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ACOUSTIC ARRAY ELEMENT INTERACTION STUDY

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In an acoustic array used for sonar, oceanographic, acoustic imaging or geophysical applications, the interaction between sources affects the radiation loading on each individual source, the power it radiates and the radiation pattern of the array. This interaction is characterized by the mutual radiation impedance which is an essential parameter determining array behavior. Low-frequency transducers in a volumetric array with small size requirements are subject to much larger mutual interaction and scattering than in conventional arrays. Recent studies have provided new insights into the acoustic and elasto-acoustic array interaction behavior. This paper provides an overview of a parametric study of elliptic cylindrical class IV flextensional transducers. An investigation of the modal components of mutual radiation impedance is presented and a simple equivalent circuit model of the in situ projectors, including the interaction effect, is proposed.

1. INTRODUCTION

In an array of acoustic transducers, the interaction between sources affects the radiation loading on each individual source, the power it radiates and the beam pattern of the array. A complete review of the physical behavior, the problems associated with element interaction and a bibliography on the current state-of-the-art has been published by Richards *et al.*[1]

The sound pressure acting at the radiating face of the transducer can be expressed in terms of a radiation impedance Z_{rad}. Foldy[2] presented a definition of radiation impedance that is based on radiated power. His effort was limited to projectors with a fixed, although nonuniform velocity distribution, characterized as a single degree-of-freedom mechanical system.

$$Z_{Rad} = R_r + j X_r = \frac{1}{V}\int_s p\, \beta dS \tag{1.1}$$

where the velocity $v(r) = V\beta(r)$; V is a reference velocity selected such that radiated power is $\frac{1}{2}R_r|V|^2$ and β is a function of position, which is independent of how the surface is driven.

The acoustic interaction is characterized by the mutual radiation impedance. Sherman[3] presents an extension of Foldy's definition to include the case of an array of transducers by expressing the interaction between every pair of transducers.

$$Z_{ij}=\frac{1}{V_i}\int_{S_j}\left(p_i(r_j)\beta^*(r_j)\right)dS_j \tag{1.2}$$

The total radiation impedance on one transducer in an array is the sum of the self impedance and all the mutual radiation impedances.

$$Z_j=\sum_{i=1}^{N}\left\{\frac{1}{V_iV_j^*}\int_{S_j}\left(p_i(r_j)v_j^*(r_j)\right)dS\right\}\frac{V_i}{V_j} \tag{1.3}$$

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Pritchard[4] describes a method of calculating the mutual acoustic impedance between circular pistons in an infinite rigid baffle. This quantity is expressed in the form of an infinite series, but may be approximated, for the case of a transducer that is small with respect to a wavelength, by a trigonometric equation involving the element spacing and the self radiation resistance. This trigonometric expression has been used successfully to design arrays of tonpilz transducers in rigid baffles. However for close-packed arrays of flextensional type transducers, which are multi-degree-of-freedom systems, without a rigid baffle, the formulation breaks down, particularly in the extremities of the frequency bands and steering directions of interest for sonar systems.

This paper presents an investigation into the physics of close-packed array interaction. Numerical methods are applied to characterize the acoustic and elasto-acoustic interaction behavior of class IV flextensional transducers. Comparisons with Pritchard's analytical expression are presented. The components of a multi-degree-of-freedom mutual radiation impedance are developed and the results provide for a simple equivalent circuit model of the in situ projector. Extension of this model to include array element interaction is proposed.

2. DISCUSSION

2.1 Class IV parametric study

To characterize the array interaction effects, numerical models of a British Aerospace (BAe) class IV flextensional were developed. The parameters considered to play a role in the mutual interaction are ka, where k is the acoustic wavenumber and a is the effective radius of the radiating surface; kd, where d is the distance between projectors; the shape of the radiating surface; the array geometry; and the velocity distribution.

Acoustic coupling is defined as the interaction occurring when the acoustic forces are small relative to the excitation force. Elasto-acoustic coupling occurs when the acoustic forces are not negligible. A class IV shell is typically thick-walled and thus acoustic coupling will be assumed. The acoustic radiation may be addressed in terms of a normal velocity distribution.

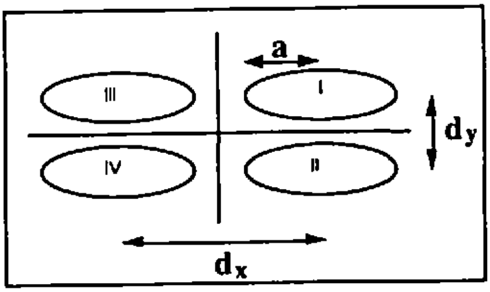

Figure 1: Flextensional array geometry

Figure 1 depicts the array geometries chosen for consideration. A Helmholtz integral equation code, Lavie[5], was applied to compute the acoustic radiation impedance for the case of a single isolated transducer and the mutual impedance due to the coupling between two transducers. Three pairs of elements were exercised; elements I-II, I-III and I-IV. A full four element model was also executed. Pritchard's formulation for the mutual impedance between two pistons was computed for comparisons.

The prescribed velocity distribution on the surface results from an in vacuo modal analysis performed with the ATILA finite element code; Decarpigny[6]. To look at the effect of the velocity distribution on impedance, the first three flexural modes of a BAe unit were selected. Figure 2 depicts these modes, superimposed on a quarter model of the projector.

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Several spacings between elements and a frequency sweep up to one wavelength were exercised. The acoustic radiation impedance was computed. The mutual radiation impedance due to the coupling between pairs of bodies in the four element model may be compared with the results of the two element configurations.

Figures 3 and 4 summarize the results of the analysis in graphic form for coupling between the I-II elements in the four element array and the two element array. Figure 3 depicts the calculated mutual impedance (thin lines) and that computed by Pritchard's formula (thick lines) for flexural mode 1. Comparison with the two element I-IIconfiguration, Figure 4, shows an excellent agreement between models.

It is noted here that for the closest spacing of the I-II array,($d = a/2$), the radiation resistance displays a resonance behavior at one wavelength due to the presence of a standing wave between the two bodies; a l/2 "horn", which produces an increase in the total radiated power near its resonance.

Figures 5 and 6 summarize the results of the analysis for coupling between the I-III elements. Agreement is reasonable between the two element I-III configuration and the four element case. For the closest spacing, the four element impedance displays the resonance influence which occurs in the two element I-II configuration. Figure 7 and 8 summarize the results for a four element I-IV configuration and a two element I-IV array, respectively. These models show excellent agreement. Again, for the closest spacing, the four element impedance is influenced by the I-II configuration resonance. Similar results were found for mode 2 and 3 velocity distributions.

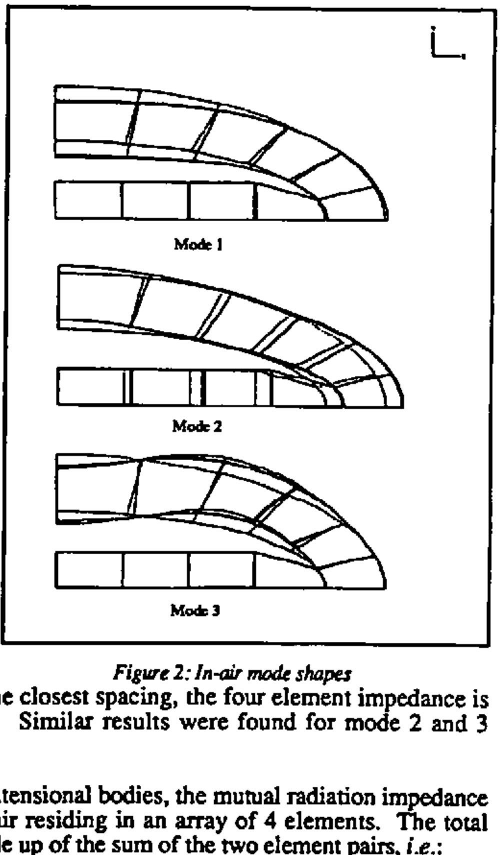

Figure 2: In-air mode shapes

The above analysis suggests that for class IV flextensional bodies, the mutual radiation impedance for a pair of elements is the same as for that pair residing in an array of 4 elements. The total radiation impedance may be considered to be made up of the sum of the two element pairs, *i.e.*:

$$Z_r = Z_{11} + Z_{12}\frac{V_2}{V_1} + Z_{13}\frac{V_3}{V_1} + Z_{14}\frac{V_4}{V_1} \qquad (2.1)$$

The thick lines in figures 3-8 represent Pritchard's analytical solution. Good agreement is shown for most configurations, except where the standing wave resonance effect is present. Figures 3 and 4 , representing the interaction of elements I-II, show significant scattering effects.

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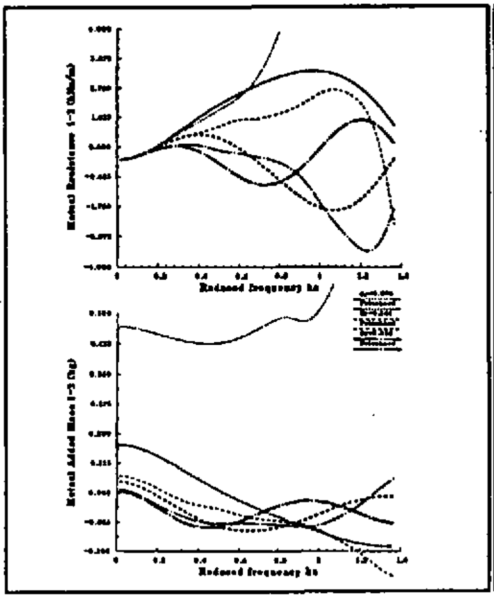

Figure 3: Z12 Projectors I-II (4 Elements)

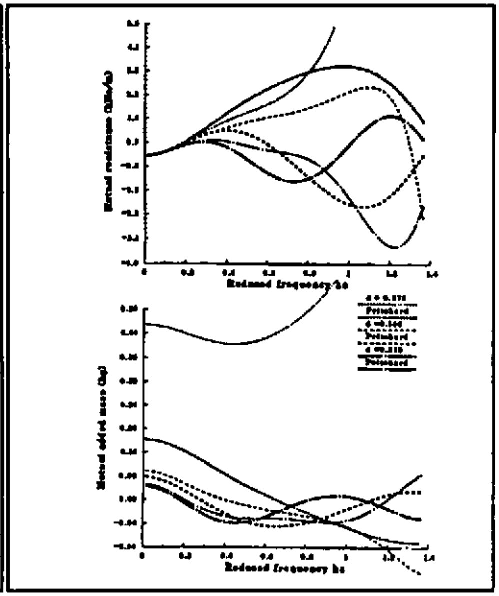

Figure 4: Z12 Projectors I-II (2 Elements)

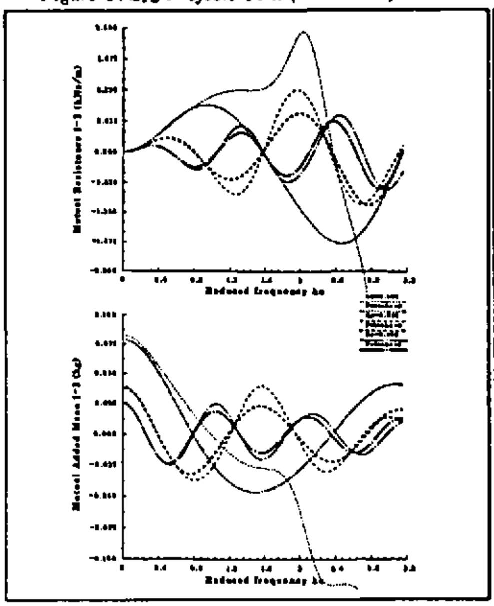

Figure 5: Z12 Projectors I-III (4 Elements)

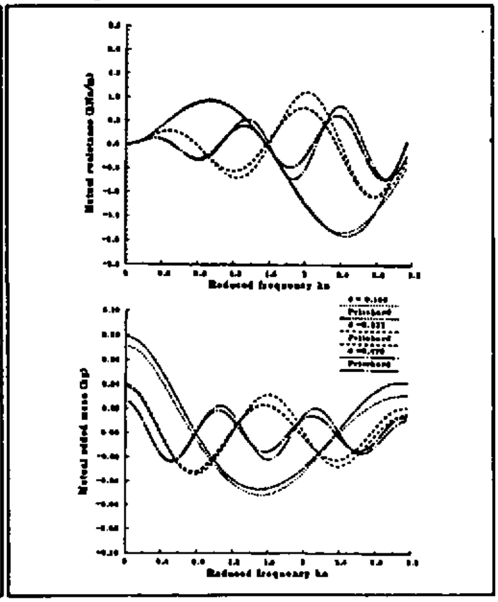

Figure 6: Z12 Projectors I-III (2 Elements)

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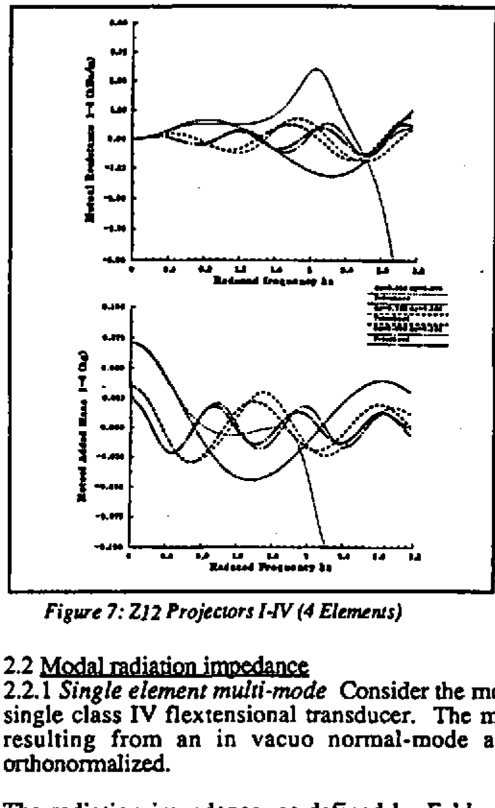

Figure 7: Z12 Projectors I-IV (4 Elements)

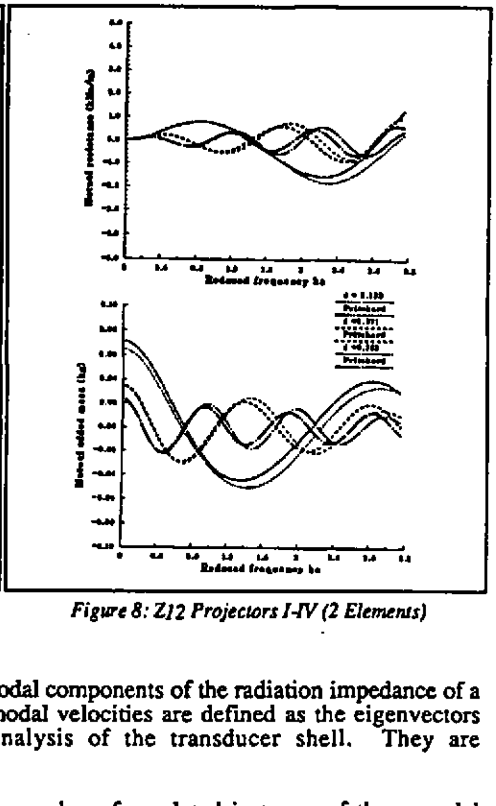

Figure 8: Z12 Projectors I-IV (2 Elements)

2.2 Modal radiation impedance

2.2.1 *Single element multi-mode* Consider the modal components of the radiation impedance of a single class IV flextensional transducer. The modal velocities are defined as the eigenvectors resulting from an in vacuo normal-mode analysis of the transducer shell. They are orthonormalized.

The radiation impedance, as defined by Foldy, may be reformulated in terms of these modal velocity distributions:

$$Z_{rad} = R_r + jX_r = \frac{1}{V_T V_T^*}\int_s p_T(r)\,v_T^*(r)\,dS \qquad (2.2)$$

where the normal surface velocity $v_T = \sum V_n \beta_n$. V_T is the total major axis reference velocity chosen such that the radiated power is $\frac{1}{2}R_r|V_T|^2$ and β_n is a set of modal eigenfunctions. The in vacuo modes which contribute to radiation are radial in orientation. Assuming these radial modes remain uncoupled, *i.e.* neglecting the axial modes, the modal pressures may be written in terms of the modal eigenfunctions.

$$p(r) = \sum^{m} P_m \beta_m(r) \qquad (2.3)$$

The total radiation impedance may now be expressed in terms of these modal components:

$$Z_{rad}=\frac{1}{V_T V_T^*}\int_s \sum^m P_m\beta_m(r)\sum^n V_n^*\beta_n^*(r)dS \tag{2.4}$$

where $V_T=\sum V_n$. For two modes, n,m=2. Substitution yields:

$$\begin{aligned} Z_{rad}&=\frac{1}{V_T V_T^*}\int_s (P_1\beta_1+P_2\beta_2)(V_1^*\beta_1^*+V_2^*\beta_2^*)dS \\ &=\frac{1}{V_T V_T}\left\{\int_s P_1\beta_1V_1^*\beta_1^*dS+\int_s P_2\beta_2V_2^*\beta_2^*dS+\int_s P_1\beta_1V_2^*\beta_2^*dS+\int_s P_2\beta_2V_1^*\beta_1^*dS\right\}\end{aligned} \tag{2.5}$$

Due to the orthogonality of the modes, the cross-modal terms are zero. The resulting expression may be written in terms of modal self impedances.

$$Z_{rad}=Z_1\frac{|V_1|^2}{|V_T|^2}+Z_2\frac{|V_1|^2}{|V_T|^2} \tag{2.6}$$

The significance of equation (2.6) is the total modal radiation impedance for a single element may be considered uncoupled. This makes the modal impedance matrix diagonal, and suitable for inclusion into existing multi-degree-of-freedom transducer equivalent circuits to completely model the transducer with fluid loading.

2.2.2 *Array multi-mode* Using (2.4) and following the concept of mutual impedance outlined by Sherman, the modal components to the radiation impedance for a pair of projectors may be developed. The total radiation, equation (1.3) may be written for the j^{th} transducer:

$${}_TZ_j=\sum_{i=1}^{N}\left\{\frac{1}{{}_TV_{iT}V_j^*}\int_{S_j}\left({}_Tp_i(r_j){}_Tv_j^*(r_j)\right)dS\right\}\frac{{}_TV_i}{{}_TV_j} \tag{2.7}$$

Where, again, ${}_Tv_j=\sum{}_mV_{j\,m}\beta_j$, however it must be noted that ${}_n\beta_i(r_j)\neq{}_n\beta_j(r_j)$ due to a coordinate transformation.

Substituting n=2 elements into equation (2.7), the total impedance may be expressed in the form of (2.6):

$${}_TZ_1={}_1Z_1^T\frac{|{}_1V_1|^2}{|{}_TV_1|^2}+{}_2Z_1^T\frac{|{}_2V_1|^2}{|{}_TV_1|^2} \tag{2.8}$$

where

$$\begin{aligned}{}_1Z_1^T&={}_1Z_1+{}_1Z_{12}\frac{{}_1V_2}{{}_1V_1}+{}_{21}Z_{12}\frac{{}_2V_2}{{}_1V_1}\\ {}_2Z_1^T&={}_2Z_1+{}_2Z_{12}\frac{{}_2V_2}{{}_2V_1}+{}_{12}Z_{12}\frac{{}_1V_2}{{}_2V_1}\end{aligned} \tag{2.9}$$

which is a sum of the modal self, modal mutual and cross-modal mutual impedances modified by the ratio of their corresponding modal reference velocities.

2.3 Equivalent circuit representation

A simple equivalent circuit of a flextensional can be obtained by assembling an equivalent circuit of the driver, described by one motional branch which takes into account its first longitudinal resonance and an equivalent circuit of the shell, which incorporates two motional branches corresponding to the first flexural mode and either the second flexural or the membrane mode, Debus *et al.*[7] (Figure 9). The radiation effects may be incorporated into the equivalent circuit by inserting in each motional branch of the shell the corresponding modal radiation impedance. It may be shown that this approach is consistent with the modal impedance definition developed in section 2.2.1.

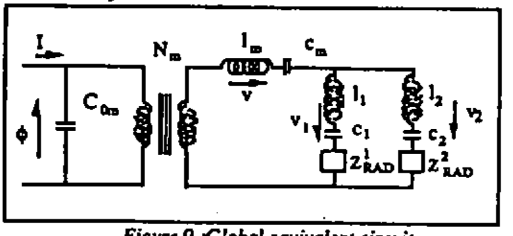

Figure 9 :Global equivalent circuit

To verify the accuracy of the equivalent circuit model, The following procedure has been implemented for a BAe 3kHz flextensional.

1) Construct, using finite element modeling, the equivalent circuit for the driver and the shell separately, using the first two modes for the shell and only the first longitudinal mode for the driver.
2) Compute the radiation effects using a Helmholtz integral equation method for each of the shell modes.
3) Construct the global equivalent circuit in water and compute its electrical impedance.
4) Compute the electrical impedance for the whole projector using a full finite element method and compare to (3) above.

The electrical impedance results presented in Figure 10 are provided for the finite element model (dotted line) and the equivalent circuit (solid line).

The radiation effects for a transducer in an array configuration may also be modeled in the same fashion as the single element. The total radiation for each mode as well as the cross-modal impedances are included in the motional branches of the shell model. It must be noted here that this approach does not provide for modeling acoustically excited mechanical modes, such as the well known "banana" mode, which is not electrically coupled.

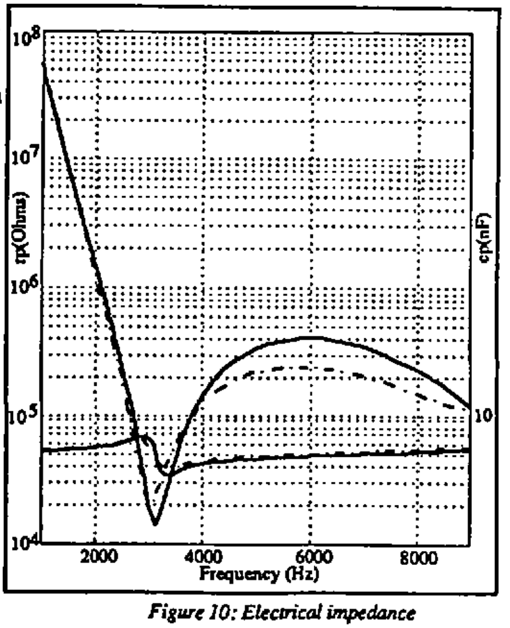

Figure 10: Electrical impedance

3. CONCLUSION

Current array design methodology typically implements Pritchard mutual radiation impedance for pistons in a rigid baffle to model array interaction effects. The parametric study presented here suggest limits to the validity of this approach for the case of class IV flextensional transducers.

The concept of a multi-degree-of-freedom radiation impedance has been developed. The results suggests that the modal contribution to the total radiation impedance is uncoupled due to the nature of the normal-mode basis used to define the modal contributions. This provides an efficient method of constructing an equivalent circuit description of a class IV flextensional in an array configuration. The modal contribution may be computed with an eigenvalue solution for the in-air normal modes, followed by an acoustic Helmholtz integral solution for the modal radiation impedance. The results of these analyses provide the lumped parameter inputs for the equivalent circuit.

Further study is necessary to validate the array element equivalent circuit model with experimental data. Extension of the model to include the banana mode effects into the model must be considered. This mode is not electrically coupled, and thus must be applied solely as an acoustic effect. Expansion of this study to include additional low frequency transducers, such as a class V flextensional is being considered.

4. REFERENCES

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