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# UNBIASED SOUND POWER DETERMINATION BY INTENSITY MEASUREMENTS

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Sound power determination is the most usual way of characterizing an acoustic source. The sound power is defined as the total net flow through a measurement surface enclosing the source :

$$W = \oint_S \vec{I}.\hat{n}\ ds \qquad (1)$$

The acoustic power flux density $I_n = \vec{I}.\hat{n}$ was conventionally approached by RMS pressure measurements. Now, are appearing methods for direct measurements of acoustic intensity. Thus, the use of sound intensity meters gives an acoustic power determination without the well known systematic errors generally attributed to normalized methods. If this determination is well unbiased, random errors still remain depending on many parameters the influence of which we must try to define.

## MEASUREMENTS UNDER FREE FIELD CONDITIONS

Free field conditions are ideal to determine sound power of sources. However, standardized methods using pressure measurements lead to the approximation

$$W' = \oint_S \frac{\langle p^2 \rangle}{\rho c}\ dS \qquad (2)$$

which specific errors over-estimate the result |1,2| :

$$W' = W + \Delta W_1 + \Delta W_2$$

These errors are :

- Near-field errors : Considering $\langle p^2 \rangle / \rho c$ is equivalent to the true energetic value $|\vec{I}|$, we suppose the impedance at the measurement point equal to the specific acoustic impedance $\rho c$ of the medium. In the near-field of industrial sources, this impedance is generally complex and this approximate expression (2) gives a near-field error

$$\Delta W_1 = \int_S \frac{\langle p^2 \rangle}{\rho c}\ ds - \int_S |\vec{I}|\ ds \qquad (3)$$

- Geometrical errors : The acoustic power flux density through the surface corresponds to the normal component of the intensity vector. If we neglect near-field error, pressure measurements supply values equivalent to the magnitude of $\vec{I}$ .

The best enclosing surface is the one in which the normal direction coincides with the sound intensity vector. Any other surface leads to an over-estimation by geometrical projection errors :

$$\Delta W_2 = \int_S |\vec{I}|\ ds - \int_S \vec{I}.\hat{n}\ ds \qquad (4)$$

However, accurate results can be obtained in an anechoïc chamber from the scalar quantity $\langle p^2 \rangle / \rho c$ if an hemisphere encloses a source of small dimensions.

In practice, we are generally obliged to choose a parallelepiped surface (which is more suitable to the geometry of the machine) situated in the near field. Experiments show that pressure measurements considerably exceed the normal intensity values |3|. With direct intensity measurements, near-field and projection errors vanish. If, the systematic phase errors in the two-microphone method are removed, the sound power determination from intensity measurements leads to an unbiased result.

In practice, the surface integral of flux is replaced by a finite number of individual measurements of normal intensity. In this case, "finity errors" appear if sampling conditions in connection with spatial fluctuations of the sound field are not satisfied. The magnitudes of these fluctuations depend on the source directivity but also on the choice of the surface shape. More precisely

$$\left\{ \begin{array}{l} \text{field to} \\ \text{sampling} \end{array} \right\} = \left\{ \begin{array}{l} \text{intensity} \\ \text{vector field} \end{array} \right\} \cap \left\{ \text{surface shape} \right\}$$

With large sources or in presence of a reflecting plane there are circulations of the intensity vector field (the curl of $\vec{I}$ is not equal to zero) in the near field of sources and this tends to increase the spatial variations.

To study sampling effects, Elliott |4| uses a spatial harmonic analysis of the polar directivity of sources. An over-sampling enables to process the spatial spectrum and to determine from the Shannon-Nyquist criterion the minimum number of measuring points. This procedure can be used as a powerful laboratory method to study the influence of the type of sound sources and the choice of surface shapes. Often, industrial constraints do not allow to know if sampling requirements are verified. Then normal intensities on the surface are considered as having a spatial random distribution. The sound power estimated over a finite number N of measuring points leads to a statistical incertitude, the normalized standard error $\varepsilon_1$ of which is associated with the variance of $I_n$ on the closed surface |5| as follows :

$$\varepsilon_1 = \frac{\left[\text{var}\ \{I_n\}/N\right]^{\frac{1}{2}}}{< I_n >} \qquad (5)$$

INFLUENCE OF THE ACOUSTICAL ENVIRONMENT

If one follows the principle of integration over a closed surface, the contribution of sources located outside is equal to zero :

$$\oint_S \vec{I}^{\,E} \cdot \hat{n}\, dS = 0 \qquad (6)$$

This important theoretical advantage is due to the law of conservation of energy flow (Gauss' law) which does not apply of course when measuring pressure. In this last case, extraneous contributions produce a bias error which can be more higher than the value due to the source radiation. In practical intensity measurements, the sampling of integral (6) keeps the characteristic of noise supression. The flux of the vector $\vec{I}^E$ which comes into and out the enclosing envelope increases the variance of the resultant $I_n$ according of the rate of variation of the perturbation field.

INFLUENCE OF STATISTICAL ERRORS IN ACOUSTIC INTENSITY MEASUREMENTS

The preceeding statistical errors was associated with the procedure of sampling the enclosing surface with a finite number N of exact values of resultant intensity $I_n$ . In fact, industrial sources produce a broad band noise, estimate of which is determined by the resolution bandwidth B of the analysis and by the time averaging T.

The intensity meters using the two-microphone method give measuring incertitude related to phase and coherence of the pressure cross-spectrum |6|. To summarize these random errors we consider the cross-spectrum averaged on the surface :

$$< \hat{S}_{2i} > = \frac{1}{N} \sum_{i} \{ \hat{S}_{21} \}_{i} \qquad (7)$$

The statistical instrumental error of the sound power determination is expressed by the following normalized standard error |5|

$$\varepsilon_2 = \frac{1}{\sqrt{2NBT}} \{(\frac{1}{\mathring{\gamma}_{21}^2} + 1) + \cot g^2 \mathring{\phi}_{21} (\frac{1}{\mathring{\gamma}_{21}^2} - 1)\}^{\frac{1}{2}} \qquad (8)$$

The coherence $\mathring{\gamma}_{21}^2$ and the phase $\mathring{\phi}_{21}$ correspond to the averaged cross-spectrum $<\hat{S}_{21}>$ . The normalized standard error is considerably sensitive to these parameters in low frequencies. They are determined by the nature of sound field and may be useful to give a concise and synthetic description of environmental conditions.

DISCUSSION

The total incertitude on the sound power determination can be expressed from two independent terms :

$$\varepsilon_W = \sqrt{\varepsilon_1^2 + \varepsilon_2^2}$$

These two terms depend both on nature of perturbating external sound field and of its spatial distribution and also of the number of measuring points. The normalized standard error $\varepsilon_2$ depends also on the B.T factor, equal to the number of ensemble averages used in FFT processing. The optimal condition for measurement is obtained when the importance of two terms $\varepsilon_1$ and $\varepsilon_2$ are equivalent. In practice, the value of the normalized standard error $\varepsilon_1$ is not know but $\varepsilon_W$ can be obtained from a variance estimate of the measuring values $\hat{I}_n$ . Coherence function and phase angle spectrum give an estimate of $\varepsilon_2$. Then, it is possible to determine an optimum value for T. Finally, the desired accuracy for the sound power determination leads to choose the number of measuring points.

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