Proceedings of the Institute of Acoustics

THE EFFECT OF ACOUSTIC DIFFUSERS ON ROOM MODES

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1. INTRODUCTION

Studios, concert halls, control and dubbing rooms, reverberation chambers, or any place where high quality acoustics are required need to have a diffuse reverberant sound field. The means of achieving this has vexed acousticians throughout history. Traditionally, plaster mouldings, niches and other decorative surface irregularities have been used to provide diffusion in an "ad hoc" manner. More recently diffusion structures based on patterns of wells whose depths are formally defined by an appropriate mathematical sequence have been proposed and used [1-3].

However, in many practical applications the acoustic designer is faced with limited space in which to work. Unfortunately, diffusers take up space and the designer may face an uncomfortable compromise on the effectiveness of the diffusion and floor area of the final design. In a previous paper [4] we described a diffuser design which had a reduced depth for the same effect as conventional designs and so reduced these constraints. Another way of avoiding the constraints of space is to install the minimum amount of required diffusion material. However in order to do this we need to understand more fully the effect of diffusers in rooms, particularly at the lower frequencies where modal behaviour becomes significant.

The purpose of this paper is to discuss the effect of diffusers on room modes. The paper will first discuss what is meant by a diffuse field. It will then discuss the effect of diffusers on a room mode with a simple theoretical model. We will then discuss the implications of this model on the bandwidth and density of room modes. Finally we will present some simulations and measurements on an electromagnetic scale model of a reverberant room and will discuss the implication of the results for the design of studios, performance spaces and reverberation chambers.

2. WHAT DO WE MEAN BY A DIFFUSE FIELD?

One of the requirements of high quality acoustic spaces is the presence of a diffuse field. But what do we mean when we use this term? Qualitatively one can say that the sound is approaching the measurement point from all possible directions with equal probability. One can also say that the intensity of such a field should be independent of the position in the room. At a more quantitative level we require the following two requirements on the modal structure to be met.

1) **In terms of the frequency response.** Modes overlap considerably with respect to their bandwidth. A minimum overlap, suggested by Schroeder [5] for use in acoustic reverberation chambers, is a factor of 3:1. That is, the typical spacing between modes is ≤ 1/3 of their bandwidth.

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2) **In terms of the spatial variation.** Ideally the reverberant field amplitude should be same irrespective of the room position of source or receiver. There are two factors at work here, one due to modal behaviour and the other due to absorption. Modes clearly have extreme spatial variation. However, this variation is reduced if the modes are lossy. The presence of absorption will result in a net energy flow and thus an energy gradient towards the absorbing material. This will increase the spatial variance. If one thinks of a mode as having an amplitude variation between 0 and 2 with a mean of one, and if one assumes that the modes are randomly distributed, then one could say the following. Variation is reduced as $\frac{1}{\sqrt{N}}$ where N is the number of modes which are excited by the frequency component under consideration. This means that between 4 to 18 modes must be excited by a single frequency component for a spatial variation of plus or minus 1dB to plus or minus 0.5dB respectively.

Both these requirements translate into a requirement for considerable modal overlap over the frequency range of interest, if a diffuse field is required. They also indicate that adding excess absorption into the room is not necessarily the way to achieve a diffuse field in a given space.

There is a relationship between the modal density of a room and its size but how does this relate to the above requirements?

3. THE MODAL DENSITY IN "ORDINARY" ROOMS

For a room the modal frequencies are given by the following equation:

$$f_{xyz} = \frac{c}{2}\sqrt{\left(\frac{x}{L}\right)^2 + \left(\frac{y}{W}\right)^2 + \left(\frac{z}{H}\right)^2} \qquad (1)$$

From the above equation we see that we get a higher modal density if we choose non-commensurate ratios for the walls. One set of favourable room dimensions are shown in table 1.

	Height	Width	Length	Volume	Surface Area	Sum of Edges	Mean Free Path
A	1.00	1.14	1.39	1.58	8.22	14.12	0.77
B	1.00	1.28	1.54	2.82	11.28	15.28	1.00
C	1.00	1.60	2.33	3.73	15.32	19.72	0.97

Table 1 Some Favourable Room Dimensions

By considering modes given by equation (1) as vectors in frequency space the total number of modes in a room up to a given frequency can be given by equation (2) [5] assuming the room modes are not degenerate:

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$$N = \frac{4\pi V}{3c^3} f^3 + \frac{\pi S}{4c^2} f^2 + \frac{E}{8c} f \qquad (2)$$

where V = the room volume, S = the surface area, and E = the sum of the edges of the room.

By redefining the dimensions of the room in terms of the wavelength equation (2) can also be expressed as:

$$N = \frac{4\pi V_\lambda}{3} + \frac{\pi S_\lambda}{4} + \frac{E_\lambda}{8} \qquad (3)$$

where V_λ = the room volume, S_λ = the surface area, and E_λ = the sum of the edges of the room measured in terms of the wavelength.

By further normalising the dimensions with respect to the number of wavelengths we get:

$$N = \frac{4\pi V'_\lambda n^3}{3} + \frac{\pi S'_\lambda n^2}{4} + \frac{E'_\lambda n}{8} \qquad (4)$$

where V'_λ = the normalised room volume, S'_λ = the normalised surface area, E'_λ = the normalised sum of the edges of the room measured in terms of the wavelength, and n is the number of wavelengths.

By differentiating equation (2) with respect to f we can get an approximate value for the number of modes for a given bandwidth which is given by:

$$\Delta N \approx \left[\frac{4\pi V}{c^3} f^2 + \frac{\pi S}{2c^2} f + \frac{E}{8c}\right]\Delta f. \qquad (5)$$

Again we can express this in terms of the normalised wavelength $n\lambda$ to give:

$$\Delta N \approx \left[\frac{4\pi V'_\lambda n^3}{1} + \frac{\pi S'_\lambda n^2}{2} + \frac{E'_\lambda n}{8}\right]\frac{\Delta f}{f} \qquad (6)$$

The bandwidth of room modes are related to reverberation time T_{60} by the following equation:

$$\Delta f = \frac{2.2}{T_{60}} \qquad (7)$$

For reverberation times between .5s to 1s this corresponds to a modal bandwidth of 4.4Hz to 2.2Hz. For a diffuse field, as defined previously, we require an overlap of about 4 to 1 which implies a ΔN of 4 within a typical modal bandwidth. At low frequencies $\frac{\Delta f}{f}$ is about 0.1 which translates into an $n\lambda$ of about one wavelength for a room of dimensions similar to B in Table 1. This translates into a minimum dimension of about 10m for a diffuse field at 34Hz. This represent a large room such as a concert hall. Unfortunately many recording and measurement spaces must be much smaller. One could deaden the room further and thus increase the modal

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bandwidth however, this increases the spatial variance, is undesirable aesthetically, and only gives a small improvement in size.

So one requires a means of broadening the bandwidth of modes without introducing any loss into the system. The answer is to use psuedorandom diffusing structures to break up the coherence of reflections. But what effect would such structures have on mode bandwidth and modal density?

4. THE EFFECT OF DIFFUSION ON MODES

Consider the following situation, two parallel reflecting surfaces separated by a distance L metres (see fig 1):

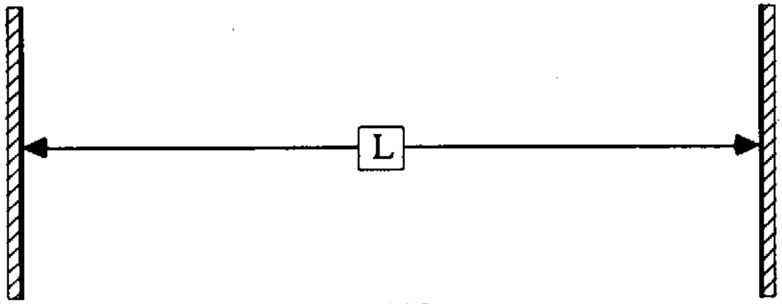

Figure 1 A Modal System

If we consider the impulse response of this arrangement we see that it is a regular train of impulses separated by $\frac{2L}{c}$ seconds. This gives us a frequency response consisting of a infinite set of resonances with zero bandwidth at frequencies given by:

$$\frac{nc}{2L} \text{ where } n = 1,2,\ldots\infty. \tag{8}$$

Now suppose that the reflectors only reflect part $\frac{1}{N}$ of their energy back in the normal direction the rest being scattered away from this cavity. So after n reflections the energy (ε) shuttling between the reflectors is given by:

$$\varepsilon = \varepsilon_0 \left(\frac{1}{N}\right)^n \tag{9}$$

The time taken for this is given by:

$$t = \frac{nL}{c} \tag{10}$$

so

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$$n = \frac{tc}{L} \tag{11}$$

thus the energy after n reflections can be rewritten as:

$$\varepsilon = \varepsilon_0 \left(\frac{1}{N}\right)^{\frac{tc}{L}}. \tag{12}$$

This can be converted into the more convenient form:

$$\frac{\varepsilon}{\varepsilon_0} = e^{\left(t\left(\frac{c}{L}\right)\ln\left(\frac{1}{N}\right)\right)} \tag{13}$$

which can be rewritten as:

$$\frac{\varepsilon}{\varepsilon_0} = e^{-t\frac{c}{L}\ln(N)} \tag{14}$$

which we can compare to:

$$\frac{\varepsilon}{\varepsilon_0} = e^{-\frac{t}{\tau}} \tag{15}$$

the equation for a simple exponential decay.

Thus we can say that the effect of diffusion is to broaden the mode bandwidth, **assuming none of the scattered energy returns coherently**. From the above equations we can say that:

$$\tau = \frac{L}{c\ \ln(N)} \tag{16}$$

and from this we can say that the bandwidth of the mode is given by

$$Bw = \frac{c\ \ln(N)}{\pi L} \tag{17}$$

we can use this bandwidth to define the "Q" of the lowest mode, the higher order modes will have a higher "Q" because the bandwidth is constant. This "Q" is given by:

$$\text{"Q"} = \frac{2\pi}{\ln(N)} \tag{20}$$

for reasonable ratios (say 7) this gives a "Q" of about 3. Assuming that scattered energy does not return coherently we can adopt the above analysis for a room but using the mean free path instead of L. The mean free path of a room is given by:

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$$mfp = \frac{4V}{S} \tag{21}$$

where V=volume and S=surface area

If we put

$$\Delta f = \mathrm{B}w = \frac{\mathrm{c}\ln(N)}{\pi L} \tag{22}$$

and

$$L = \frac{4V}{S} \tag{23}$$

in and substitute into equation (5) we get

$$\Delta N \approx \left[\frac{4\pi V}{c^3} f^2 + \frac{\pi S}{2c^2} f + \frac{E}{8c}\right] \frac{\mathrm{c}\ln(N)}{\pi} \frac{S}{4V}. \tag{24}$$

This gives us the approximate number of modes within the modal bandwidth of a diffuse room. If we change the measurement of length into normalised wavelength then we get:

$$\Delta N \approx \left[\frac{4n^2 V'_\lambda}{1} + \frac{n\, S'_\lambda}{2} + \frac{E_\lambda}{\pi 8}\right] \frac{\ln(N)}{1} \frac{S'_\lambda}{4V'_\lambda}. \tag{25}$$

For room B in Table1 with n=1 (height =λ) and N=7 we find that $\Delta N = 34$. Now we require ΔN to be a minimum of 4 for room B this corresponds to a height of 0.2λ, a worthwhile size reduction.

5. DISCUSSION

The above analysis must be treated with some caution as it assumes that the scattered energy does not return to the modal system coherently. Unfortunately as the energy is still trapped in the room it will inevitably return and some of this energy will be coherent thus causing a mode. However, the likelihood is that the path of the energy return will be much longer and thus act as if it comes from a larger room so providing a higher mode density.

5.1 Appropriate sequences

Either quadratic residue sequences which scatter energy in all directions evenly, including the specular direction, or pseudo-random sequences (prg's), which have a null in the specular direction may be used. One could argue that prg's based on $\mathrm{GF}\left(p^q\right)$ are likely to give better performance, but they are more complicated to build. However they are more likely to meet the requirement of no return of coherent reflections.

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5.2 Extending the frequency range of the diffusers.

The frequency range of the diffusers is set by several factors. Firstly the width of the wells must be less than $\lambda/2$ at the highest frequency of use. Secondly the maximum well depth must be $\lambda/2$ at the lowest frequency of use, although it con be slightly less. The frequency range is also determined by the sequence length which must be greater than the ratio of maximum to minimum frequency. Finally the quantum step of well depths must be less than $\lambda/2$ at the highest frequency of use because when the steps equal $\lambda/2$ the diffuser looks like a flat surface. In general this means that the number of levels in a sequence must be large for a given frequency range which implies a complex and expensive manufacture. There are two ways of coping with this:-

1. **Fractal diffusers:** The boundaries for the bottoms of the wells could be scaled down (and thus higher frequency) versions of the diffuser. The could be manufactured via extrusions or stamping and so could be less expensive. This technique is probably the only way to achieve a very wide frequency range.

2. **Relatively prime opposing diffusers:** If one has two diffusers on opposite walls both having the same maximum depth but with relatively prime sequence lengths then when one sequence is looking specular the other is diffusing and vica versa. this can result in a frequency range which is the product of the two sequences. so a seven and eleven sequence would give a 77:1 frequency range. One could also apply this idea to sequences with the same number of levels but different lengths $GF\left(p^{q_1}\right)$, $GF\left(p^{q_2}\right)$ by making the step depths non-commensurate and this should achieve a similar result.

6. SIMULATION AND RESULTS

To test the above ideas we simulated an electromagnetic model of a reverberant room using the electromagnetic modelling package TLM [6] in the frequency range 0 - 3GHz. We also then measured the electromagnetic mode density in physical models of the simulated system in the frequency range 0 - 1GHz. For ease of construction and simulation we used two level pseudo-random sequences. These are less than ideal having a restricted frequency range. However they should give us some idea of how the system works. The box simulated is shown in Fig. 2 and was 0.9m by 0.45m by 0.45m with treatment on all the walls and ceiling but not the floor. The sequences were all bi-level and were all parts of the same sequence. The sequence depth was 0.045m. The simulated results are shown in Graph 1 and they clearly show a marked increase in the mode density of the treated room. The reflected energy from a source in the measured boxes is shown in Figs. 3 and 4 for the untreated and the treated rooms respectively. These are less clear cut but still show an increase in modal activity in the treated room.

7. CONCLUSION

We have presented an analysis of the effect of diffusers on room modes which demonstrates that they are effective in increasing the mode density and the diffuse field in a room. The analysis showed that it was possible to achieve an acceptably diffuse field in a room which was considerably smaller than the equivalent untreated room. Simulation and measurements also indicate an improvement.

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8. REFERENCES

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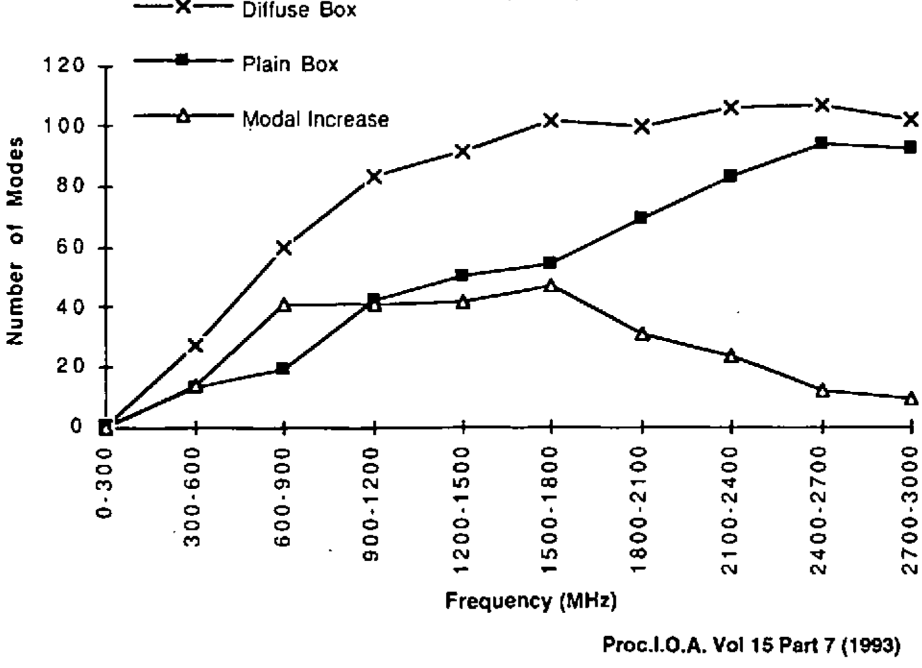
Graph 1 Average Number of Modes as a Function Frequency



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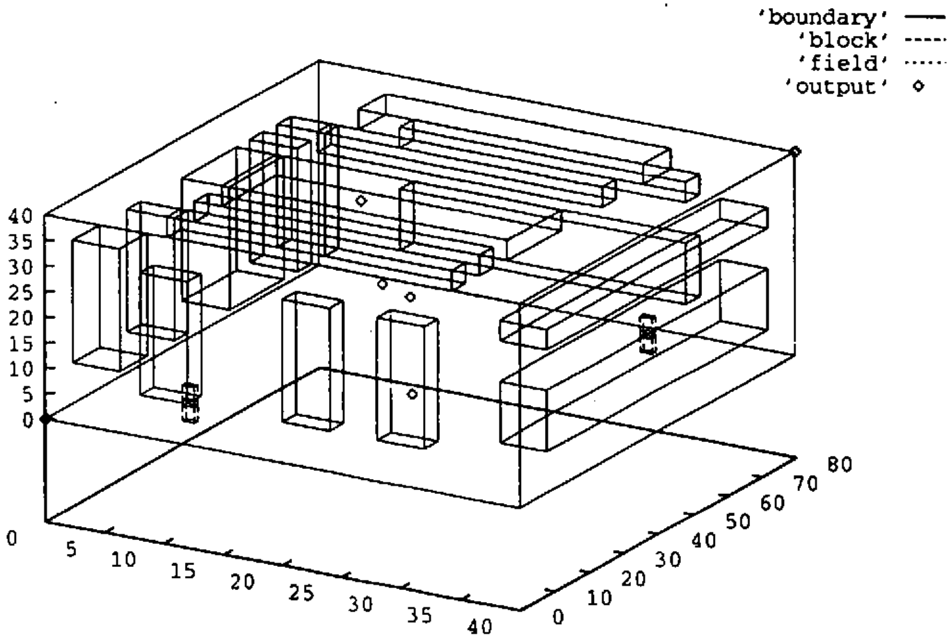

Fig. 2 A Wire Frame Drawing of the Simulated Box

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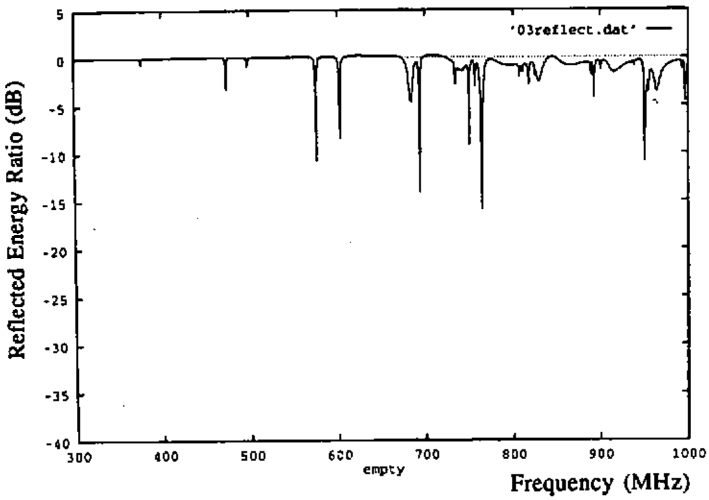

Fig. 3 Reflected Energy as a Function of Frequency for the Untreated Box.

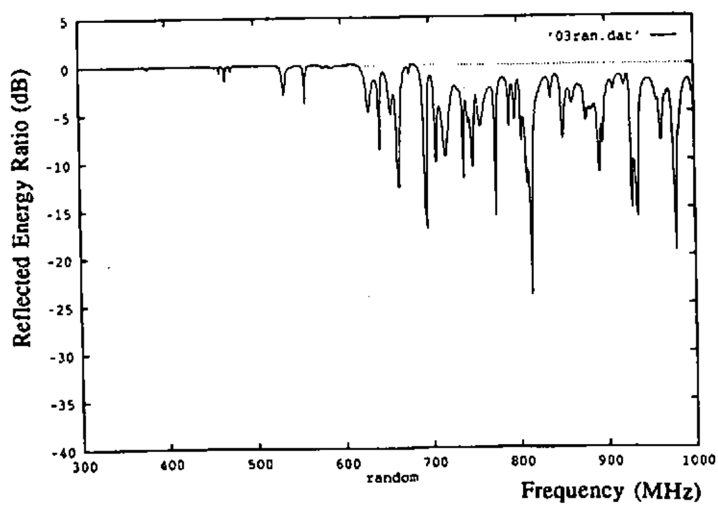

Fig. 4 Reflected Energy as a Function of Frequency for the Treated Box.