NONLINEAR VIBRATION CHARACTERISTICS ANALYSIS OF THE AXIALLY MOVING PRINTING MEMBRANE

Ji-mei Wu, Zhen Tian, Yan Wang and Ming-yue Shao
Xi'an University of Technology, Xi'an 710048, China
email: wyaiyh@163.com

Xu-xia Guo
Baoji University of Arts and Sciences, Baoji 721016, China

The transverse vibration of high-speed membrane in the printing process and the "fold" phenomenon caused by vibration will seriously affect the printing overprint precision and printing quality. The nonlinear vibration characteristics of the axially moving membrane are studied. The model of the moving membrane was established. Then, the Von Karman equations expressed by the deflection function and the internal force function of axially moving membrane are derived based on the theory of elasticity. The time and spatial variables are separated by Galerkin method. Then the ordinary differential equations were analyzed by the method of multiple scales. The dynamic behaviors were identified based on the time history, phase chart and amplitude frequency curves. Periodic, quasi-periodic and multi-periodic were occurred under different speed and length-width ratio for the transverse vibration of the axially moving membrane.
Keywords: nonlinear vibration, multi-scales method, moving membrane

1. Introduction

In the actual printing production process, printing film, textile fiber cloth, printing tape, etc. can be modelled as the axially moving membrane. The lateral vibrations are inevitably produced when the membrane in the high speed printing process. The vibration can cause the phenomenon of wrinkles will seriously affect the printing of the overlay accuracy, thus affecting the quality of printed materials. .

In recent years, the research on the transverse vibration and stability of the axial motion system has made great achievements. Chen et al. [1-3] researched the bifurcation and chaos of axially moving linear beam, the nonlinear partial differential equation of the transverse vibration of an axially moving viscoelastic beam was obtained based on the constitutive relation and geometric relation. And the asymptotic analysis of axially accelerating viscoelastic string was made. Their[4] also derived nonlinear lateral vibration control equation of strings by using generalized Hamilton principle and Kelvin viscoelastic model, the governing equations are solved by using multi-scales method and the nonlinear problem of the longitudinal tension and internal resonance was studied. The nonlinear free vibration and stability of axially moving strings were studied by Wang [5], the motion differential equation of nonlinear free vibration was derived and the nonlinear response of the system was obtained. Differential quadrature method was applied to analyze the axially moving viscoelastic beam dynamics behaviour of nonlinear vibration plane by Wang [6]. Kulachenko et al. [7-8] studied the nonlinear dynamics of the fold and stability caused by the transverse vibration of the membrane by using the finite element. The nonlinear dynamics of an axially moving plate was studied by Ghayesh et al.[9-10].

2. Vibration model and solution of equation by multi-scales method

2.1 Nonlinear vibration model

Figure.1 shows the mechanical model of the axially moving membrane. The membrane has a length *a*. width *b*, and thickness *h* in the *x*, *y* and *z* direction respectively. T_x and T_y are the tensions on the boundary. $\overline{w}(x, y, \overline{t})$ is transverse vibration displacement of the membrane.

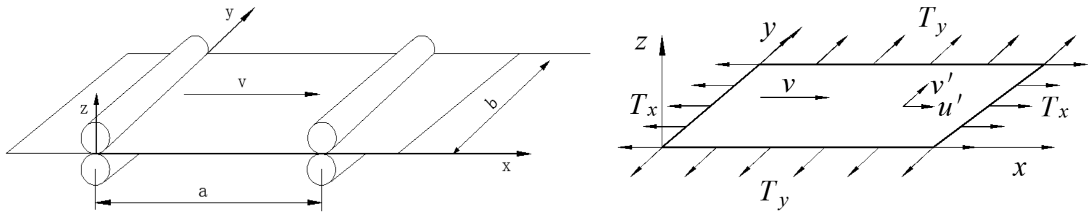


Figure1: The mechanical model of the axially moving membrane

In the problem of large deflection of membrane, the internal force and the displacement of the center plane can not be ignored. Based on the large defection theory, the Von Karman large deflection vibration equations of the moving membrane can be respectively expressed as

$$\begin{cases} \rho(\dfrac{\partial^2 \overline{w}}{\partial \overline{t}^2} + 2v\dfrac{\partial^2 \overline{w}}{\partial x \partial \overline{t}} + v^2 \dfrac{\partial^2 \overline{w}}{\partial x^2}) - \dfrac{\partial^2 \varphi}{\partial y^2}\dfrac{\partial^2 \overline{w}}{\partial x^2} - \dfrac{\partial^2 \varphi}{\partial x^2}\dfrac{\partial^2 \overline{w}}{\partial y^2} = 0 \\ \dfrac{\partial^4 \varphi}{\partial x^4} + \dfrac{\partial^4 \varphi}{\partial y^4} = Eh[(\dfrac{\partial^2 \overline{w}}{\partial x \partial y})^2 - \dfrac{\partial^2 \overline{w}}{\partial x^2}\dfrac{\partial^2 \overline{w}}{\partial y^2}] \end{cases} \tag{1}$$

Introduce the dimensionless quantities

$$\begin{aligned} &\xi = \frac{x}{a},\ \eta = \frac{y}{b},\ w = \frac{\overline{w}}{h},\ t = \overline{t}\sqrt{\frac{Eh^3}{\rho a^4}}, \\ &c = v\sqrt{\frac{\rho a^2}{Eh^3}},\ r = \frac{a}{b},\ f = \frac{\varphi}{Eh^3} \end{aligned} \tag{2}$$

Eq.(1) takes the form

$$\begin{cases} (\dfrac{\partial^2 w}{\partial t^2} + 2c\dfrac{\partial^2 w}{\partial \xi \partial t} + c^2 \dfrac{\partial^2 w}{\partial \xi^2}) - r^2 \dfrac{\partial^2 f}{\partial \eta^2}\dfrac{\partial^2 w}{\partial \xi^2} - r^2 \dfrac{\partial^2 f}{\partial \xi^2}\dfrac{\partial^2 w}{\partial \eta^2} = 0 \\ \dfrac{\partial^4 f}{\partial \xi^4} + r^4 \dfrac{\partial^4 f}{\partial \eta^4} = r^2(\dfrac{\partial^2 w}{\partial \xi \partial \eta})^2 - r^2 \dfrac{\partial^2 w}{\partial \xi^2}\dfrac{\partial^2 w}{\partial \eta^2} \end{cases} \tag{3}$$

Boundary condition

$$\begin{aligned} &\xi = 0,\ 1:\ \frac{\partial^2 f}{\partial \xi^2} = 1, w = 0 \\ &\xi = 0,\ 1:\ \frac{\partial^2 f}{\partial \eta^2} = 1, w = 0 \end{aligned} \tag{4}$$

The equations are derived using the Galerkin method. The dimensionless transverse displacement approximation function $w(x, y, t)$ and the dimensionless internal force function $f(x, y, t)$ can be respectively expressed as

$$w(\xi,\eta,t)=W(\xi,\eta)q(t) \tag{5}$$

$$f(\xi,\eta,t)=F(\xi,\eta)q^2(t) \tag{6}$$

Substituting Eq.(5)(6) into Eq.(3), using Galerkin method yields the following equation

$$\begin{aligned}\iint_s \Bigg\{ \Bigg(& W\frac{\partial^2 q(t)}{\partial t^2}+2c\frac{\partial W}{\partial \xi}\frac{\partial q(t)}{\partial t}+c^2\frac{\partial^2 W}{\partial \xi^2}q(t) \\ & -r^2\frac{\partial^2 F}{\partial \eta^2}\frac{\partial^2 W}{\partial \xi^2}q^3(t)-r^2\frac{\partial^2 F}{\partial \xi^2}\frac{\partial^2 W}{\partial \eta^2}q^3(t)\Bigg\} W(\xi,\eta)ds=0\end{aligned} \tag{7}$$

The nonlinear vibration differential equation can be expressed as

$$A\ddot{q}+B\dot{q}+Cq+Dq^3=0 \tag{8}$$

where

$$A=\iint_s W^2 ds$$

$$B=2c\iint_s \frac{\partial W}{\partial \xi}W ds$$

$$C=c^2\iint_s \frac{\partial^2 W}{\partial \xi^2}W ds$$

$$D=-r^2\iint_s\left(\frac{\partial^2 F}{\partial \eta^2}\frac{\partial^2 W}{\partial \xi^2}+\frac{\partial^2 F}{\partial \xi^2}\frac{\partial^2 W}{\partial \eta^2}\right)W ds$$

The displacement function satisfying the boundary condition is given by

$$W(\xi,\eta)=\sin\pi\xi\sin\pi\eta \tag{9}$$

Eq.(8) takes the form

$$\ddot{q}+\omega_0{}^2 q-kq^3=0 \tag{10}$$

Where $\omega_0{}^2=-\pi^2c^2$, $k=-\frac{\pi^4}{16}\left(1+r^4\right)$

2.2 Multi-Scales method

In this paper, the multi-scales method is used to study the nonlinear vibration of printed membrane. Assuming that the motion of the system varies according to different time scales. $T_0=t$, $T_1=\varepsilon t$. Where ε is small. At the same time, the formula (10) is introduced into a small quantity ε.

$$\ddot{q}+\omega_0{}^2 q=\varepsilon kq^3 \tag{11}$$

Assuming the solution of the Eq. (11) is

$$q=q_1(T_0,T_1)+\varepsilon q_2(T_0,T_1) \tag{12}$$

Substituting Eq.(12) into Eq.(11), yields the following equation

$$\left[D_0^{\,2}+2\varepsilon D_0 D_1\right]\left[q_1+\varepsilon q_2\right]+\omega_0^{\,2}\left(q_1+\varepsilon q_2\right)=\varepsilon k\left(q_1+\varepsilon q_2\right)^3 \tag{13}$$

Let the same power coefficients of ε be equal, and the linear differential equations of the order can be expressed as

$$D_0^{\,2}q_1+\omega_0^{\,2}q_1=0 \tag{14a}$$

$$D_0^{\,2}q_2+\omega_0^{\,2}q_2=kq_1^{\,3}-2D_0D_1q_1 \tag{14b}$$

The solution of the Eq. (14a) is given by

$$q_1=A\left(T_1\right)e^{i\omega_0T_0}+\overline{A}\left(T_1\right)e^{-i\omega_0T_0} \tag{15}$$

Substituting Eq.(15) into Eq.(14b), yields the following equation

$$D_0^{\,2}q_2+\omega_0^{\,2}q_2=\left(3kA^2\overline{A}-2i\omega_0D_1A\right)e^{i\omega_0T_0}+kA^3e^{3i\omega_0T_0}+cc \tag{16}$$

The amplitude-frequency characteristic relation can be expressed as

$$a_s=\sqrt{\left(s-1\right)\frac{8\omega_0^{\,2}}{3\varepsilon k a_0^{\,2}}} \tag{17}$$

where $a_s=\dfrac{a}{a_0}$, $s=\dfrac{\omega}{\omega_0}$

Approximate solution of nonlinear differential equation for moving membrane can be expressed as

$$q_1=\frac{1}{2}a_0e^{i\left(-\frac{3\varepsilon k a_0^{\,2}}{8\omega_0}t+\theta_0\right)}e^{i\omega_0t}+cc=a_0\cos\varphi \tag{18}$$

$$q_2=-\frac{ka_0^{\,3}}{64\omega_0^{\,2}}a_0e^{3i\left(-\frac{3\varepsilon k a_0^{\,2}}{8\omega_0}t+\theta_0\right)}e^{3i\omega_0t}+cc=-\frac{ka_0^{\,3}}{32\omega_0^{\,2}}\cos3\varphi \tag{19}$$

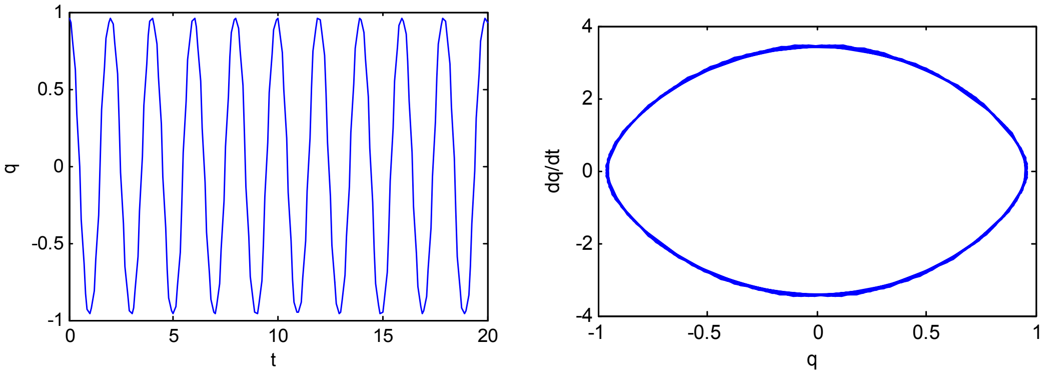
where $\varphi=\left(-\dfrac{3\varepsilon k a_0^{\,2}}{8\omega_0}+\omega_0\right)t+\theta_0$

3. Calculation results and analysis

3.1 Time history and phase diagram

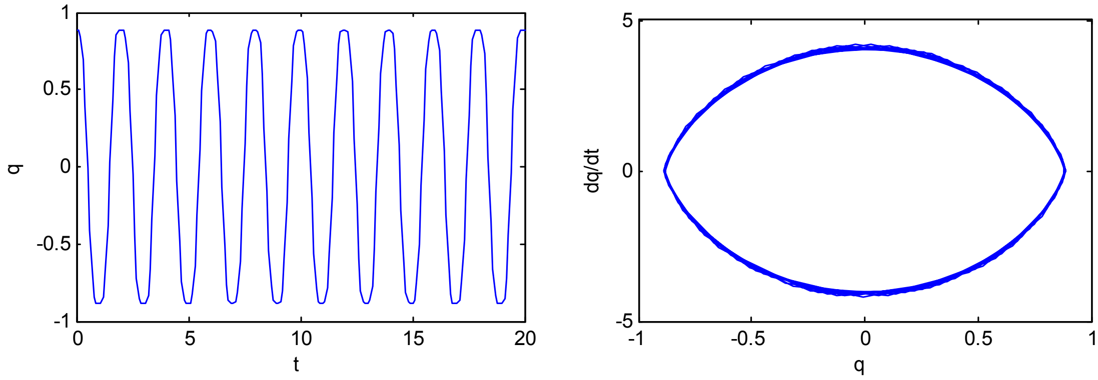
According to the large deflection differential equation of the printing membrane, and the dimensionless parameters are given, $\varepsilon=0.01$, $a_0=1$, $\theta_0=0$. Figure 2 shows the nonlinear time-history and phase diagram of the moving printing membrane when dimensionless velocity $c=1$ and the length width ratio is respectively $r=1,\ 1.5,\ 2$. When the dimensionless velocity of the membrane is kept constant and the aspect ratio increase, the time-history shows a small cycle fluctuation, and the amplitude is slightly reduced. At the same time, the phase diagram gradually split into two circles, there is a saddle and two centers. It indicates that when the aspect ratio

$r = 2$, the vibration from the original single cycle movement gradually becomes into the doubling cycle movement.



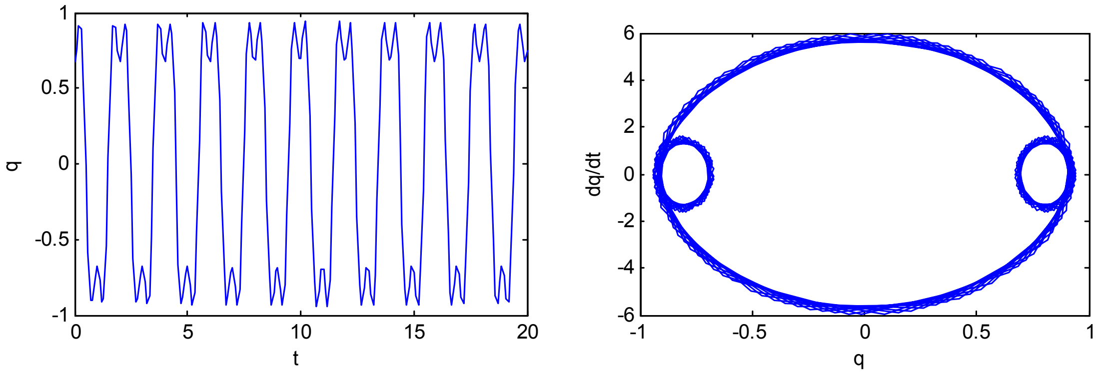
（a） Time history （b） Phase chart

c=1， r=1



（a） Time history （b） Phase chart

c=1， r=1.5



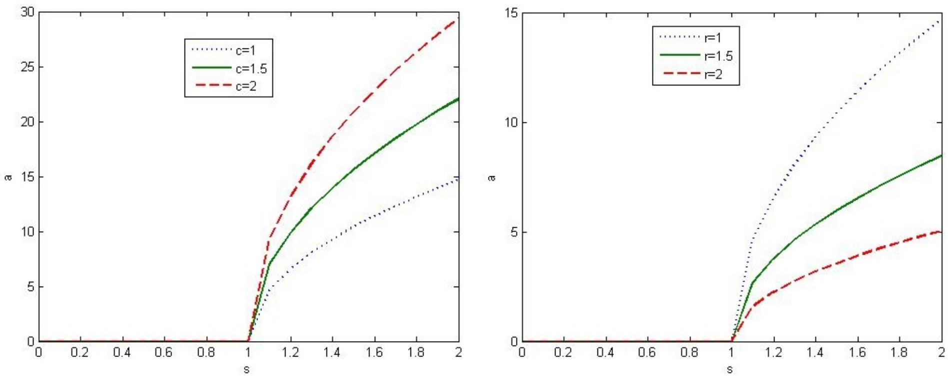
（a） Time history （b） Phase chart

c=1， r=2

Figure2: The time history, the phase chart for different length-width ratio

3.2 Amplitude frequency curves

In the case, dimensionless parameter are give， $a_0 = 1$， $\theta_0 = 0$. Figure 4a show the amplitude frequency curves of the nonlinear free vibration of the axially moving printing membrane with the aspect ratio $r = 1$ and the dimensionless velocity $c = 1,\ 1.5,\ 2$ respectively. When the frequency $s < 1$, the amplitude only virtual part, when the amplitude have a real part, the imaginary part is zero. At the same time, it can be seen that the amplitude of the system is increasing as the dimensionless the increase of velocity, and the slope of the curve increases. It indicated that the increase of dimensionless speed makes the system become more unstable. Figure 4b shows the amplitude frequency curve of the nonlinear free vibration of axially moving printing membrane with the dimensionless velocity $c = 1$, and the aspect ratio $r = 1,\ 1.5,\ 2$ respectively. It indicated that the amplitude of the system decreases with the increase of the aspect ratio, and the slope of the curve decreases. It is shown that the increase of aspect ratio makes the system become more stable.



(a) r=1 (b) c=1

Figure3: Amplitude frequency curves under different speed and different length-width ratio

4. CONCLUSION

The nonlinear vibration properties of the moving membrane are studied, and the Von Karman equation expressed by the defection function and the internal force function is deduced by the elastic theory. The nonlinear vibration characteristic of the moving printed membrane is analyzed. The time variables and spatial variables, displacement functions and stress function are separated by Galerkin method, and then the ordinary differential equations are obtained and analysed by multi-scale method. Through the system's time histogram, phase diagram and amplitude-frequency characteristic curve, the results of this study can be summarized as follows:

(1) As the aspect ratio increases, the system enters the doubling period vibration, and the system amplitude decreases and the system becomes more stable. It indicated that the aspect ratio of the printed membrane is increased, can improve the stability of the membrane motion system, and also can effectively ensure that the membrane work in a stable state.

(2) The greater the dimensionless speed, the more obvious the large deflection nonlinear vibration, the system becomes into multiple cycles. While the system amplitude increases, the system becomes more unstable. Therefore, the speed of the printing membrane can be appropriately reduced, the stability of the system can be improved, and the generation of nonlinear vibration with large deflection can be effectively avoided.

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