24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

PREDICTION OF VIBRATION TRANSMISSION ACROSS PLATE JUNCTIONS WITH A PERIODIC RIBBED PLATE USING SEA AND ASEA

Jianfei Yin[1], Carl Hopkins[2], Haibin Yang[1]

[1]*Vibration and Acoustics Research Group, Laboratory of Science and Technology on Integrated Logistics Support, National University of Defense Technology, Changsha, Hunan, P. R. China 410073.*
Email: nmhsyjf@hotmail.com

[2] *Acoustics Research Unit, School of Architecture, University of Liverpool, Liverpool, UK*
Email: carl.hopkins@liverpool.ac.uk

This paper considers low- and mid-frequency bending wave transmission across L-junctions of an isotropic homogeneous plate and a periodic ribbed plates with symmetric ribs as well as when both plates are periodic ribbed plates with symmetric ribs. Different analytical approaches within the framework of Statistical Energy Analysis (SEA) are used to calculate the coupling loss factors and the results are compared with laboratory measurements and Finite Element Method (FEM). The analytical approaches include treating the ribbed plates as isotropic plates with different bending stiffness's or angle-dependent bending stiffness as well as using wave theory that incorporates Bloch theory as proposed by Tso and Hansen. For ribs parallel to the junction line, it is shown that wave theory incorporating Bloch theory gives closest agreement with FEM and measurements.

Keywords: ribbed plate, SEA, ASEA, vibration

1. Introduction

The ability to predict sound and vibration transmission in built-up structures such as buildings, ships, trains and automobiles is important for human comfort, health and safety. For structural reasons, engineering structures often incorporate periodic ribbed plates to increase the strength, static stiffness and stability whilst reducing the weight. In terms of vibration propagation, periodic ribbed plates often exhibit a stop/pass band feature where waves at certain angles of incidence cannot propagate in certain frequency bands (stop bands) and will attenuate exponentially and propagate freely in other frequency bands (pass bands) [1]. To-date there has been more focus on isolated ribbed plates rather than wave transmission in built-up structures when periodic ribbed plates are incorporated in the junctions.

Statistical Energy Analysis (SEA) is well suited to the prediction of structure-borne sound transmission on complex built-up structures. However, the inclusion of rib-stiffened plates within the framework of SEA is a challenge in the field of engineering noise control. Recent works [2-5] have used Advanced SEA (ASEA) to predict high-frequency transmission of bending waves across various structural junctions. At and above the fundamental local mode frequency of each bay (i.e. the bays formed between adjacent ribs) ASEA was able to predict the significant decrease in vibration that occurred for vibration transmission across the ribbed plate. At low and mid-frequencies where the wavelength is similar or larger than the bay spacing, the use of SEA is also challenging due to the orthotropic nature of the ribbed plate. Therefore, a pragmatic approach to model such an orthotropic plate is to treat it as an isotropic plate with an effective bending stiffness that is equal to the geometric mean of the bending stiffness in the two orthogonal directions [6]. This approach is sometimes described as ‘smearing’ the stiffness in the two orthogonal directions and in this paper this model is referred to as an ‘effective isotropic plate’. Bosmans *et al* [7] used wave theory to determine transmission coefficients across junctions of isotropic and orthotropic plates. The results indicated that an SEA model could give good estimates when the plates are orientated similarly but can significantly overestimate the transmission when they are orientated dissimilarly (particularly when the modal density is low). However, the ‘smearing’ method and the Bosmans *et al* method did not consider the periodic nature of the ribbed plate. This aspect was considered by Tso and Hansen [8] who used Bloch theory in combination with wave theory to determine the transmission coefficient across an L-junction formed by an isotropic, homogeneous plate and a periodic ribbed plate where each plate was treated as an individual subsystem. Their approach successfully incorporated pass/stop band behaviour. The experimental validation used free plate boundaries which is not realistic for many engineering structures. However, the comparison of experimental and predicted coupling loss factors showed good agreement up to the first stop band. At higher frequencies the measured data showed deviations from SEA which were attributed to the frequency-dependent nature of the assumptions involved in the theory. The present authors have shown that at frequencies above the fundamental mode of the bays there can be a significant decrease in vibration across successive bays [2]. Hence Tso and Hansen’s approach treating the periodic ribbed plate as a single subsystem is more likely to be appropriate as a solution for the low- and mid-frequency ranges and this is how it is considered in this paper. Yin and Hopkins [9] incorporated Bosmans *et al* and Tso and Hanson’s methods in SEA to give a systematic approach for low- and mid-frequency prediction and then used ASEA at high-frequencies [3].

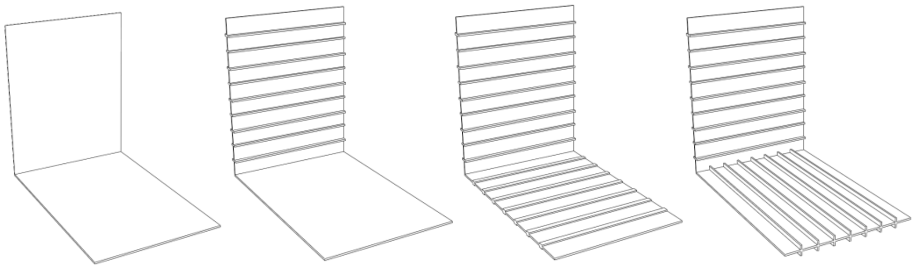
In this paper an L-junction of plates is considered where one or both plates are a periodic ribbed plate as shown in Fig. 1 where each plate is modelled as an individual SEA subsystem. All plate boundaries are pinned (i.e. constrained displacement in the three coordinate directions) to ensure that the boundary conditions were representative of engineering structures typically used for noise control. The different SEA models are assessed by comparing the predicted energy level differences between source and receiver plates with FEM and measured data to identify the most appropriate SEA model when the ribs are orientated either parallel or perpendicular to the junction line. Four approaches are used to determine angular-average transmission coefficients for the coupling loss factors in the SEA models:

(1) Treating the periodic ribbed plate(s) as an ‘effective isotropic plate’ by using the effective bending stiffness [6].

(2) Treating the periodic ribbed plate(s) as an ‘equivalent isotropic plate’ by using only the bending stiffness in one particular direction as defined by Bosmans *et al* [7].

(3) Using angle-dependent bending stiffness following the approach of Bosmans *et al* [7].

(4) Using a combination of Bloch theory and wave theory. This follows the approach of Tso and Hansen [8] for an L-junction formed by an isotropic, homogeneous plate and a periodic ribbed plate and extends the theory to (a) allow calculation of the transmission coefficient in both directions without needing to estimate the modal density of the ribbed plate and (b) model an L-junction formed from two periodic ribbed plates.



(a) L-junction No.1 (b) L-junction No.2 (c) L-junction No.3 (d) L-junction No.4

Figure 1 – L-junctions.

2. Test structures

As a comparator, an L-junction of isotropic, homogeneous plates (Fig. 1a) is used to assess the agreement between both FEM and measured data with SEA. Different arrangements of ribbed plates are considered with ribs that are either parallel or perpendicular to the junction line. When the ribs are perpendicular to the junction, it is only the plate thickness that is physically connected to the other plate along the junction line, because the rib length is shortened by ≈2mm to leave a gap.

The material used for all plates is Perspex for which the properties are given in Table 1 with the geometrical details in Table 2. Plate 1 has dimensions L_x = 1.2 m and L_y = 0.8 m, and plate 2 has dimensions L_x = 1 m and L_y = 0.8 m. Plates 1 and 2 are connected to form an L-junction across the L_y dimension. For all the ribbed plates, the majority of the plate area is formed by the bays rather than the ribs. Hence the experimental and FEM set-ups only consider the displacement in the bays (i.e. excluding the rib displacement in the calculation of the average energy for the ribbed plate).

Table 1 - Plate properties.

Junction No.	Plate	Density (kg/m^3)	Longitudinal wavespeed (m/s)	Poisson's ratio (-)	Internal loss factor (-)
1	1	1180	2350	0.3	0.06
	2	1218	2045	0.3	0.06
2,3,4	1	1218	2045	0.3	0.06
	2	1218	2045	0.3	0.06

Table 2 – Details for plates forming the L-junctions.

Junction No.	Plate	Ribbed	Rib orientation to junction line	Plate thickness, h_p (mm)	Rib spacing, l (mm)	Rib width, b_b (mm)	Rib height, h_b (mm)
1	1	No	N/A	13	N/A	N/A	N/A
	2	No	N/A	10	N/A	N/A	N/A
2	1	No	N/A	10	N/A	N/A	N/A
	2	Yes	Parallel	10	100	10	60
3	1	Yes	Parallel	10	150	30	25
	2	Yes	Parallel	10	100	10	60
4	1	Yes	Perpendicular	10	100	10	60
	2	Yes	Parallel	10	100	10	60

3. Theory

An SEA model for bending wave transmission between two coupled plates forming an L-junction requires calculation of angular-average transmission coefficients from which the coupling loss factors can then be calculated. This section gives an overview of the different calculation methods used to determine the transmission coefficient between the two plates of the L-junctions.

3.1 Effective isotropic plate

An effective isotropic plate is defined by modelling the ribbed plate using an effective bending stiffness, B_{eff}, given by:

$$B_{eff} = \sqrt{B_x B_y} \tag{1}$$

where the bending stiffness in the two orthotropic directions, B_x and B_y for the ribbed plate are given in [6].

3.2 Equivalent isotropic plate

The definition of an equivalent isotropic plates uses the approach of Bosmans *et al* [7] by assuming that the bending stiffness in one direction can be considered as representative of the bending stiffness for the entire plate.

For one isotropic plate connected to an orthotropic plate (L-junction No.2) where the stiffest coordinate direction is parallel to the junction line (i.e. ribs parallel to the junction), the orthotropic plate uses the Young's modulus that is perpendicular to the junction for the equivalent isotropic plate.

For two orthotropic plates (L-junction No.3) where the stiffest coordinate direction on each plate is oriented in the same direction on both plates, the Young's modulus for each equivalent isotropic plate corresponds to the Young's modulus of each orthotropic plate in the direction perpendicular to the junction line.

The last case concerns two orthotropic plates (L-junction No.4) where the stiffest coordinate direction on each plate is oriented in the opposite direction to the stiffest coordinate direction on the other plate. The plate with the stiffest coordinate direction perpendicular to the junction uses the Young's modulus that is parallel to the junction for the equivalent isotropic plate. The plate with the stiffest coordinate direction parallel to the junction uses the Young's modulus that is perpendicular to the junction for the equivalent isotropic plate.

3.3 Bloch and wave theory

Bloch and wave theory developed by Tso and Hansen [8] is used to determine the angular-average transmission coefficient from an incident bending wave field on an isotropic, homogeneous plate to a periodic ribbed plate that form an L-junction where the ribs are parallel to the junction. Work by the current authors [9] has extended this approach to solve the problem of an L-junction of two ribbed plates with ribs parallel to the junction.

4. Finite element models

ABAQUS 6.10 software was used for the L-junctions. The rectangular four-node shell element, S4R, is used to model both the isotropic, homogeneous plate and the periodic ribbed plate with seven elements per wavelength at the highest frequency. All the nodes along the plate boundaries and the junction line connecting the two plates have constrained displacement in the three coordinate directions which is commonly referred to as being pinned. This ensures that with excitation of bending waves on the source subsystem, no in-plane waves are generated at the junction of the two plates.

Rain-on-the-roof excitation is applied to all unconstrained nodes on the source plate with forces of unity magnitude and random phase. However, with discrete numerical realization of rain-on-the-roof, it is possible to have different sets of random phase values for the unity magnitude input forces. Therefore, ten different sets of random phase values are used so that the ensemble output can be considered as representative of different physical realizations of rain-on-the-roof [10]. FEM data from ten sets of rain-on-the-roof are used to calculate a mean value with 95% confidence intervals. For each one-third octave band between 100Hz and 1kHz, FEM is used to determine the mean-square response at five discrete frequencies within the band and averaged to give a single value.

5. Laboratory experiments

Four L-junctions were formed from two Perspex plates using cyanoacrylate glue to connect the plates along the junction line and to connect the ribs on both sides of the plate. Pinned boundary conditions are simulated by supporting the plate edges along the centreline with an array of pointed pins that are fitted into a heavy steel frame as shown in Fig. 2. These boundary conditions were validated in [2]. The L-junction in supported in a frame formed by two U-shaped frames as shown in Fig. 4b which allows all edges of the two plates to be pinned except for the junction line.

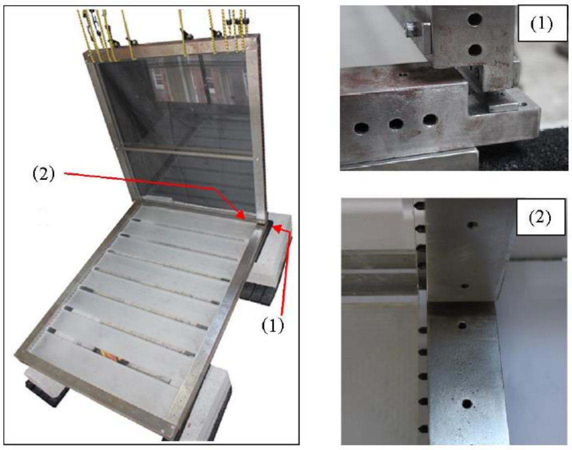


Figure 2 – Experimental frame used to support each L-junction.

6. Results

Comparison of SEA, FEM and measurements for the four L-junctions in one-third octave bands are shown in Figs. 3 to 6 with subplot (a) for source plate 1 and subplot (b) for source plate 2. The top of each graph shows the bending mode count, N_B, for each plate and the geometric-average modal overlap factor, M_{av}, for the two plates.

Results for L-junction No.1 are shown in Fig. 3 (100Hz and 5kHz). Fahy and Mohammed [11] previously showed that wave transmission coefficients only tend to give accurate estimates when the geometric mean of the modal overlap factors, $M_{av} \geq 1$, and the mode count in each band satisfies, $N \geq 5$, although these values have been found to be too strict [12]. For L-junction No.1, $M_{av} \geq 0.5$ and $N \geq 2$ in one-third octave bands at and above 160Hz where mean energy level differences for measured and/or FEM data are typically within 2dB of the SEA prediction, mean values for measured and FEM data are typically within 2dB of each other and the 95% confidence limits for measured and FEM data tend to overlap each other and be within 1dB of the SEA prediction.

Results for L-junction No.2 are shown in Fig. 4. Above 125Hz, measured and FEM data for E_1/E_2 and E_2/E_1 have mean values that are typically within 2dB of each other and overlapping 95% confidence intervals. For E_1/E_2 the peak around the 125Hz/160Hz bands and the trough around the 250Hz/315Hz bands are not caused by pass or stop bands and could be attributed to the modal behaviour of the periodic ribbed plate. For transmission from the isotropic homogeneous plate to the periodic ribbed plate there is negligible difference between using the Bloch theory model to calculate the coupling loss factor in both directions and using Bloch theory in one direction and the consistency relationship in the other direction with predicted modal densities. However, for transmission from the periodic ribbed plate to the isotropic homogeneous plate, significantly better agreement is obtained between FEM, SEA and measurements when Bloch theory is used to calculate the coupling loss factor in both directions. For E_1/E_2, SEA using an effective isotropic plate predicts energy level differences that are within 5dB of measured and/or FEM data. However, for E_2/E_1 they are within 2dB which is a similar accuracy to L-junction No.1. Hence this approach could be used for SEA path analysis for transmission from the isotropic homogeneous plate to the periodic ribbed plate.

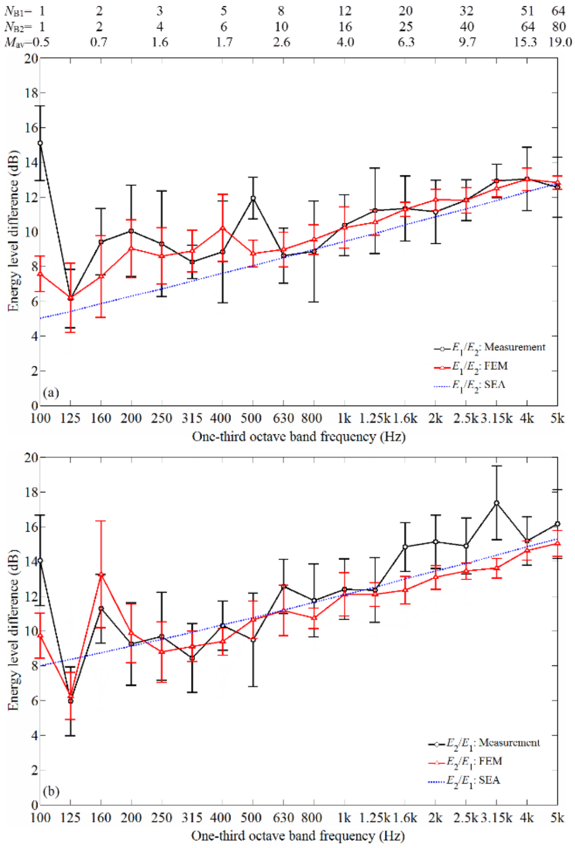


Figure 3 – L-junction No.1

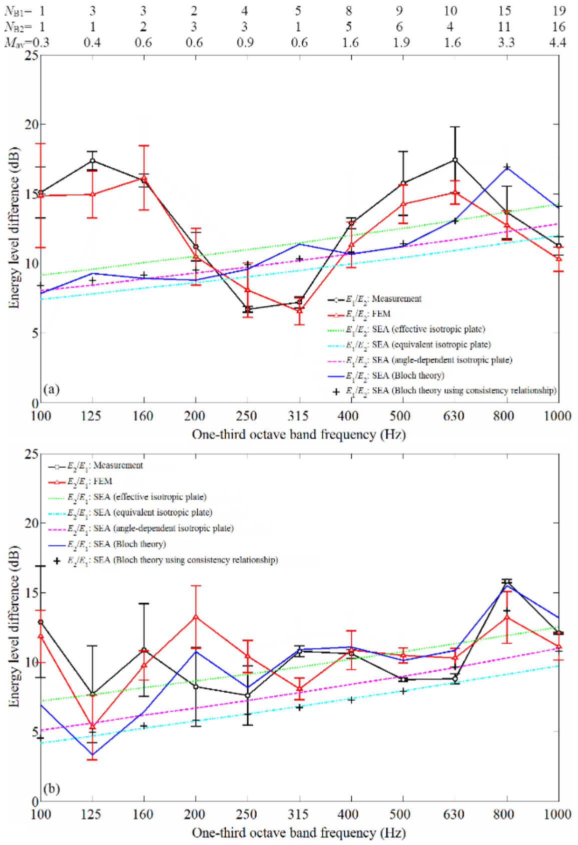


Figure 4 – L-junction No.2

Results for L-junction No.3 are shown in Fig. 5. SEA predictions for an effective isotropic plate, an equivalent isotropic plate and angle-dependent bending stiffness are typically within 2dB of each other. For E_1/E_2 and E_2/E_1 the SEA prediction using Bloch theory tends to give better agreement with measured and FEM data than the three other SEA predictions. However, there is no evidence of the stop band at 800Hz in either the measured and FEM data although it is predicted with SEA using Bloch theory – see Fig. 5b.

Results for L-junction No.4 are shown in Fig. 6. For E_1/E_2 and E_2/E_1 the measured and FEM data show closer agreement with each other than the SEA predictions using an effective isotropic plate, an equivalent isotropic plate or angle-dependent bending stiffness. All the SEA approaches tend to underestimate the energy level difference. SEA using angle-dependent bending stiffness gives the highest energy level differences, but compared to measured and FEM data in individual one-third octave bands it underestimates by up to 13dB for E_1/E_2 and up to 9dB for E_2/E_1. This underestimate was also observed by Bosmans *et al* [7] for L-junctions of naturally orthotropic plates with low modal density where the plates have mutually perpendicular directions of maximum stiffness.

In summary, SEA prediction using Bloch theory is potentially the most useful approach for L-junctions where one or both rectangular plates are ribbed, and the ribs are parallel to the junction. In the low- and mid-frequency range where the bays on the ribbed plate would not support local modes the errors can be expected to be similar to SEA using wave theory for the same plates with the ribs removed (i.e. two isotropic homogeneous plates). At higher frequencies where the bending wavelength is smaller than the bay spacing on the ribbed plate, ASEA can be used to model the bays on the ribbed plates as individual subsystems [2].

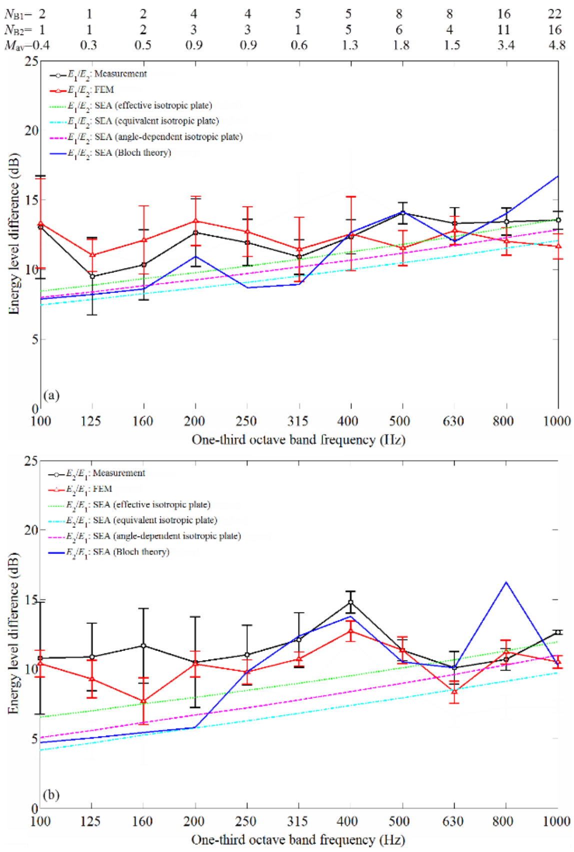


Figure 5 – L-junction No.3

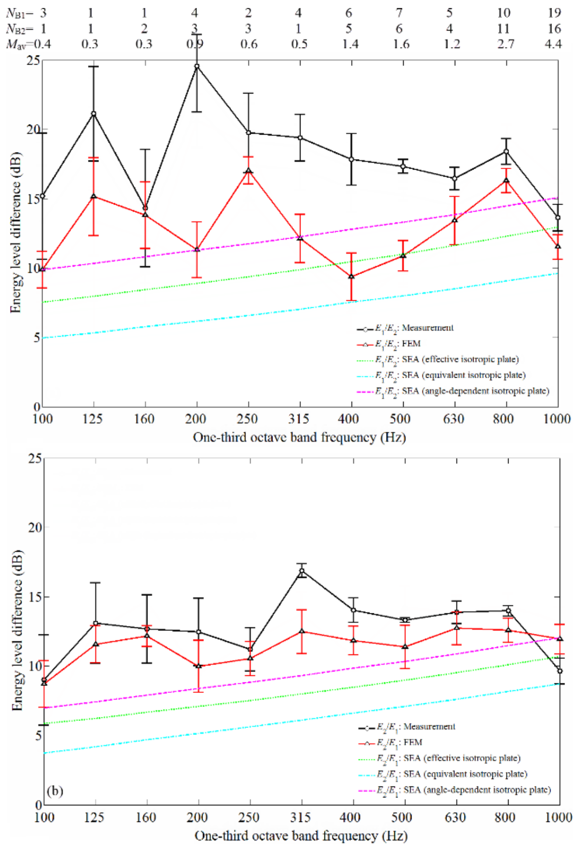


Figure 6 – L-junction No.4

7. Conclusions

Coupling loss factors for an L-junction of plates where one or both plates are a periodic ribbed plate were calculated from angular-average transmission coefficients by treating the periodic ribbed plate(s) as an ‘effective isotropic plate’ by using the effective bending stiffness, or treating the periodic ribbed plate(s) as an ‘equivalent isotropic plate’ by assuming the bending stiffness in one direction is representative of the overall bending stiffness, or using angle-dependent bending stiffness or using a combination of Bloch theory and wave theory. To assess and validate these approaches, SEA models were compared against measurements and FEM.

When the L-junction had one or two ribbed plates with ribs parallel to the junction, the closest agreement between measurements and FEM is with SEA models using a combination of Bloch theory and wave theory. In the low- and mid-frequency ranges where the bending wavelength is larger than the bay spacing, the errors are similar to SEA using wave theory for two isotropic homogeneous plates. At higher frequencies where the bending wavelength is smaller than the bay spacing on the ribbed plate, ASEA can be used to model bays on the ribbed plates as individual subsystems [2].

For L-junctions formed from two ribbed plates, one with ribs orientated perpendicular to the junction and the other with ribs orientated parallel to the junction, SEA models using an effective isotropic plate, or an equivalent isotropic plate or angle-dependent bending stiffness all underestimated the energy level difference.

Results indicated that higher energy level differences can be achieved for transmission from a source plate with ribs parallel to the junction line to a receiver plate with ribs perpendicular to the junction line, compared to a receiver plate with ribs parallel to the junction line.

ACKNOWLEDGEMENTS

Jianfei Yin is grateful for the support provided by the National Natural Science Foundation of China (Grant No. 51405502), China Scholarship Council, the Acoustics Research Unit (ARU) at the University of Liverpool and the National University of Defense Technology. All authors are grateful to Gary Seiffert for his assistance and advice with the experimental work in the ARU laboratories.

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