulated directly (Direct linear), a nonlinear model simulated directly (Direct nonlinear), the linear IBS substructuring model (IRF–FE linear), and the non-linear IRF–FE substructuring model (IRF–FE nonlinear).

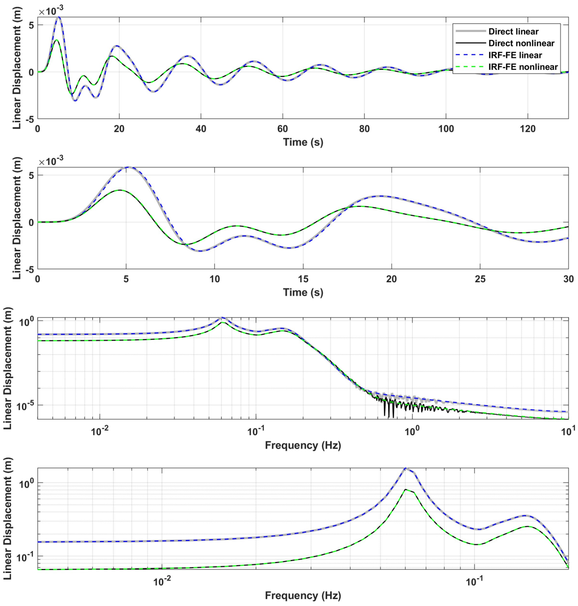
The results demonstrate good agreement between the direct solutions and the substructuring model. The non-linear implementation properly reduces to the linear case when the non-linearity is absent and, with strong non-linearity, the IRF-FE non-linear model continues to match the direct non-linear solution with only minor discrepancies related to machine precision. The results verify that the built IBS code, giving confidence to the following transmission suite case presented below.

3.4 Impact noise reduction model

The identified impact force can now be combined with an IBS model of the transmission suite and hypothetical resilient matting. With this, the parameters of the resilient matting can be modified to examine how its properties affect the transmission of impact noise.



(a) Diagram showing the mass spring system modelled. K_{nl} denotes the non-linear stiffness.



(b) system response in both the time and frequency domain calculated with: Direct linear, Direct non-linear, IRF-FE linear, and IRF-FE calculation methods.

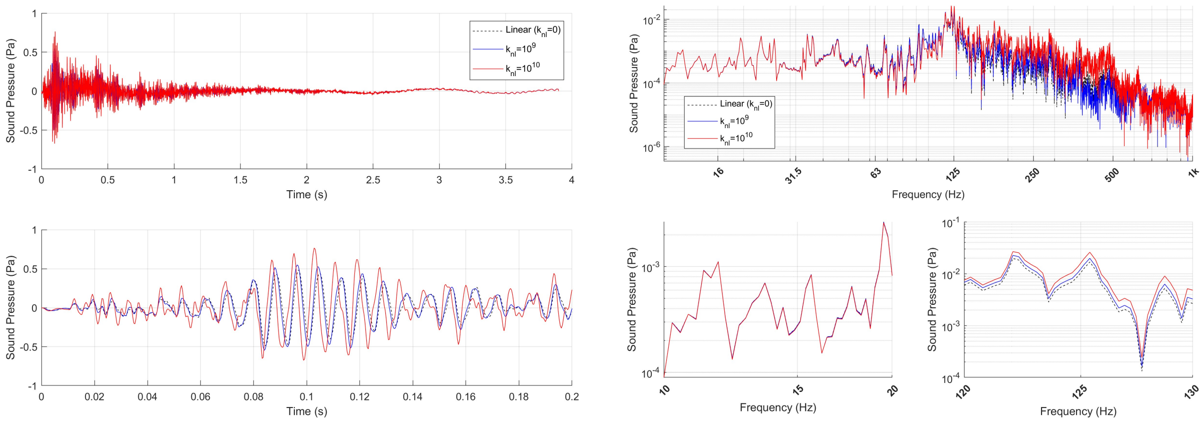
Figure 7: Validation results for the non-linear IBS model.

To build the transmission suite IBS model a representation of the resilient matting must first be developed. To illustrate the method, we have chosen to model the (hypothetical) matting as having a simple cubic nonlinear stiffness, its underlying equation of motion being given by,

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) + K_{nl}^3 u(t) = f(t). \tag{10}$$

Figure 8a demonstrates how the pressure response changes with increasing non-linear stiffness values. As stiffness increases, the peak pressure amplitudes rise significantly, while the decay rate also increases, resulting in a shorter response. This behaviour is expected, since the higher stiffness allows for more efficient energy transfer, producing larger initial pressure peaks, and enables faster energy dissipation through radiation and internal damping. In the frequency domain, shown in Figure 8b, we see that stiffness variations have a frequency-dependent effect, with minimal impact on frequencies below approximately $80Hz$ and above $500Hz$. The minimal variation at frequencies outside this range indicates that other parameters, such as floor mass, are dominating the response and that the matting as a negligible effect. This behaviour is relevant for the design of resilient isolation systems, as it suggests that optimising stiffness properties (including non-linearities) can effectively target specific frequencies of concern.

Due to time constraints, an experimental validation of the transmission suite IBS model was not possible. This would require a detailed characterisation of the chosen matting to infer an appropriate governing equation.



(a) Time domain prediction. **(b)** Frequency domain response.

Figure 8: Transmission suite IBS pressure response inside the room for: the linear case, and with nonlinear matting stiffnesses of $K_{nl} = 1e9$ and $K_{nl} = 1e10$.

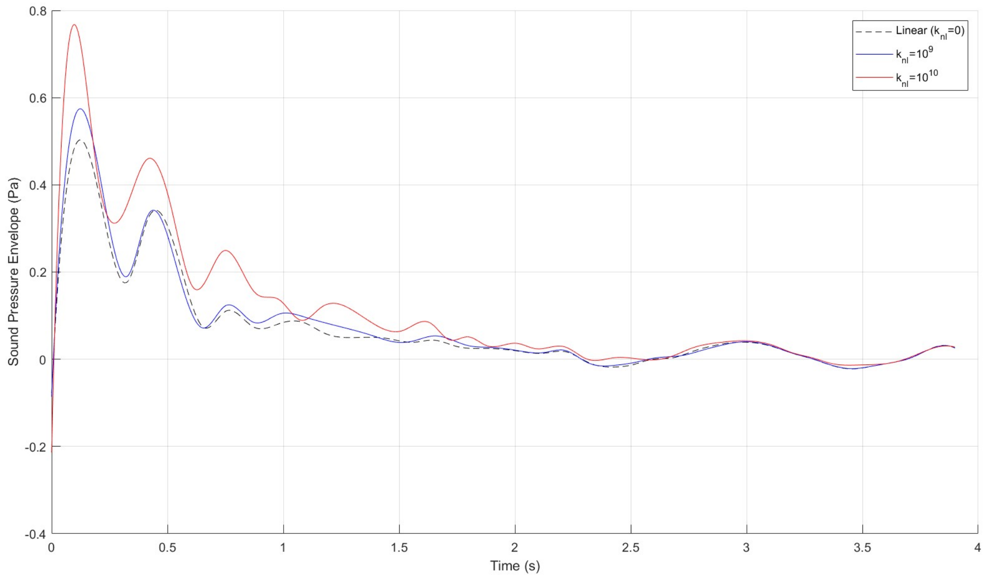


Figure 9: Substructuring models predicted pressure response envelopes inside the room for: the linear case, and with nonlinear matting stiffnesses of $Knl = 1e9$ and $Knl = 1e10$. Shown in the time-domain over the whole response period.

4 Conclusions

This project successfully developed a methodology, combining inverse force identification with non-linear impulse-based substructuring to predict sound pressure levels in a transmission suite. The three main objectives were achieved: accurately reconstructing unknown impact forces, developing an impulse-based substructuring model for non-linear systems, and integrating these components to predict sound pressure levels in the receiving space. The model was seen to effectively capture the effect of a resilient matting's properties on its vibro-acoustic performance/impact noise reduction. This clearly illustrates the potential of such an IBS approach for designing/specifying resilient matting systems for impact noise problems.

Impulse based substructuring as a tool for time domain analysis is known to work well in theory, i.e. on numerical problems, but has always had stability issues when experimentally derived components are introduced into the workflow [8]. Fortunately, it seems that the problem addressed in this paper is simple enough that such IBS models have the potential to work quite well. Though, further work is still required to experimentally validate this method of impact noise prediction.

5 REFERENCES

1. M. Späh, K. Hagberg, O. Bartlomé, L. Weber, P. Leistner, and A. Liebl. Subjective and objective evaluation of impact noise sources in wooden buildings. *Building Acoustics*, 20(3):193–213, 2013.

2. M. Oliveira and J.V. Patricio. Impact noise of non-homogeneous floors: analysis of different input parameters for computational modeling predictions. *Journal of Civil Engineering and Architecture*, 11(3):274–281, 2017.

3. D.J. Ewins. *Modal Testing: Theory, Practice and Application*. Engineering dynamics series. Wiley, 2000.

4. D. de Klerk, D.J. Rixen, and S.N. Voormeeren. General framework for dynamic substructuring: History, review and classification of techniques. *AIAA Journal*, 46(5):1169–1181, May 2008.

5. D.J. Rixen and P.L.C. van der Valk. An impulse-based substructuring approach for impact analysis and load case simulations. *Journal of Sound and Vibration*, 332(26):7174–7190, 2013.

6. P. Van der Valk and D.J. Rixen. An effective method for assembling impulse response functions to linear and non-linear finite element models. In *Proceedings of the Society for Experimental Mechanics Series*, volume 2, pages 123–135, 02 2012.

7. MathWorks. Solve nonstiff differential equations — medium order method - matlab ode45 - mathworks united kingdom, 2024.

8. O.M. Zobel, F. Trainotti, and D.J Rixen. Enabling experimental impulse-based substructuring through time domain deconvolution and downsampling. *Experimental Techniques*, pages 1–35, 2024.