BRITISH ACOUSTICAL SOCIETY

71SC7

BAS Spring Meeting 1971
Underwater Acoustics

TRANSDUCER DESIGN BASED ON FILTER THEORY

J.C. MORRIS

Introduction

The degree of complexity used in the design of a transducer should bear some relationship to the accuracy with which the performance of the transducer must match a target specification. This paper is concerned with optimising the bandwidth and input admittance of piezo-electric transducers of the "two mass and a spring" type and relies on the fact that a transducer of this type can be represented by a simple equivalent circuit which can be considered as a half-section band-pass filter; the technique applies equally well to any type of transducer which can be represented by this simple circuit. Once it is established that a transducer can be represented by a simple bandpass filter circuit the analysis of the transducer performance can be done using the conventional normalised filter parameters.

Band-Pass Filter Circuit

If a simple Band-Pass filter circuit has cut-off frequencies f_1 and f_2 and a nominal resonant frequency f_o then the fractional bandwidth becomes:-

$$W = \frac{f_2 - f_1}{f_o} \qquad (1)$$

and the performance of the circuit at any other frequency can be defined in terms of a frequency variable:-

$$y = \frac{\frac{f}{f_o} - \frac{f_o}{f}}{W} \qquad (2)$$

A general form of the band-pass filter circuit[1] is shown in fig.1 where L and C are related to the resonant frequency and nominal design impedance R_N by:-

$$R_N = \sqrt{\frac{L}{C}} \qquad (3)$$

$$\omega_o = 2\pi f_o = \frac{1}{\sqrt{L.C.}} \qquad (4)$$

In the general form of the circuit the terminating impedance R_1 does not necessarily have to be the design impedance and it can be shown that the characteristics can be improved by terminating the mid-series terminals of the filter with a resistance of value βR_N and the mid-shunt terminals with $\frac{R_N}{\beta}$ (this is sometimes called "rho" or "anti-rho" matching, although "rho" has not been used here because it also connotes density).

Another important parameter of the circuit is the coupling coefficient "k" defined as the square root of the ratio of the series capacitance to the total circuit capacitance (series plus shunt)

$$k = \sqrt{\frac{W^2}{1 + W^2}} \qquad (5)$$

In the design of transducers for high power applications it is usual to specify the bandwidth over which the transducer must work efficiently, and this is governed by the transfer characteristic. But at the same time in order to limit the size of the driving amplifier it is necessary to control the transducer's input admittance over the frequency range of interest. Control over the input admittance locus is achieved by varying the matching factor β, the input admittance being given by:-

$$Y_{IN} = \frac{1}{R_N} \left[\frac{\beta}{\beta^2 + y^2} + jy \frac{(\beta^2 + y^2 - 1)}{\beta^2 + y^2} \right] \qquad (6)$$

To illustrate the effect of changing β a series of graphs are shown in fig.2 for $\beta = 0.6, 0.8, 1.0, 1.5$ over the full nominal bandwidth (y from -1.0 to +1). The shape of the curves varies considerably with the value of the matching factor and the choice of a particular curve for design purposes depends on the constraints imposed on it by the amplifier design. One of the most useful values for the matching factor is $\beta = 0.8$, for in this case the amplifier sees a load which is limited to a 2:1 change of admittance modulus and a power factor always greater than 0.8. Besides changing the input admittance the matching factor also affects the transfer characteristics, and by chosing the right factor a flatter response can be achieved in the pass band. The transfer admittance $\frac{i_{out}}{e}$ is given by the equation:-

$$\frac{i_{out}}{e} = \frac{\beta}{R_N} \frac{1}{(\beta^2 + 1 - y^2 + j2\beta y)} \qquad (7)$$

and the effect that β has on this transfer admittance is shown in fig.3, where the value of $\left|\frac{i_{out}}{e}\right|$ is plotted in dB relative to the minimum loss. The fact that a matching factor $\beta = 0.8$ gives a much flatter curve than most is another good reason for its choice in design.

Transducer Equivalent circuit

Since the concept of transducer design using filter theory is based on the fact that the transducer can be represented by the simple filter circuit this must be verified and the assumptions clearly stated. Initially it is assumed that (a) the transducer is made up of a rigid piston in contact with the medium, driven by a composite prestressed stack constructed of piezoelectric ceramic and spacers and having a centre bolt to keep the stack in compression. and a countermass. (b) The piston (head) and countermass (tail) are lumped masses and that the stack is short compared to a wavelength in the material. (c) The stack is massless, although later, correction terms will be added to the head and tail masses to take into account this mass.

The equivalent circuit for this transducer can be derived fairly easily and is shown in fig.4, where the main parameters are as follows[2]:

$$C_o = \frac{A_c n^2}{l_c} \varepsilon^S_{33} = \frac{A.n^2}{l_c} .\varepsilon^T_{33} (1 - k^2_{33}) \tag{8}$$

$$\phi = \frac{A_c n.k_{33}}{l_c} \left[\varepsilon^T_{33}.Y^E_{33}\right]^{\frac{1}{2}} \tag{9}$$

n is the number of ceramic rings

l_c is the total ceramic length

A_c is the ceramic area

$C_c = \frac{l_c}{Y^E_{33}.A_c}$, compliance of the ceramic

C_s = total compliance of any joints, spacers, insulators

C_B = compliance of the centre (pretensioning) bolt

M_T = Mass of the tail (countermass)

M_H = Mass of the head (piston)

R = Radiation resistance

By a series of network transforms this circuit can be reduced to a form effectively that of the simple filter circuit with the exception of the shunt coil, and this has to be added as a discrete electrical component to form the complete circuit. The values of the components in this new circuit are:-

$$C_e = C_o + p^2 \phi^2 C_1$$

$$C_1 = C_s + C_c$$

$$p = \frac{C_c}{C_s + C_c}$$

$$b = \frac{C_B}{C_1}$$

$$C_m = \left(\frac{m}{1 + m}\right)^2 \left(\frac{b}{1 + b}\right) C_1$$

$$M_m = M_H \left(\frac{1 + m}{m}\right)$$

$$t = \frac{1 + m}{m}$$

$$L_e = \frac{1}{\omega_o^2 C_e} \quad \text{(tuning inductor)}$$

$$m = \frac{M_T}{M_H}$$

and the effective coupling coefficient becomes:-

$$k_e = \left[\frac{k_{33}^2 \, b \, C_c}{C_1(1 + b) - k_{33}^2 \, C_c}\right]^{\frac{1}{2}} \qquad (10)$$

At this stage the load impedance and corrections due to the mass of the stack have to be taken into account. When designing transducers to work as single elements the radiation load used is the self radiation impedance[3]:-

$$Z_{11} = \rho c \pi a^2 \left[1 - \frac{J_1(2ka)}{ka} + j \frac{S_1(2ka)}{ka}\right] \qquad (11)$$

where a is the head radius

J_1 is a Bessel function of the first kind
S_1 is a Struve function

But when designing transducers for arrays it is necessary to calculate the average radiation impedance by assuming a velocity distribution over the array and calculating the interaction effects due to all the elements in the array. An approximation to this value is given by Toulis[4] for unsteered arrays of small pistons. In all cases the radiation load consists of resistive and reactive components.

In the equivalent circuit the resistive component of the radiation load is represented by R and the reactive component can be considered as being an effective water mass (M_W) and combined with the head mass. There is also a correction which has to be applied to the head mass to allow for the finite mass of the stack. If the mass of the stack is M_0 then it can be shown that a good approximation to the effect of this mass is achieved by adding $\frac{M_o}{3}\left(\frac{m}{1+m}\right)$ to the actual head mass and $\frac{M_o}{3}\left(\frac{1}{1+m}\right)$ to the actual tail mass. Hence if

M_H' is defined as being the actual head mass

then $M_H = M_H' + M_W + \frac{M_o}{3}\left(\frac{m}{1+m}\right)$

and M_T' is defined as the actual tail mass then

$$M_T = M_T' + \frac{M_o}{3}\left(\frac{1}{1+m}\right)$$

The circuit is now complete and incorporates corrections for the reactive part of the radiation load and also for the finite mass of the stack. To design a transducer the components of transducer equivalent circuit are equated to components in the simple filter circuit using the radiation load R as the basis, the only other information required to complete the design is the maximum field that the ceramic is required to handle.

Conclusions

A transducer can be represented by an equivalent circuit which can be reduced to the form of a half-section band-pass filter. Using the basic filter equations a transducer can be designed which optimises the input admittance variations for a particular bandwidth.

References

1. J.H.Mole "Filter Design Data for communication engineers" Spon 1952.
2. H.W.Katz "Solid State Magnetic and Dielectric Devices" Wiley (1959).
3. R.W.B.Stephens & A.E.Bate "Acoustics and Vibrational Physics" Arnold (1966).
4. W.J.Toulis "Radiation Load on arrays of small pistons" J.A.S.A., 29, No.3, p.346 (1957).

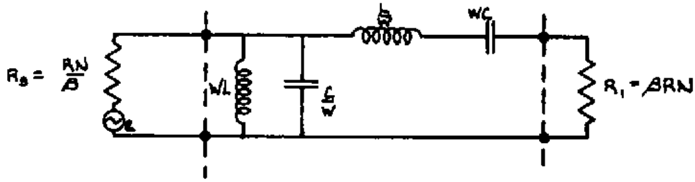

FIG.1. Band-Pass Filter Circuit

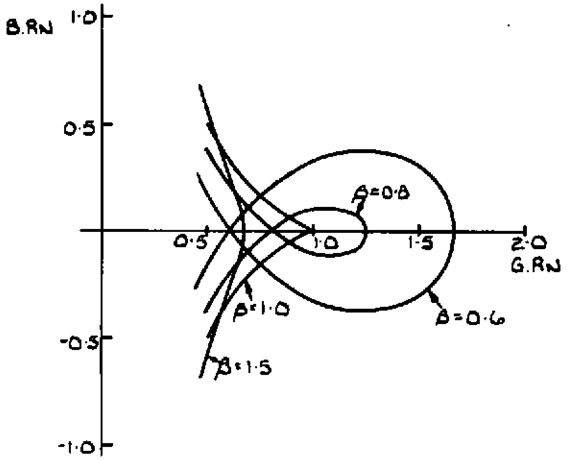

FIG.2. Input Admittance

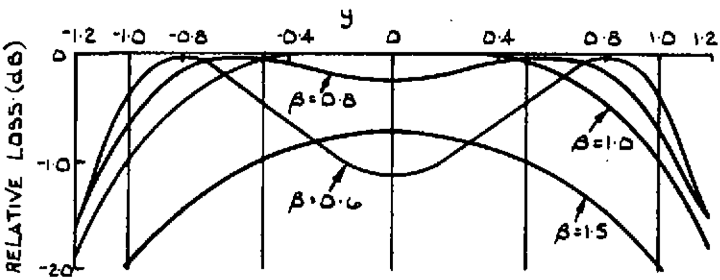

FIG.3. Transfer Admittance

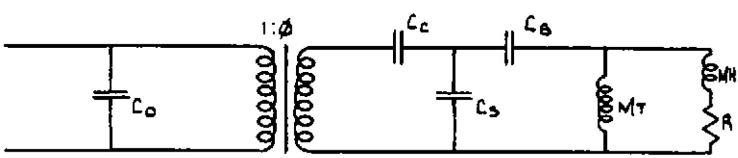

FIG.4. Transducer Equivalent Circuit

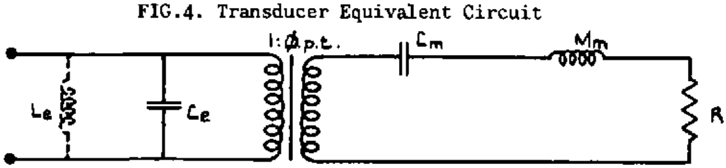

FIG.5. Simplified Transducer Equivalent Circuit