ALTERNATIVE DIFFUSER SEQUENCES

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INTRODUCTION

Studios, concert halls, control and dubbing rooms or any place where high quality acoustics are required need to have a diffuse reverberant sound field. The means of achieving this has vexed acousticians throughout history. Traditionally, the use of plaster mouldings, niches and other decorative surface irregularities have been used to provide diffusion in an "ad hoc" manner. More recently diffusion structures based on patterns of wells whose depths are formally defined by an appropriate mathematical sequence have been proposed and used [1-3].

However, in many practical applications the acoustic designer is faced with limited space in which to work. Unfortunately, diffusors take up space and the designer may face an uncomfortable compromise on the effectiveness of the diffusion and floor area of the final design.

The purpose of this paper is to present a modification of Schroeders [1-2] diffusor design which offers an equivalent performance with a modest but useful reduction in depth over the conventional sequence. The paper will first discuss how conventional diffusers work. It will then discuss the new design and finally discuss an actual implementation of the design and some practical considerations in implementing such diffusers.

HOW DIFFUSERS WORK

Consider a hard surface consisting of bumps of height d. Also consider an acoustic wavefront approaching it from a normal direction (Figure 1). The way this wavefront is reflected will depend on the height (d) of the bumps relative to its wavelength. Let us consider three cases:

i) In the case of $d \ll \lambda$ the surface will behave like a flat surface and specularly reflect the wavefront.

ii) The case of $d = \lambda/4$ the wavefronts which are reflected from the front of the bumps are reflected $\lambda/2$ earlier than those from the surface. This means that in the normal direction the wavefronts cancel and so no sound pressure is propagated in this direction. However, there has been no energy loss in the system so the wavefront must be reflected in some direction. In fact as one moves away from the normal direction the relative path lengths between the bump and the surface become less and the amplitude of the wavefront increased as one moves off the normal direction [Figure 2]. This is the basic principle behind diffusion using hard reflectors. That is, the diffusing surface modifies the phase of the wavefronts so that the reflected wave such that it must propagate in other directions than the specular direction.

iii) In the case of $d = \lambda/2$ the wavefronts from the bumps and surface are delayed by λ and so arrive back in phase. Thus the bumps disappear and the surface behaves as if it were flat. That is, it behaves like a specular reflector.

So, one has a problem, a regular sequence of bumps will diffuse but only at frequencies at which it is an odd multiple of $\lambda/4$. Note also that these frequencies will depend on the angle of incidence of the incoming wavefront.

What is required is a pattern of bumps which alter the phases of the incident in such a way that two objectives are satisfied.

i) The sound is scattered in some "optimum" manner.

ii) The scattering is optimum over a range of frequencies.

These objectives can be satisfied by several different sequences, however they share two common properties.

i) The Fourier transform of the sequence is constant except for the dc component which may be the same or lower. This satisfies objective (i) because it can be shown that reflection surfaces with such a property scatters energy equally in all directions. The effect of a reduced dc component is to further reduce the amount of energy which is reflected in the specular direction.

ii) The second desirable property of these sequences is that the Fourier transform is unaffected if the wavelength of the incident sound varies. This has the effect of changing the scale of the sequence but again one can show that the resulting sequence still has the same properties as the original sequence.

Both the above properties arise because the sequences work by perturbing the wavefronts over a full cycle of the waveform.

To make this a little clearer, let us consider the two sequences which are used for diffusers.

i) **Quadratic residue sequences**
well depth = n^2 mod p where p is a prime number if p = 5 this gives a set of well depths of

0, 1, 4, 4, 1, 0, 1 ....etc

so the sequence repeats with a period of 5;

ii) **Primitive root sequences**
well depth = a^n mod p where p is a prime and a is a suitable constant called a primitive root. For a = 2 and p = 5 we get the sequence

1, 2, 4, 3, 1, 2 ......etc

Here we have a sequence which has a period of 4 (5 - 1).

At the lowest design frequency for these examples a well of depth 5 would correspond to $\lambda/2$. Figure 3 shows what happens to a coherent wavefront at the lowest frequency and one at an octave higher when they are reflected by these example gratings. One can see that at both frequencies the wavefronts are spread in phase over the waveform cycle in both cases. It is this spreading that makes these sequences work effectively. Note that when the frequency gets high enough so that $\lambda/2$ becomes equal to the minimum difference in depths (1) then the surface again becomes equivalent to a flat surface.

DISCUSSION

As we have seen, these sequences achieve their performance by spreading the phase of the reflected wavefront over at least one cycle of the incident wavefront. In order to do this, their maximum depth must be $\lambda/2$ at the lowest design frequency. This means that to achieve diffusion a reasonable depth is required. For example, to have effective diffusion down 500Hz a depth of 34cm (13.5 inches) is required. To get down to 250Hz one would need double this depth. However, as we have seen, a simple bump of $\lambda/4$ can provide diffusion, albeit somewhat frequency dependently. This is half the depth of the above sequences and represents the ultimate limit for a diffusing object.

If one could have a sequence which achieve the phase scatter required for good diffusion using a depth closer to $\lambda/2$ at the lowest frequency it would allow better performing diffusers in restricted spaces.

AN ALTERNATIVE DIFFUSER SEQUENCE

The two sequences mentioned previously achieve similar objectives but have radically different structures. The quadratic residue sequence is symmetric and only uses about half the possible well depths, whereas the primitive root sequence is non symmetrical and uses all the possible well depths except for zero. However, they both achieve similar performances. Given this, the author speculated that perhaps one could modify the quadratic residue sequence to have a reduced range

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of depths but still have the same performance. The way to do this is to recognise that a phase shift of one cycle plus ø (2π + ø) is equivalent, for our purposes, to a phase shift of ø. Consider the effect of adding 1 to the example quadratic residue sequence. This is equivalent to moving the diffuser back one unit [Figure 4]. If one does this, one notices that the sequence changes from

0,1,4,4,1,0,1,.........

to

1,2,5,5,2,1,2,..........

However, a well of depth 5 is equivalent to a depth 0 so the above sequence becomes

1,2,0,0,2,1,2,.........

Note that this sequence has a half the depth variation of the original sequence but it still perturbs the phases of the wavefront in the same way [Figure 5].
In other words, one can have a diffuser which has the same performance but a reduced depth. The example quoted is the best one! Unfortunately it does not have a very broad frequency range. As one goes to longer sequences, the gain becomes less but nevertheless it can still be useful. Figures 6 and 7 show a diffuser based on a length 13 sequence which achieves a 75% reduction in depth. Some sequence lengths admit a greater depth reduction than others. For example, the standard length of 7 sequences is already optimum having maximum depth of 4.

In case all of this seems to good to be true, the following section give a mathematical justification of this modification

MATHEMATICAL JUSTIFICATION

The effect of each well is to phase shift the wavefront incident on each well by an amount dependent on the well depth. Thus one can view the reflected wavefront on a sequence of phase shifts, or in other words, a complex sequence [Figure 8]. This can be represented mathematically as

$$r_n = e^{\frac{j2\pi n^2}{p}}$$ where n and p are as defined previously.

This sequence is periodic like the grating with p terms per period all with magnitude 1, representing perfect reflection differing only in phase shifts. For an ideal diffuser, the autocorrelation of this sequence should be zero for every phase shift other than zero. That is, we want

$$c_k = \sum_{n=0}^{p-1} r_n r^*_{n+k} = 0 \quad \text{for} \quad k \not\equiv 0 \bmod p$$

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This is equivalent to

$$c_k = \sum_{n=0}^{p-1}\left(e^{\frac{j2\pi n^2}{p}}\right)\left(e^{\frac{-j2\pi(n+k)^2}{p}}\right) = 0 \quad \text{for} \quad k \not\equiv 0 \bmod p$$

$$c_k = \sum_{n=0}^{p-1} e^{\frac{j2\pi(n^2-n^2-2nk-k^2)}{p}} = 0 \quad \text{for} \quad k \not\equiv 0 \bmod p$$

$$c_k = e^{\frac{-j2\pi k^2}{p}}\sum_{n=0}^{p-1} e^{\frac{-j4\pi nk^2}{p}} = 0 \quad \text{for} \quad k \not\equiv 0 \bmod p$$

The above equation is indeed zero for the above condition because the summation represents adding up a sine wave over a whole cycle which of course adds up to zero.

Now consider the effect of adding a constant x mod p to our sequence as we have for our alternative sequence

$$r_{a_n} = e^{\frac{j2\pi n^2+x}{p}}$$

which gives

$$c_k = \sum_{n=0}^{p-1} r_{a_n} r^*_{a_{(n+k)}} = 0 \quad \text{for} \quad k \not\equiv 0 \bmod p$$

giving

$$c_k = \sum_{n=0}^{p-1} e^{\frac{j2\pi(n^2+x-n^2-2nk-k^2-x)}{p}} = 0 \quad \text{for} \quad k \not\equiv 0 \bmod p$$

$$c_k = e^{\frac{-j2\pi k^2}{p}}\sum_{n=0}^{p-1} e^{\frac{-j4\pi nk^2}{p}} = 0 \quad \text{for} \quad k \not\equiv 0 \bmod p$$

This is the same as the result obtained without adding a constant. Thus the two sequences have identical performance in that the zero order coefficient ($c_k, k \equiv 0$ mod p) is given by: $c_0 = p$ and all other coefficients c_k, $k \not\equiv 0$ mod p equal zero. This gives our desired Fourier transform result that $|R_m|^2 = p$ and thus spreads the incident energy equally in all directions.

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CONCLUSION

By adding a constant mod p to quadratic residue diffusing sequences one can obtain new sequences with the same properties as the old ones but with a reduced depth. This adds a new tool to the acoustic designers armoury for tackling real acoustic designs which have physical as well as theoretical constraints.

REFERENCES

1 Schroeder, M R, "Diffuse sound reflection by maximum-length sequences." *Journal of the Acoustical Society of America*, 57, January 1975.

2 Schroeder, M R. "Progress in architectural acoustics and artificial reverberation: Concert Hall acoustics and number theory," *Journal of the Audio Engineering Society*, 32, No 4, April 1984, pp 194-203.

3 D'Antonio, P and Konnert, J H, "The reflection phase grating diffusor: design theory and application." *Journal of the Audio Engineering Society*, 32, 4, April 1984, pp 228-236.

Figure 1: Normal Wavefront on a Diffusing Surface

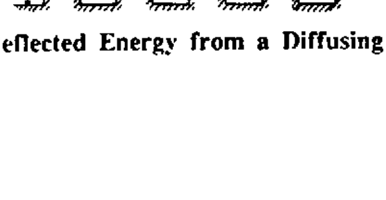

Figure 2: Reflected Energy from a Diffusing Surface

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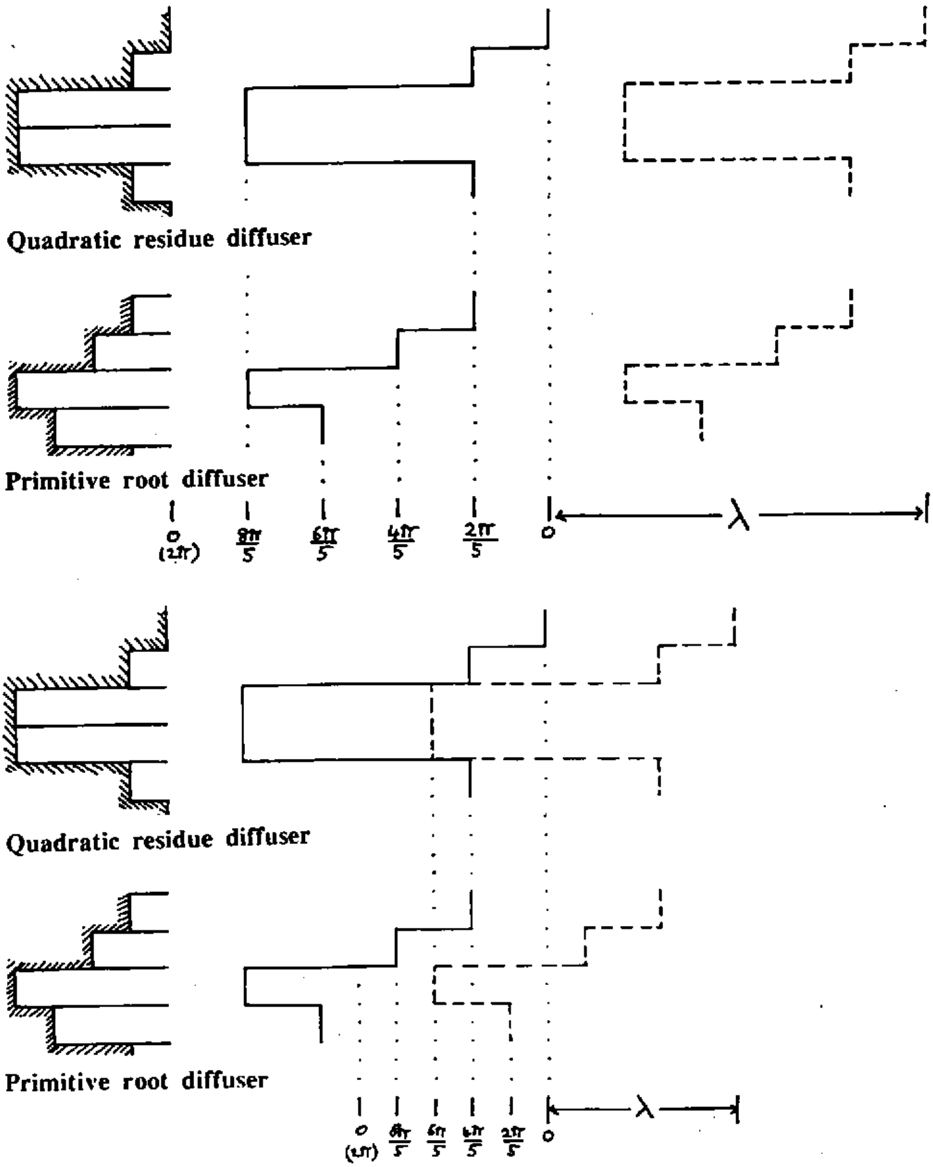

Figure 3: Reflected Wavefronts from Two Different Diffusers at the Lowest Design Wavelength and an Octave Above

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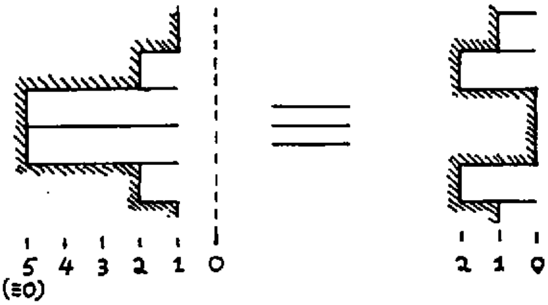

Figure 4: The Effect of Adding 1 To a Quadratic Residue Diffuser

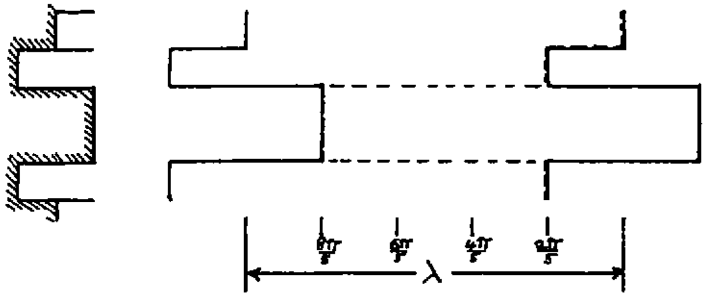

Figure 5: The Reflected Wavefront from the Modified Diffuser at the Lowest Design Wavelength

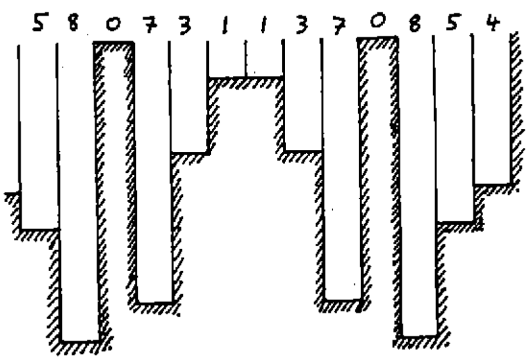

Figure 6: A 13 Well Modified Diffuser

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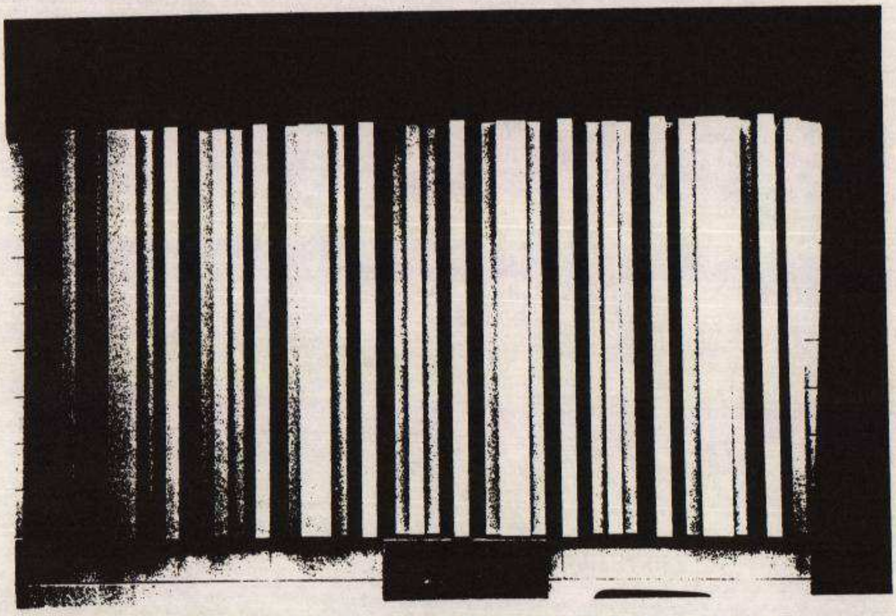

Figure 7: A Photograph of a, 13 Well, Modified Diffuser Implementation

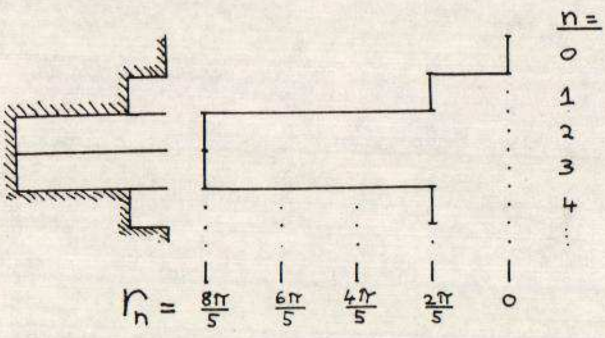

Figure 8: A Visual Representation of the Sequence r_n