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TESTING FOR ISOTROPY IN ADAPTIVE PASSIVE SONARS

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Introduction

One of the functions of a passive sonar is the detection of plane waves immersed in medium noise. If the medium noise is assumed to be isotropic the detection test reduces to a statistical measurement of the probability that the observed noise field, in the presence of sampling error etc., fits an isotropic hypothesis. If it is decided that the noise field is anisotropic it may be concluded that a target, ie a noise source producing a plane wave, is present.

Scharf and Schmid[1] have discussed the optimal detection of an isotropy when a conventional (non-adaptive) beamformer is used to form a number of beams spanning 360° in azimuth and show that the best detector consists of a measurement of the output powers in the beams in decibels whose sample variance is calculated. If the variance exceeds a threshold, the noise field is assumed to be an isotropic ie one or more plane waves due to signals are present.

In this paper the performance of detection system when the beamformer is made adaptive is discussed. Since it is known that adaptive beamforming gives no improvement in conditions where the interference is uncorrelated between elements of the array attention is focused on the case where the array is operating at a low frequency and the elements are only a fraction of a wavelength apart since the medium noise field is highly correlated under these circumstances and adaptive beamforming may be expected to offer a slight processing gain as has been shown theoretically[2,3,4].

Effects of Array Errors

In the absence of array construction errors and if the medium noise is perfectly isotropic then it is obvious that by averaging the beam powers for a sufficiently long time the errors caused by sampling will vanish and the time variance of the beam powers will tend to zero. In such conditions it is clear that there is no lower limit on the strength of a plane wave which is detectable. In order to analyse a more realistic case it is assumed that array and transducer constructional errors are present which perturb the received signals by about 10% of their nominal values. This is an interesting case becasuse it is known that adaptive beamformers aggrevate the effects of array error.

The conventional and adaptive beamformer mean power estimates are given by

Conventional: $$f_c = C^T RC/n^2 \tag{1}$$

Adaptive: $$f_a = (C^T R^{-1} C)^{-1} \tag{2}$$

where C is a complex steering vector whose elements are unity modulus beamformer coefficients for the beam direction required (a sequence of different C vectors is required to scan the array), R is the covariance matrix of the narrow-band array data, and 'n' is the number of elements in the array. The above powers are expected values corresponding to averaging over an infinite time interval. For the adaptive case it is assumed that the norm of the weighting vector will be bounded to reduce its sensitivity to errors. This can be done by injecting uncorrelated noise into the element outputs at a suitable level[5] for the purposes of mathematical computations.

Although the array errors directly affect the signals it is valid to assume that the signals are error-free, the errors arise in the steering vectors C, and they can be represented by adding to C a vector E whose elements are zero mean random variables. For each beam direction a different C vector is used and it is assumed that the errors' effects are uncorrelated from beam to beam. Although this is not the true picture it is equivalent statistically to the case where the errors are fixed but affect a sequence of orthogonal steering vectors.

The conventional and adaptive output powers with errors become

$$\tilde{f}_c = (C+E)^T R (C+E)/n^2 \qquad (3)$$

$$\tilde{f}_a = \left[(C+E)^T R^{-1} (C+E)\right]^{-1} \qquad (4)$$

and on the assumption that the norm of E is small relative to the norm of C then terms involving E twice may be neglected and expanding (3) yields

$$\tilde{f}_c = C^T RC + 2\mathrm{Re}(E^T RC) \qquad (5)$$

and the variance of $\tilde{f}_c$ is ($\langle\cdot\rangle$ is an expectation)

$$\langle \tilde{f}_c^{\,2}\rangle - \langle \tilde{f}_c\rangle^2 = \langle 4\left[\mathrm{Re}(E^T RC)\right]^2\rangle = \langle 2C^T REE^T RC\rangle = 2C^T R\langle EE^T\rangle RC \qquad (6)$$

on the assumption that $\langle EE^T\rangle = \sigma_e^{\,2} I$ then we obtain

$$\mathrm{var}(\tilde{f}_c) = 2C^T R^2 C\, \sigma_e^{\,2} \qquad (7)$$

Normalisation of $\mathrm{var}(\tilde{f}_c)$ by the mean conventional power squared to get a dimensionless measure of variance gives

$$\mathrm{var}(\tilde{f}_c)/f_c^2 = 2C^T R^2 C\sigma_e^{\,2}/(C^T RC)^2 \qquad (8)$$

In order to evaluate this expression we assume that correlation matrix R contains a number of eigenvectors with large, almost equal, eigenvalues[3,4] λ and that C lies mostly in the subspace spanned by these eigenvectors. Then $RC = \lambda C$, $R^2 C = \lambda^2 C$ and (8) reduces to

$$\mathrm{var}(\tilde{f}_c)/f_c^2 = 2\sigma_e^{\,2}/C^T C = 2\sigma_e^{\,2}/n \qquad (9)$$

For the adaptive array using the same assumptions on E

$$\tilde{f}_a = \left[C^T R^{-1} C + 2\mathrm{Re}(E^T R^{-1} C)\right]^{-1} \qquad (10)$$

and using the binominal approximation for the reciprocal we have

$$\mathrm{var}(\tilde{f}_a) = 2C^T R^{-2} C\, \sigma_e^{\,2}/(C^T R^{-1} C)^4 \qquad (11)$$

and

$$\mathrm{var}(\tilde{f}_a)/f_a^2 = 2C^T R^{-1} C\sigma_e^{\,2}/(C^T R^{-1} C)^2 \qquad (12)$$

Noting that the optimal weighting vector $W_o = R^{-1}C/C^T R^{-1} C$, and $W_o^T W_o = C^T R^{-2} C/(C^T R^{-1} C)^2$ then

$$\mathrm{var}(\tilde{f}_a)/f_a^2 = 2W_o^T W_o \sigma_e^{\,2} \qquad (13)$$

The optimal beamformer has usually been operated with a fixed bound on the magnitude of $W_o^T W_o$ on the order of 10 times the convention weight vector value the latter being $W_c^T W_c = C^T C/n^2 = 1/n$. This is done to limit the sensitivity of the adaptive array to errors.[5] Taking $W_o^T W_o = b^2 W_c^T W_c$ we get

$$\mathrm{var}(\tilde{f}_a)/f_a^{\ 2} = 2b^2 \sigma_e^{\ 2}/n$$

and comparing with (9) the relative variance of the estimated beam powers are b^2 times greater in the adaptive case.

Fig 1 shows the output of a simulation program in which a 24 element circular array with interelement spacing of 0.15λ was operated in spherically isotropic noise of unit power per channel and with 10% random channel gain mistakes. The upper curve shows the output power of the beam as it is scanned through the range 0^o to 360^o while the lower curve is the output of the corresponding adaptive processor. A signal of power 0.02 per channel (-17 dB) is present at 300^o azimuth and is just detectable in both cases. However although the output power change is greater in the adaptive case (as a decibel shift there is a corresponding increase in the randomness of the beam power as it is scanned through directions where there is only noise, thus detectability is not greatly enhanced. Measurements on the standard deviation of these beam powers show that the increase in signal differentiation is just about cancelled by the increase in variance of the beam powers in this example.

In a robust adaptive beamformer[5] the weight vector norm can be bounded at any desired value and it is of interest to determine whether there is an optimum norm which maximises the detectability. Fig 2 shows the mean gain in SNR offered by a typical adaptive array as a function of the weight vector norm in the presence of 10% channel mismatches. Since the change in beam output power due to the presence of a signal is the signal power, if the beam powers are normalised to unity for the noise only case, then the change in beam power becomes the output signal to noise ratio of the beam former. A test for the detectability of the signal is the ratio output SNR ÷ standard deviation of noise-only beam powers. This function is also plotted in fig 2. It is seen that there is an improvement of around 2.5 dB for weight vector norms just greater than unity but this falls away if the weight vector norm is allowed to become too great. The best result compares reasonably with the directivity calculations of Pritchard[2] and Vanderkulk[3].

Conclusions

The detectability of plane waves by a circular array in the presence of spherically isotropic medium noise and array errors of about 10% has been investigated and it has been shown that small improvements are offered by adaptive processing which are not incompatible with early theoretical predictions assuming an error-free array. The improvement for medium noise with cylindrical isotropy has been found in simulation programs to be significantly less than the spherical case, ie probably about half measured in decibels, and since it is likely that cylindrical noise isotropy will occur more frequently than spherical noise the 3 dB improvements obtained by the adaptive processor are probably somewhat over optimistic.

Acknowledgements

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References

1 Scharf L L, Schmid C E 'Testing for isotropy' JASA Vol 49 (1971) pp 1092-1099

2 Pritchard R L, 'Maximum directivity index of a linear point array' JASA 26 (1954) pp 1034-1039

3 Vanderkulk W, 'Optimum processing for acoustic arrays' J Brit IRE Oct 1963 pp 285-292

4 Lewis J B, Schultheiss P M 'Optimum and conventional detection using a linear array'. JASA 49 (1971) pp 1083-1091

5 Griffiths J W R, Hudson J E 'An introduction to adaptive processing in a passive sonar system' in 'Aspects of Signal Processing' Editor G Tacconi. Reidel 1978. (Proc. 1976 NATO ASI on signal processing).

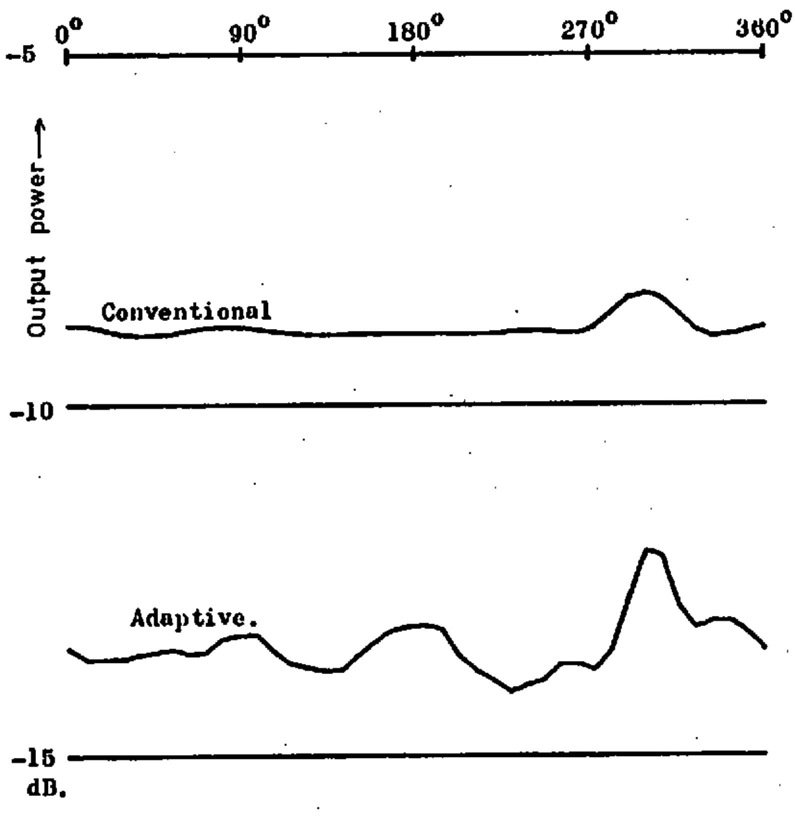

Fig.1 Power/bearing scans for 24 element circular array approx. 2/3 wavelengths radius.

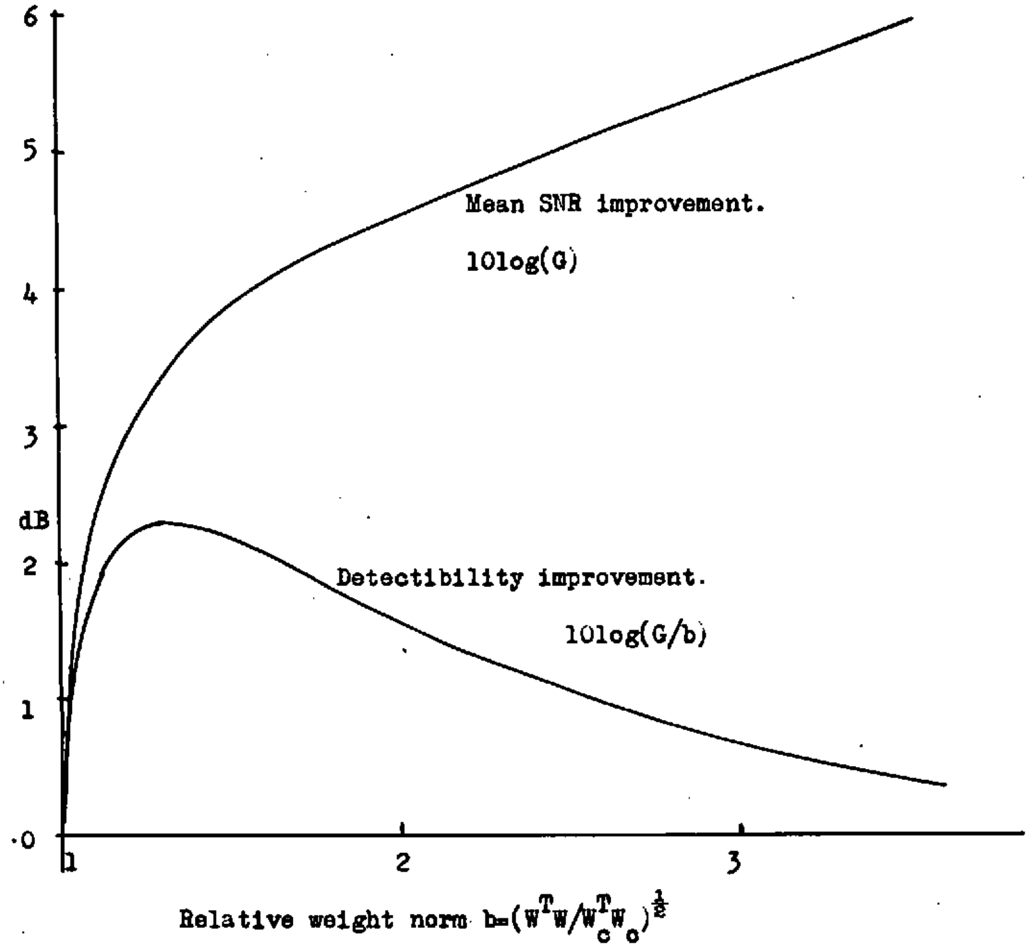
6
5
4
3
dB
2
1
·0
1
2
3
Mean SNR improvement.
10log(G)
Detectibility improvement.
10log(G/b)
Relative weight norm b=(W^T W/W_o^T W_o)^½

Fig. 2.