Performance Limitations in Adaptive Beamforming

1.3

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1. Introduction.

The properties and limitations of the conventional beamformer are of course well known and need no amplification here. The resolution of the conventional beamformer is limited mainly by the size of its sensor array in wavelengths. Suggestions have been made from time to time that the beamwidth of an array can be reduced by adopting some form of superdirective taper function, but such efforts have normally been thwarted by the presence of errors of gain and placement in the sensors. The sidelobe levels of an array can also be reduced by using special fixed taper functions. The Chebyshev taper for example produces arbitrarily small uniform amplitude sidelobes but only at the cost of increasing the beamwidth. Such fixed taper functions are effected by sensor errors which increase the mean sidelobe level. For example it is probably almost impossible to achieve an rms. sidelobe level of -40 dB unless the array has 50 to 100 elements.

Adaptive processors can improve resolution and reduce sidelobe levels at the same time.[1,2,3] The purpose of this paper is to analyse in a simple fashion the limits of performance for the adaptive processor so that it can be compared quantitatively to its non-adaptive counterpart.

Adaptive processors are characterised by requiring signal measurements for a finite time interval in the signal environment. This time is used to obtain parameters of the statistics of the signal field and until stable measurements are obtained the adaptive system is not in equilibrium. It is assumed here that equilibrium has been attained and the processor is in an 'optimal' state. It is now shown that under certain conditions this state is easily predicted if the signal field is known and simple equations can predict the nature of the 'optimal' solution.

2. Adaptive Processor Operation

Only one type of adaptive processor is considered here. This is the Least Mean Square or LMS system. In hardware it consists, as shown in Fig. 1, of a sensor array detecting a time varying signal vector $\underline{X}(t)$, a vector of coefficients $\underline{W}$, and an adder which produces the output signal y(t).

The criterion of optimality of the LMS processor is very simple; it is that its mean-square output $\langle y^2(t) \rangle$ is as small as possible. This is affected by variation of the weight vector $\underline{W}$ and in an adaptive processor there exist some algorithm which automatically seeks this state and maintains it. The LMS processor must be prevented from converging to the undesirable (though logical) state of having all weight coefficients of zero. In order to 'look' at a broadside signal for example we require constant gain in that direction and this is achieved by introducing the constraint $\sum_{k=1}^{N} w(k) = \text{constant}$, where the $w(k)$, $k=1,2,..N$ are the weights.

This constraint over-rides the LMS algorithm.

In vector form we can introduce a broadside signal vector $\underline{S}_o = \{1,1,1,..1\}^T$ which represents a broadside unity amplitude plane wave, and can constrain the gain in the broadside direction to be unity by the linear equation $W^T S_o = 1$.*

The presence of the constraint does not impair the ability of the system to minimise its output power due to signals which are not near broadside, and in general we expect it to steer nulls toward such sources by variation of W in the manner now analysed. It should be noted at this point that if we require to 'look' in a different direction than broadside this is most easily done by phasing the signals from the sensor array by a beam switching technique.

The conventional weight vector for the system i.e. its starting state, is the 'matched filter' vector $\underline{W}_c$ which is equal to $\frac{1}{N} S_o$ for an 'N' sensor array. The system can be considered to adapt by addition of a 'difference' vector $\underline{\omega}$ to W_c. We have $W = W_c + \omega$. Because of the constraint $(W_c + \omega)^T S_o = 1$, and since $W_c^T S_o = 1$ then $\omega^T S_o = 0$ and, in consequence, $\omega^T W_c = 0$. Therefore the difference vector ω is orthogonal to the conventional weight vector W_c.

When some arbitrary signal vector $\underline{S}$ is input to the system, and adaptation is complete, the vector ω is such that $W^T S = (W_c + \omega)^T S$ is minimum in the mean square sense.

3. Effects of Sensor Error

We can now analyse the effects of constant errors in the sensors upon the broadside gain of the adaptive system. Let S_o be the broadside acoustic signal vector representing the signal but now imagine that, owing to errors in the sensors, the system perceives an electrical signal vector S which is different to S_o by the addition of an error vector $\underline{E}$: $S = S_o + E$. In reality S_o is time varying, and the errors are a multiplicative effect but if S_o is assumed constant, the result is still valid.

* X^T = complex conjugate transpose of X

We also assume that E is orthogonal to S_o (and hence W_c) or in other words the errors do not change the look direction gain. This does not affect the generality of the argument.

The constraint is not now sufficient to ensure that the adaptive system gain in the broadside direction remains constant. In fact the system will seek a difference vector such that

$y = (W_c + \omega)^T(S_o + E)$ is minimised. If this expression is expanded, noting that $W_c^T S_o = 1$, and $W_c^T E = \omega^T S_o = 0$, we obtain $y = W_c^T S + W_c^T E + \omega^T S + \omega^T E = 1 + \omega^T E$

Using the Schwartz inequality for vectors we can then write,

$$1 - (\omega^T \omega E^T E)^{\frac{1}{2}} \leq y \leq 1 + (\omega^T \omega E^T E)^{\frac{1}{2}}$$

and because of the nature of the optimality criterion the gain will usually lie near the lower limit. Suppose that y must not be less than y_{min}. Then

$$\omega^T \omega \leq (1-y_{min})^2/E^T E$$

The quantity $E^T E$ can be related to the variance of the sensor gains σ^2 (averaged on the array only, and with nominal sensor gain of unity), by $E^T E = N\sigma^2$. Thus $E^T E$ will be equal to some constant which can be estimated by measurement. For 3.5° rms. phase error and 7% amplitude error we would have $\sigma^2 \approx 10^{-2}$.

The super-gain ratio of the weight vector is introduced and defined as 'mean square weight value'/'mean square conventional weight value'

$$= W^T W/W_c^T W_c = 1 + \omega^T \omega/W_c^T W_c = 1 + N\omega^T \omega \quad (W_c^T W_c = {}^1/_N)$$

It is easy to show that $W^T W/W_c^T W_c < 1 + (1 - y_{min})^2/\sigma^2$ to maintain the gain criterion. For the sensor errors quoted and with $y_{min} = 0.707$ the super gain ratio would have to be less than 9.5.

4. Two Target Resolution

We now turn to the question of resolving two discrete noise sources by means of three adaptive processors. Referring to Fig. 2, system 1 steers a beam b_1 directly at t_1, system 3 steers a beam directly at t_2, while system 2 is steered to the centre of the sources. The steering is done by the phasing technique mentioned earlier. The three systems however operate completely independently. The supergain ratio of each processor is bounded as described above.

If the sources are to be resolved unambiguously, the central beam b_2 must output less power than either b_1 or b_3 so that we see a dip between the noises. If one of the sources is very weak, the output of beam b_2 must be correspondingly weaker to maintain the dip. The only reasonable output power is in fact zero. Thus in beam b_2 we require to form two nulls in the direction of sources t_1 and t_2 as shown in Fig. 3. To form two nulls like this demands a certain supergain ratio in the weight vector for beam b_2. It can be calculated as,

$$SGR = \frac{1 + d(2\theta)}{1 + d(2\theta) - 2d^2(\theta)}$$

in which $d(\theta)$ is the conventional system gain for beam b_2 for each of the sources t_1 and t_2, and $d(2\theta)$ is the gain at twice the spatial frequency. This function is plotted in Fig. 4 for a $\sin(x)/x$ pattern.

Since the supergain ratio has been bounded the nulls cannot be formed too close to the look direction of b_2. Consequently the angular separation of sources t_1 and t_2 must exceed some lower limit if they are to be resolved as indicated. Fig. 5 plots this angle as a function of sensor rms. error (1 = angle of first zeroes of conventional pattern).

It is seen that the resolution of the adaptive beamformer is limited and only if the sensor errors become vanishingly small is there a large improvement in resolution. Note also that a best case target situation has been chosen. If there were more than two sources, the supergain ratio of the beamformer (b_2) would have to increase to null them all and its resolution would decline.

5. Sidelobe Levels

Complete analysis of the side lobe levels of an adaptive processor is not possible here. The results are simply quoted as follows. If the number of discrete sources illuminating the array is small, then those sources which lie outside the region of the look direction are nulled until their output is below the level of any incoherent noise at the processor output regardless of sensor errors. Essentially the contribution of discrete interferences to the system output is zero.

This simple result is complicated in one implementation of the system by the presence of an A/D converter in the signal path from the sensors. This generates a quantisation noise. In order to ensure efficient operation of the A/D converter it may be preceded by an AGC amplifier whose purpose it is to maintain the rms. signal input to the converter at a fairly constant level (see Fig. 6). The combined effect is rather unfortunate. In the presence of a strong interference the gain of the AGC amplifier is reduced and with it the level of a weak signal which may be present. Eventually the signal is swamped by the quantisation noise if the interference is strong enough. In other words, the signal input to the A/D converter becomes smaller than the least significant bit at the binary output.

If one channel only is considered we get the following results for the ratio of Interference power/Signal power at the sensor (P_i/P_s) The aGC amplifier holds the converter input rms voltage at $\frac{1}{3}$ of the clipping level, and 2's compliment binary representation is assumed. The signal power (P_s) is equated to the quantisation noise power. ($P_i/P_s = \frac{1}{3}.2^{2B}$)

B (no. bits)	P_i/P_s (dB.)
4	19
6	31
8	43
10	55

The processor itself improves the situation by 10 log(N) dB.

If full advantage is to be taken of the interference reducing capability of an adaptive processor we must provide high grade signal channels as shown above.

6. List of References

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2. Frost O.L. "An algorithm for linearly constrained adaptive array processing" Proc.IEEE Vol.60 no. 8 (August 1972) pp.926-935.

3. Cox H. "Resolving power and sensitivity to mismatch of optimum array processors" JASA. Vol.54 No.3 (1973) pp.771-785.

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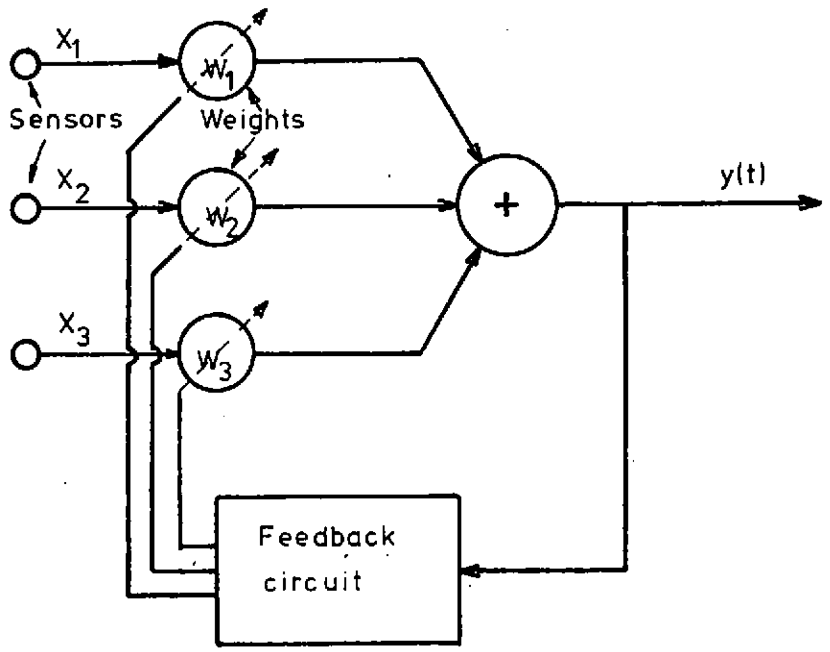

FIG. 1

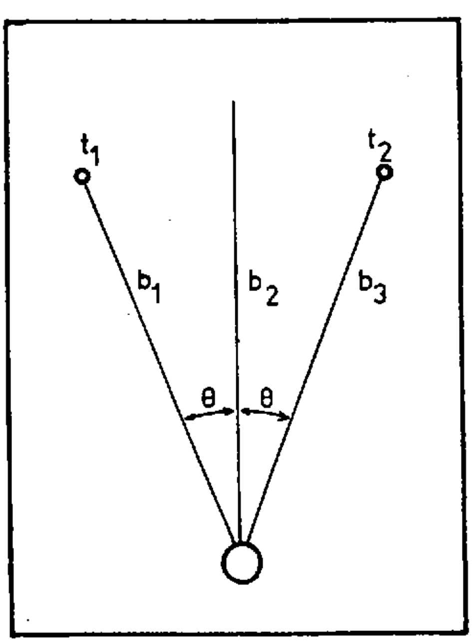

FIG 2

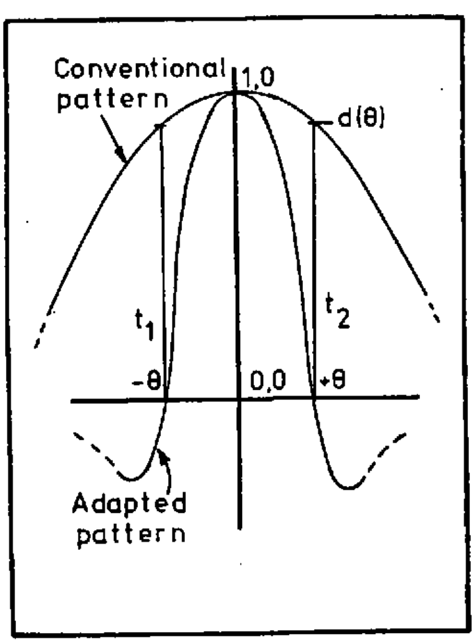

FIG 3

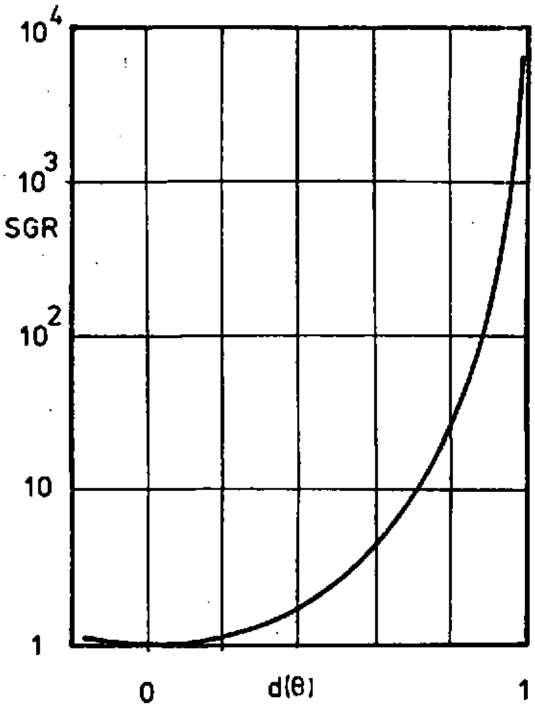

FIG 4

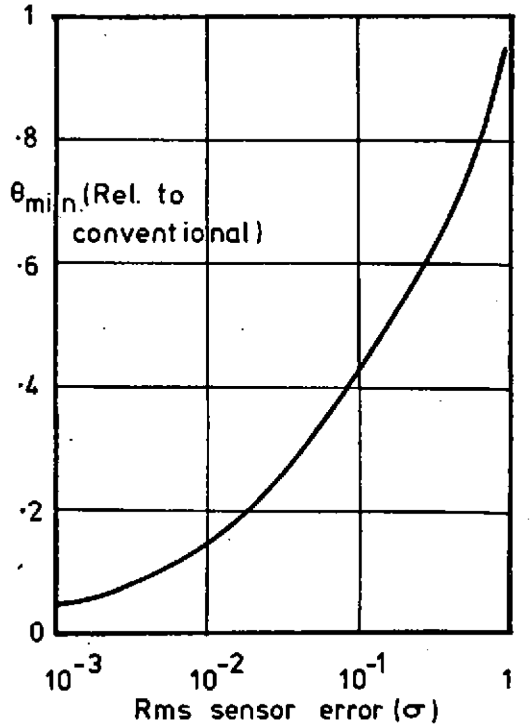

FIG 5

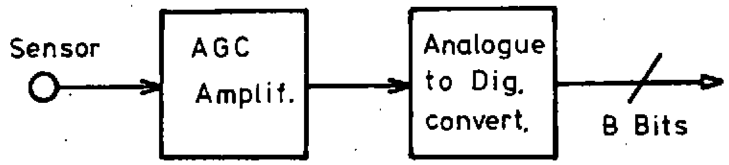

FIG. 6