Probability Distribution of Intensity for Acoustic Propagation

J.A. DeSanto and R.N. Baer
Naval Research Laboratory
Washington, D.C. 20375 USA

ABSTRACT

This paper is in two parts. In the first part we discuss a simple method of correcting the parabolic approximation (PA), and demonstrate how this corrected parabolic approximation (CPA) is an improvement on the PA by comparing the respective results to a normal mode (NM) example. The improvement is most marked in the phase results. In the second part of the paper we use this CPA to propagate an acoustic field through a sound speed structure generated by mesoscale variation in the ocean using data described in a separate paper to this conference by Drs. J. Dugan and W. Emery. A numerical statistical experiment on this data is described and its results compared to other work.

1. A correction to the Parabolic Approximation

The parabolic approximation (PA) method has been shown to be an effective method of computing acoustic fields in regions where the sound speed is a function of both range and depth. Recent analytic results using the PA enable one to construct exact solutions to the Helmholtz equation via conformal mapping techniques[1], and to exactly relate the solutions of the Helmholtz and parabolic equations by means of an integral transform[2]. The results (for cylindrical coordinates) can be stated quite simply as follows. Given that Ψ is a solution of the Helmholtz equation

$$[\nabla^2 + k^2 K(r,z)]\Psi = 0 , \qquad (1)$$

where $k=\omega/c_o$, ω is the frequency, c_o is a reference sound speed, and $K=[c_o/c(r,z)]^2$ with $c(r,z)$ the range- and depth-dependent sound speed. Given that p satisfies the parabolic partial differential equation

$$2ik\, p_r + p_{zz} + k^2(K-1)\, p = 0 \;, \tag{2}$$

then Ψ and p can be exactly related using the integral transform

$$\Psi\,(r,z) = A \int_0^{\infty} p(t,z)\; R(r,t,z) \exp[(ik/2t)\;(r^2+t^2)]\; t^{-1}\, dt \;, \tag{3}$$

where R satisfies the differential equation listed in Ref. 2, Eq. (9), and A is a constant. The transform has a stationary phase point at t=r and the resulting stationary phase evaluation of the integral yields a correction to the parabolic approximation (CPA) given by

$$\Psi_{CPA}(r,z) = \left[p(r,z) - (ir/2k) p_{rr} \right] (kr)^{-1/2} \exp(ikr) \;. \tag{4}$$

The PA result follows by dropping the p_{rr} term in Eq. (4). Comparison with an exact normal mode (NM) example[3] using the error function

$$E(r) = \left\{ \frac{1}{J} \sum_{j=1}^{J} \left| \frac{\Psi_{NM}(r,z_j) - \Psi(r,z_j)}{\Psi_{NM}(r,z_j)} \right|^2 \right\}^{1/2} \;,$$

where Ψ is either the PA or CPA yields the result shown in the figure. The CPA is shown to give a marked improvement in the phase results. Amplitude and dB error results show similar improvement.

2. Propagation through mesoscale variation in the ocean

The above method has been used to treat acoustic propagation through an analytic model of an eddy[4], with the position of the source relative to the eddy causing transmission loss changes up to 20 dB. Here we discuss the use of the CPA to propagate an acoustic field through a sound speed structure generated by mesoscale variation in the ocean using data described in a separate paper to this conference by Drs. J. Dugan and W. Emery. The data contains considerable variation in the sound speed with range. A single track of the data is about 2500 km long, with a sound speed profile roughly every 25 km. A single realization of the CPA is defined as propagation at 50 Hz through 250 km of this data using a Gaussian initial field and a shallow source. The initial range of the incident field is assumed to be a random variable distributed uniformly between 0 and 2250. A total of 100 realizations define the ensemble. The resulting probability distribution function (PDF) and variance of intensity are graphed as a function of range and depth. Preliminary results indicate that the PDF appears to be Rayleigh and that it saturates quickly in range with a variance approaching 6 dB. The mean changes strongly with position in the field. The results will be discussed in the light of work done by Dyer and Shepard[5] and others.

REFERENCES

(1) J.A. DeSanto, Proc. NATO Conf. on Oceanic Acoustic Modeling, ed. by W. Bachmann and R.B. Williams (SACLANTCEN, La Spezia, Italy, 1975) pp. 43-1 to 43-17.

(2) J.A. DeSanto, J. Acoust. Soc. Am. 62, 295-297 (1977).

(3) J.A. DeSanto, J.S. Perkins, R.N. Baer, "A Correction to the Parabolic Approximation", submitted to J. Acoust. Soc. Am.

(4) R.N. Baer, "Calculations of Sound Propagation Through an Eddy", submitted to J. Acoust. Soc. Am.

(5) I.Dyer, G.W. Shepard, J. Acoust. Soc. Am., 61, 937-942 (1977).

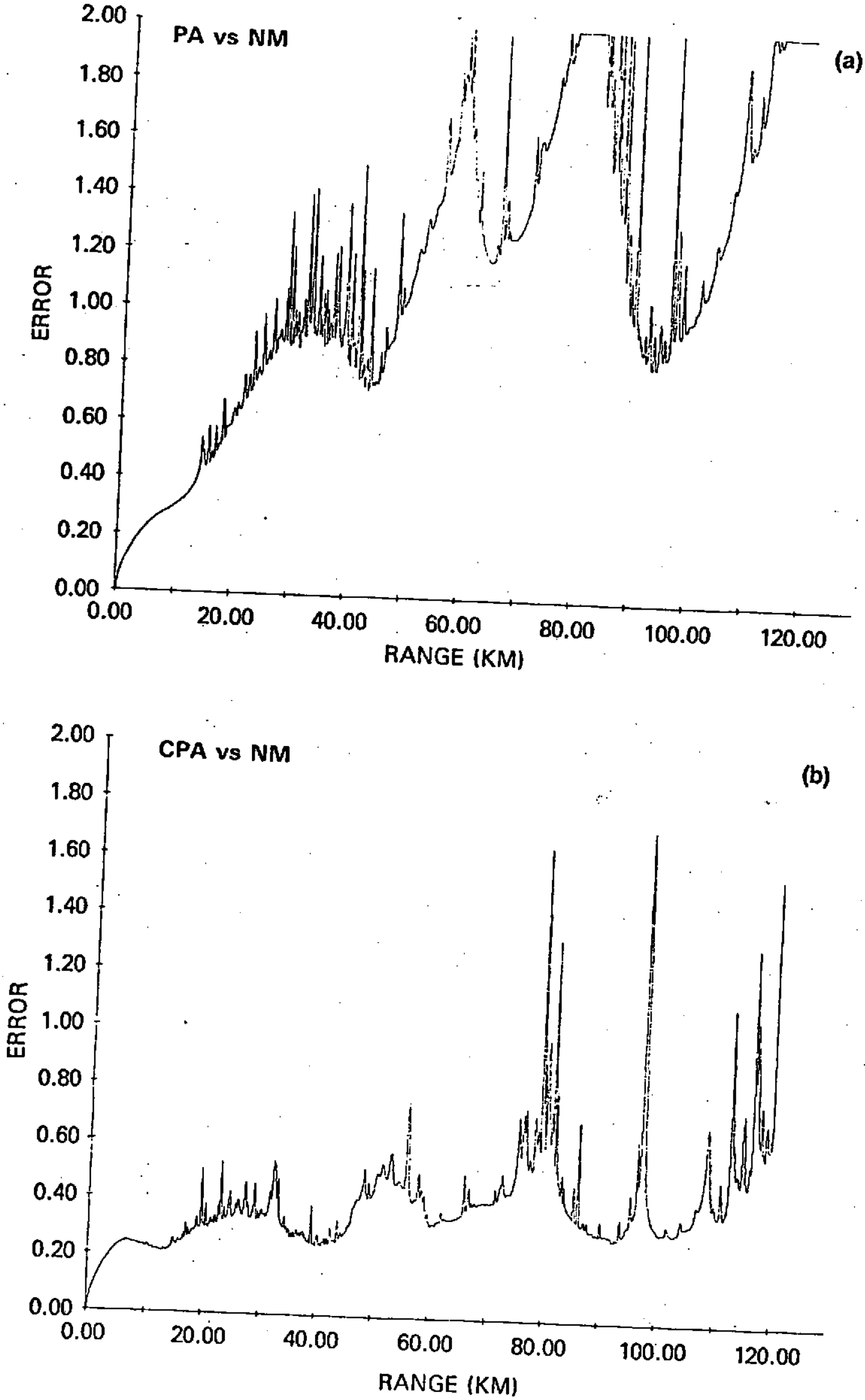

Fig. 1 Error results for (a) the parabolic approximation (PA) versus a representative normal mode (NM) example and (b) the corrected parabolic approximation (CPA) versus the NM example.