APPLICATIONS OF FINITE ELEMENT AND BOUNDARY ELEMENT ACOUSTICS MODELS IN NOISE CONTROL FOR VEHICLES, MARINE AND APPLIANCE DESIGN

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1 INTRODUCTION

The ever-increasing power of computers for engineering analysis means that numerical modelling is becoming increasingly attractive as a means to predict acoustic as well as structural behaviour at the design stage, before prototyping. Finite element (FE) modelling and (to a limited extent) boundary element (BE) modelling have become commonplace in the past ten or twenty years for predicting structural behaviour. These numerical methods have been applied to acoustics and acoustic-structure problems in recent years, and are now gaining acceptance alongside other numerical and experimental methods.

In this paper, we review the basic approaches of FE and BE acoustics, compare them with some other approaches and discuss their application and integration with other methods, illustrated by practical examples.

2 THEORETICAL BASIS

2.1 Physical Problem

The structural and acoustic behaviour of an object are closely related (see Figure 1): vibrations of the surfaces produce vibrations in the surrounding fluid and hence noise, and pressure waves impinging on the object produce vibrations in it. The pressure waves may have been generated by some external source, or by the object itself. In the latter case, there is a continuous feedback between the structural and acoustic behaviour.

In numerical modelling, it is common to consider only one of the above interactions: either, we assume the structural behaviour is known and we use the the vibration data to calculate the noise field around the object, or we assume that the incident acoustic field is known (eg, from defined sources, such as plane or spherical waves) and we calculate the pressure loading on bodies placed in the field. This class of problems are often termed uncoupled or one-way coupled, since the feedback between the acoustic and structural systems is not taken into account: the results of one analysis are just used as boundary conditions for the other (see Figure 2).

In many important cases, however, the two systems cannot be uncoupled in this way, since the structural behaviour is strongly influenced by the presence of the fluid and this affects the pressure field generated by the vibrating object. A good example is a vibrating ship hull where the added-mass effect of the water is not negligible. Similar cases exist where the fluid is air. These are usually called coupled problems. The acoustic and structural problems are solved together, taking coupling constraints into account: the vibrations of the structure induce pressure waves in the fluid, and the pressure distribution on the surface acts as a supplementary loading. The mathematical representation of these effects is described in more detail elsewhere, for example by Coyette [1,2].

2.2 Basic Equations

The acoustic field is governed by Helmholtz' derivation from the wave equation:

$$\nabla^2 p + k^2 p = 0$$

where k is the wave number ($= \omega/c$) and ∇ is the Laplace operator (usually in three dimensions).

Bondary conditions on different parts of the surface S can be defined:

$p = \overline{p}$ on S1; $dp/dn = -i\rho\omega\overline{v_n}$ on S2; and $dp/dn = -i\rho\omega\overline{A_n}.p$ on S3

where v_n denotes a prescribed normal velocity and A_n a prescribed normal admittance.

the Sommerfeld radiation condition may also need to be satisfied, given by:

$$r\ (dp/dr + ikp) \rightarrow 0 \quad \text{as} \quad r \rightarrow \infty$$

where r is a spherical polar radius.

For scattering problems, the total pressure field Pt can be separated into two components - the incident pressure field Pi (known) and the scattered field Ps, hence:

$$Pt = Pi + Ps$$

The equations can thus be adapted to formulate the problem in terms of the scattered pressure field alone.

In passing, we note that the Helmholtz equation is formulated in the frequency domain, implying that we are solving phenomena which are harmonic, or stationary with time: transient effects such as impulses are generally not addressed, unless transformed to the frequency domain.

2.3 Finite Element Acoustics

The usual residual formulation on which the FE method is based incorporates the field and boundary residuals into a single set of weighted equations. The divergence theorem allows volume integrals to be converted to surface integrals over S1, S2 and S3, leading to:

$$\left[[K] - i\rho\omega[C] - \omega^2[M]\right] \{q\} = i\rho\omega\{F\}$$

The fluid domain is divided into acoustic finite elements, whose 'stiffness', 'mass' and 'damping' matrices (using terminology from structural FE) are assembled into the global equation system. This can be solved frequency by frequency through the range of interest. Acoustic eigenmodes can also be found. The eigenfrequencies are of interest as key characteristics of the acoustics of a closed cavity, and the eigenmodes are also of value since the response may also be described as a linear combination of eigenmodes, each mode multiplied by a participation factor. Modal superposition has a greatly reduced number of unknowns (just a few modes) compared to a direct solution.

We note that the finite element method is applicable to closed domains: if the domain is open ('free field') the mesh of elements must nonetheless be given a fictional boundary, which inevitably requires approximation to the actual (unknown) conditions at the chosen location (pressure equal to zero, absorbent surface, ...).

2.4 Boundary Element Acoustics

The direct integral formulation is based on the standard Kirchhoff-Helmholtz integral theorem, using the Green function to relate the pressure at a point in the field to the pressure and normal velocity on a closed boundary surface. The surface must be divided into boundary elements and appropriate interpolation functions selected for pressure and normal velocity, leading to a relation between nodal pressures P and nodal normal velocities Vn:

$$[\ A(k)\ ]\ (P) = [\ B(k)\ ]\ (Vn)$$

where A and B are frequency-dependent matrices, fully-populated, non-symmetric and complex. A conventional collocation scheme may be used to solve the equation system.

The indirect integral formulation allows interior and exterior fields to be handled at the same time, as well as open surfaces (where the topology makes 'interior' and 'exterior' meaningless terms). This is done using single and double layer potential densities, which are related to the jump of pressure and of the normal derivative of pressure through the boundary surface S. The resulting equation system involves hyper-singular terms and application of the traditional collocation solution scheme is not straightforward. A variational solution scheme has been used, which also ensures symmetry of the matrices. The mathematical basis may be followed more fully elsewhere (for example, Pierce [3], Filippi [4], Stalybrass [5] and Coyette et al [2]).

We can summarize the benefits and drawbacks of the direct/collocation and indirect/variational schemes in the following table.

	advantage	disadvantage
direct formulation	unknowns are physical variables (p, v)	needs CLOSED surfaces
indirect formul'n	can have OPEN surfaces	unknowns are layer potentials
collocation	faster for small problems	non-symmetric matrices
variational	faster for large problems symmetric matrices	slower for small problems

2.5 Fluid-Structure Interaction

To include the influence of the fluid on structural behaviour, and vice versa, FE or BE acoustics can be coupled to conventional FE models of the structure. The FE structural model may be defined in terms of physical coordinates, for example by importing mass, stiffness and damping matrices from standard structural FE software, or in terms of modal coordinates, importing modes from a structural eigensolution without the fluid, and then using a modal synthesis approach. An efficient approach, called the fluid methodology, reduces the equation system by eliminating all the structure-only unknowns prior to solution, so the final system is of the same order as a purely acoustic system. Modal approaches are possible, deriving the modes of the coupled system and using the modal superposition technique to calculate forced response. For FE acoustics coupled to an FE structural model, a very efficient solution technique using Ritz vectors is also possible (eg, Coyette [8]).

We note that an FE acoustics/FE structure model is limited to interior regions, a BE collocation/FE structure model may be interior or exterior to a closed structure, with fluid on one side of the structure only, and a BE variational/FE structure model is interior and exterior, closed or open, with fluid on both sides of the structure. An automatic method has been implemented to handle the compatibility requirements for the layer potentials at 'junctions' where several elements meet at one node. Naturally in all cases not all structural elements need to be in contact with the fluid.

2.6 Application and Limitations of FE and BE Acoustics

2.6.1 In summary, we note that FE acoustics is effectively limited to closed, interior regions, whereas BE acoustics handles both interior and exterior cases: the collocation approach caters for one or the other, the variational approach caters for both simultaneously and allows for cases with interior/exterior inter-connections and thin structures such as fins projecting into the fluid.

2.6.2 Frequency Range: The main limitation of both BE and FE approaches is in terms of frequency range. As a rule-of-thumb, it is found that six elements are needed per wavelength, for a reasonable accuracy. Hence, at high frequencies the mesh becomes inordinately large and the numerical problem is difficult for even today's powerful engineering workstations to solve within reasonable times. However, in the BE collocation method the asymptotic approximation may be used for acoustic radiation from many structures at high frequencies, with good accuracy.

2.6.3 Comparisons: One can remark that acoustic FE and BE are very deterministic and can be seen as complementary rather than competitive with other numerical methods such as ray-tracing or the conical beam method (see the paper by de Geest, also at Euronoise'92) since these methods are inaccurate at low frequencies when wave effects and similar phenomena are significant.

2.6.4 Absorbent Materials: We also note that for the BE approach a homogeneous fluid is implied, whereas in an FE model each element can have unique acoustic material properties. This also gives the possibility in FE acoustics for some (volume) elements to have special behaviour based on absorbent material properties, for example representing a volume of absorber such as foam or fibrous material, whereas only surface absorption can be represented in BE acoustics (as a boundary admittance condition).

2.6.5 Wave Envelope Elements: A new approach, based on wave envelope elements, allows FE acoustics to be extended to exterior applications without the approximations mentioned earlier. A volume FE mesh is used for the near field around the object and has on its outer surface a mesh of 'infinite' elements, based on the wave envelope approach. The basis for these elements is described in detail elsewhere (eg, by Astley et al [7,8]) and correctly takes into account the radiation condition. It is planned to describe some practical applications of this approach in a later paper.

3 PRACTICAL TECHNIQUES AND APPLICATIONS

3.1 Reciprocity

Since the acoustic FE or BE model (with or without structural coupling) is a linear representation of the physical problem, the reciprocity principle can be applied. For example, the influence on the sound pressure produced at the driver's ear location in a vehicle due to the vibration of different structural panels, roof, door, &c, can be predicted by placing an acoustic source at the ear position and calculating the pressures produced at the different surfaces. Plots of these pressures, or their integration over selected regions, show the relative panel influences. Phase information is also important, since some panels may cancel out the effects of others.

3.2 Using Test Data

3.2.1 Data from vibration measurements can be used to define surface velocity boundary conditions for the acoustic model, where fluid-structure interaction is not considered significant. The results of FE analysis of the structure (modes, forced vibrations) may also be used. An interesting enhancement of this is to use the structural modes of an FE model as inputs to the acoustic calculation, having corrected the structural FE model to match experimentally-derived modes (eg, see [9]).

3.2.2 Data derived from measurements on noise sources can be used to define the incident acoustic field, as a special function (eg, user-programmed) or a combination of standard spherical or other sources. This has been used, for example, to define the field emitted by open-rotor aircraft engines, which is used to find the scattered field around a fuselage, the resultant fuselage vibrations and sound transmission to the interior of the aircraft.

3.2.3 Data from tests can be used to derive acoustic property data for materials, particularly absorbers. The modelling procedure requires complex admittance data (generally, frequency-dependent) for surface absorbers, and resistivity and other parameters for volume absorbers. These can often be derived from tests, where a numerical model of the test set-up is used to correlate and derive materials data. Tests may range from simple impedance tube and flow-resistance measurements, to experiments on whole components such as car seats.

3.2.4 Correlation with testing is also of value in proving the accuracy and applicability of the FE and BE acoustics methods. This has taken place in the laboratory (for example, von Estorff et al [10]) and in industrial practice (for example, good correlation has been observed in both mode frequencies and shapes for vehicle interiors).

3.3 Applications

3.3.1 The first application (see Figures 3 and 4) shows the analysis of the response of a vehicle interior, as modes and as panel influences on the sound pressure level at the driver's ear. Knowing the panel influences, appropriate structural modifications and their influence on panel behaviour and hence acoustic response can be determined at the design stage.

3.3.2 The second application (see Figure 5) shows a plane wave impinging on a submarine hull. In this case, the hull is assumed rigid and the scattering effect is found, together with pressures on the hull.

3.3.3 The third application (see Figure 6 to 8) shows a microwave oven in which the structural response of certain parts is strongly affected by the acoustic field produced by the wave-generator. In this case, the acoustic-structure interaction is solved as one system and structural changes are made to eliminate the problem: an oven without a 'buzz'.

4 CONCLUSIONS

Finite element and boundary element acoustics models and fluid-structure interaction models allow frequency-domain acoustic behaviour to be predicted accurately. The appropriate frequency range is geometry- and problem-dependent.

The approach is deterministic and models wave effects precisely.

The indirect boundary element formulation, solved by a variational approach, is useful for geometries without inside or outside, with structures contacting the fluid on both sides, and fluid-structure interaction.

Acoustic modelling must be integrated with other predictive design techniques such as structural FE analysis, and with meaurements. Measurement data may be used to derive structural behaviour, acoustic source definitions, and acoustic materials data, as well as validation and correlation with acoustic predictions. Results in these areas are good so far. More work is needed: it is planned to report this later.

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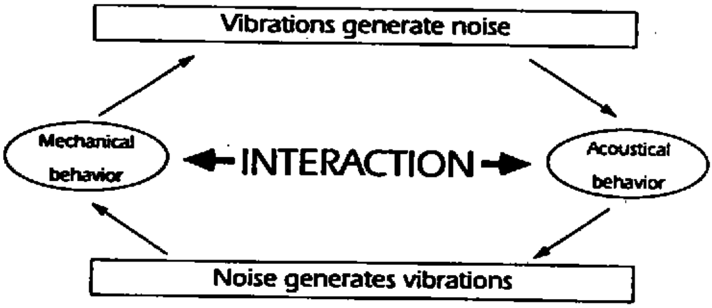
Figure 1



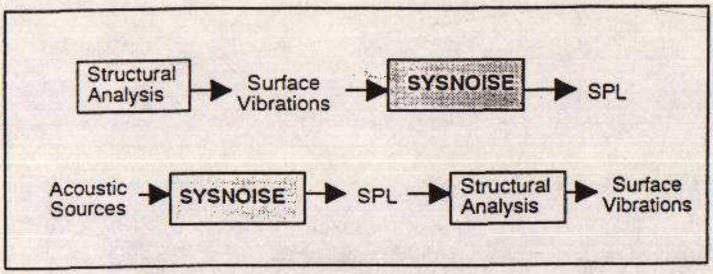


Figure 2: Procedures for uncoupled analyses

Figure 3: Vehicle interior - typical mode

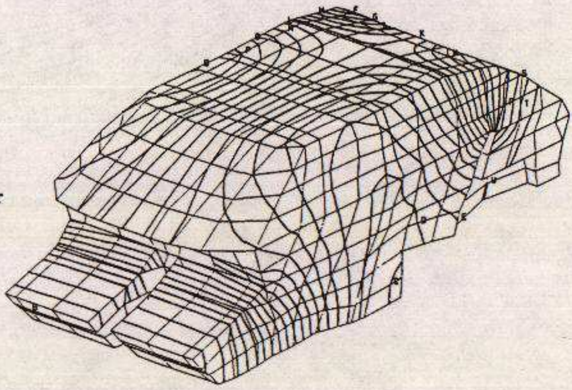
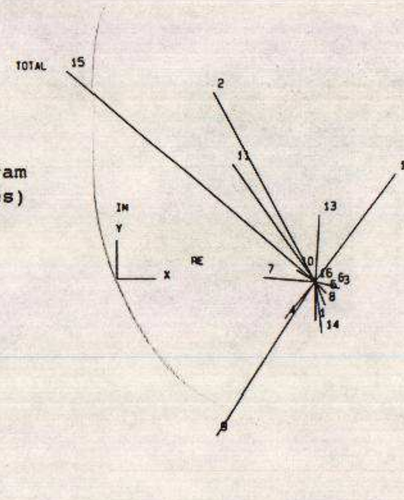


Figure 4: Typical panel influence diagram for driver's ear (with phases)



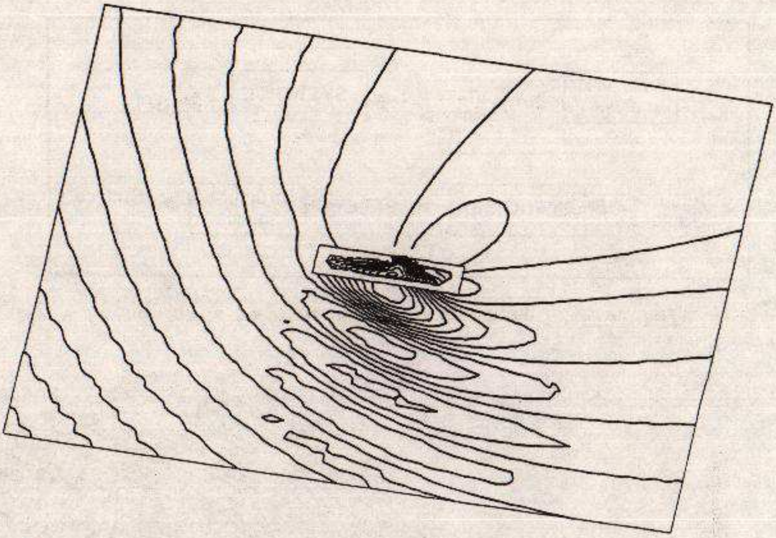
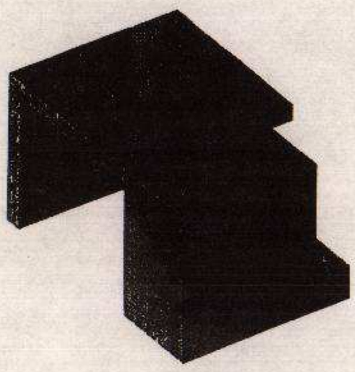


Figure 5: Submarine hull: plane wave reflection/diffraction



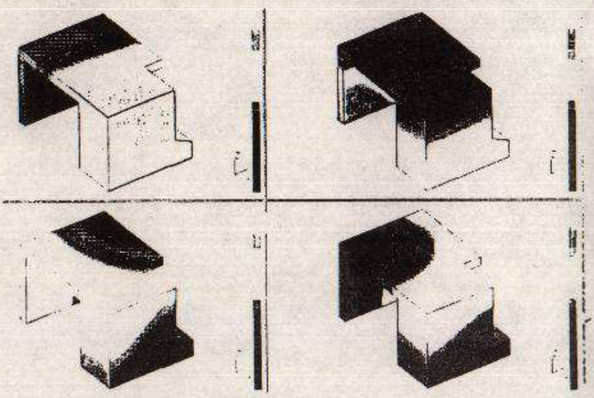


Figure 6: Microwave: FE mesh

Figure 7: Acoustic modes (uncoupled)

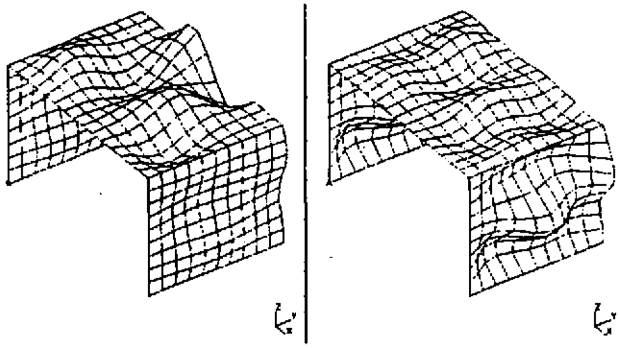


Figure 8: Microwave: typical structural FE modes (uncoupled)

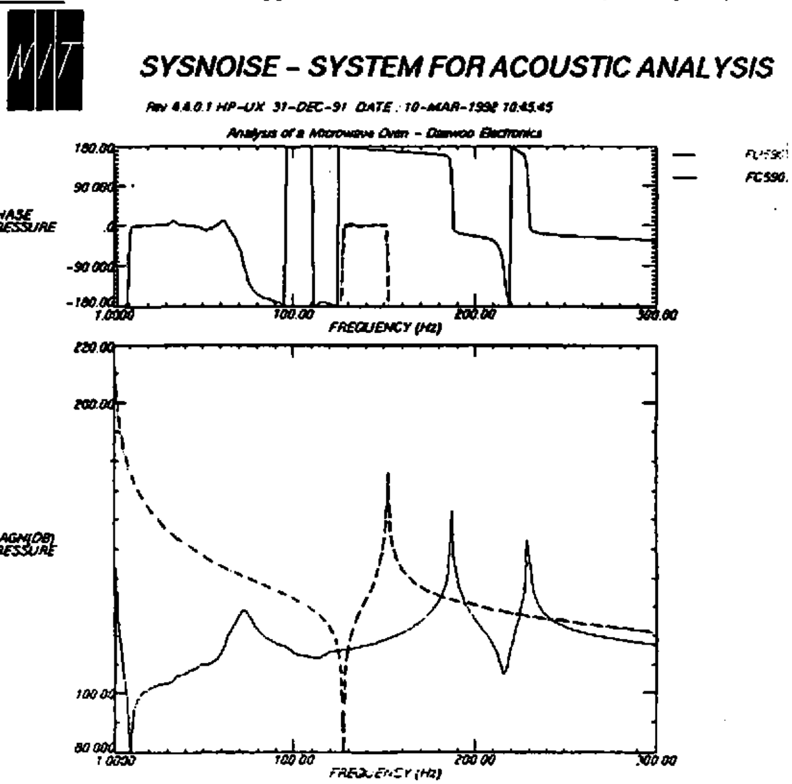


Figure 9: Microwave acoustic response, uncoupled (---) vs coupled (——)

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