Performance of a Passive Underwater Localization Systems Based on N Submerged Omnidirectional Sensors

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1. INTRODUCTION

The passive sonar problem of localizing and tracking underwater sound sources based on observations of the random sound that they radiate is an active research area. This is an extensive field that has received much attention in the literature. An area within this field that has received limited attention is the problem of localizing a source using a few submerged omnidirectional receivers [1, 2, 3, 4]. In this case, the source must be near the sensors and the multipath information can be used to reduce the error in the localization.

The information in the received signals that is used to locate a source is the difference in propagation time between the various paths. These time difference of arrivals (TDOAs) can be extracted using time average auto- and cross-correlations [5, 6, 7]. The error in the localization is mainly caused by the ocean acoustic noise and ocean waves. Ocean noise affects the time average correlations and causes errors in the TDOA estimates while the ocean waves perturb the sensors so that the frame of reference is in error.

Expressions for the variance of localization due to ocean noise have been developed for a single sensor [3, 4], a two sensor vertical structure [3, 8] and a two sensor horizontal structure [3]. The effect of sensor perturbations on localization has received attention in the area of linear arrays [9, 10], but this work only considers single path propagation. The multipath, multisensor scenario which includes the effects of both random sensor perturbation and ocean noise is considered in [11].

In this paper, the performance of a localization system based on N submerged omnidirectional sensors is determined and comparisons are made between two specific systems: one based on two sensors and the other based on three sensors. A weighted least square error estimator is used for the localization and the Euclidean variance is used as the measure for performance. The performance is determined for two types of interference: ocean acoustic noise and sensor perturbations.

2. LEAST MEAN SQUARE ERROR ESTIMATION

The system under consideration is based on N submerged omnidirectional sensors. When the source is near the sensors, the sound arrives at the sensors by more than one path. It is assumed that significant power arrives from the direct and surface reflected paths and that virtually no power arrives via a bottom bounce path. The direct path to sensor n is denoted d_n, while the surface bounce path to sensor n is denoted s_n. The path length for the direct path to sensor n, i.e. path d_n, is the Euclidean distance between the source and sensor n. The length of the surface bounce path to sensor n, i.e. path s_n, is the Euclidean distance from the source to a virtual sensor located at the mirror image of sensor n, with the ocean surface acting as the mirror. In a $(x\ y\ z)$ Cartesian coordinate system, where z is the depth, the n^{th} sensor is positioned at $\mathbf{p}_{d_n} = (x_{d_n}\ y_{d_n}\ z_{d_n})^T$ and the virtual sensor is positioned at $\mathbf{p}_{s_n} = (x_{d_n}\ y_{d_n}\ -z_{d_n})^T$, i.e. $x_{s_n} = x_{d_n}$, $y_{s_n} = y_{d_n}$, and $z_{s_n} = -z_{d_n}$.

The signals received by the N sensors are given by

$$r_n(t) = g_{d_n} s(t - D_{d_n}) + g_{s_n} s(t - D_{s_n}) + n_n(t),\ n = 1, 2, ..., N, \qquad (1)$$

where

$r_n(t)$ is the signal received by the n^{th} sensor,

$s(t)$ is the random signal produced by the source,

$n_n(t)$ is the noise received by the n^{th} sensor,

g_{d_n} is the attenuation coefficient for the direct path from the source to the n^{th} sensor and

g_{s_n} is the attenuation coefficient for the surface bounce path from the source to the n^{th} sensor.

The signal $s(t)$ and noises $n_n(t)$, $n = 1, 2, \ldots, N$ are assumed to be stationary, uncorrelated, zero-mean, Gaussian random processes.

The information of interest is the time difference of arrivals (TDOAs) in the path pairs. In an N sensor system, there are N direct and N surface bounce paths and therefore $N(2N-1)$ possible path pairs. The TDOAs are obtained from time average auto- and cross-correlation plots of the received signals. These time difference of arrivals are related to the source and sensor positions by

$$D_{jk} = h_{jk}(\mathbf{p}_c, \mathbf{p}_j, \mathbf{p}_k), \qquad (2)$$

where D_{jk} is the time difference of arrival for path j with respect to path k, $\mathbf{p}_c = (x_c\ y_c\ z_c)^T$ is the source location, $\mathbf{p}_j$ is the position of the real or virtual sensor indicated by j, $j \in \{d_1, d_2, ..., d_N, s_1, s_2, ..., s_N\}$ and $h_{jk}(\mathbf{p}_c, \mathbf{p}_j, \mathbf{p}_k)$ is the hyperboloid given by

$$h_{jk}(\mathbf{p}_c, \mathbf{p}_j, \mathbf{p}_k) = \frac{1}{c}\left[\sqrt{(x_c - x_j)^2 + (y_c - y_j)^2 + (z_c - z_j)^2} - \sqrt{(x_c - x_k)^2 + (y_c - y_k)^2 + (z_c - z_k)^2}\right], \qquad (3)$$

where c is the speed of sound in the ocean. For notational convenience, the arguments of the hyperboloids $h_{jk}(\mathbf{p}_c, \mathbf{p}_j, \mathbf{p}_k)$ are often omitted and the function is written h_{jk}.

The notation chosen for equation (3) can be misleading, especially when taking the partial derivative of h_{jk} with respect to a sensor position. The dependencies among the positions of the real and virtual sensors must be considered when taking a partial derivative. For example, before taking the partial derivative with respect to the vertical position of real sensor n, all occurrences of z_{d_n} and z_{s_n} must be replaced with a common variable, and then the partial taken with respect to that variable. In general the following substitutions are required prior to taking a partial derivative:

$$x_j = x_n,\ y_j = y_n,\ \text{for } j = d_n, s_n;\ \ z_j = \begin{cases} z_n & \text{for } j = d_n \\ -z_n & \text{for } j = s_n \end{cases}. \qquad (4)$$

These non-linear equations can be approximated for source positions near $\mathbf{p}_c = (x_c\ y_c\ z_c)^T$ by first order Taylor series expansions about x_c, y_c and z_c, which have the form:

$$\Delta D_{jk} = \frac{\partial h_{jk}}{\partial x_c}\Delta x_c + \frac{\partial h_{jk}}{\partial y_c}\Delta y_c + \frac{\partial h_{jk}}{\partial z_c}\Delta z_c, \tag{5}$$

where ΔD_{jk} is the change in the TDOA for path j with respect to path k for a change in source position $\Delta\mathbf{p}_c = (\Delta x_c\ \Delta y_c\ \Delta z_c)^T$ and the partial derivatives of h_{jk} are evaluated at $\mathbf{p}_c, \mathbf{p}_j, \mathbf{p}_k$. The full set of $N(2N-1)$ linearized equations can be represented in matrix form as

$$\Delta\mathbf{D}_N = \mathbf{A}_N \Delta\mathbf{p}_c, \tag{6}$$

where $\Delta\mathbf{D}_N$ is an $N(2N-1)$ by 1 vector containing the changes in time difference of arrivals and $\mathbf{A}_N$ is an $N(2N-1)$ by 3 matrix whose elements are the partial derivatives of the hyperboloids. The matrix $\mathbf{A}_N$ is a function of the source and sensor positions and is given by

$$\mathbf{A}_N(\mathbf{p}_c, \mathbf{p}_{d_n};\ n = 1, 2, ..., N) = \mathbf{A}_N = \left[\ \frac{\partial\mathbf{H}}{\partial x_c}\ \frac{\partial\mathbf{H}}{\partial y_c}\ \frac{\partial\mathbf{H}}{\partial z_c}\ \right], \tag{7}$$

where $\mathbf{H}$ is the vector of $N(2N-1)$ distinct hyperboloid functions, one for each distinct path pair, and is given by

$$\mathbf{H}(\mathbf{p}_c, \mathbf{p}_{d_n};\ n = 1, 2, ..., N) = \mathbf{H} = \begin{bmatrix} h_{d_1 d_2} \\ h_{d_1 d_3} \\ \cdot \\ \cdot \\ \cdot \\ h_{s_{N-1} s_N} \end{bmatrix} \tag{8}$$

The $N(2N-1)$ unique path pairs used to index the elements of $\mathbf{H}$ are given by the elements of the lower or upper triangle (excluding the diagonal) of the $2N$ by $2N$ square matrix formed from $[d_1\, d_2 \ldots d_N\, s_1\, s_2 \ldots s_N]^T [d_1\, d_2 \ldots d_N\, s_1\, s_2 \ldots s_N]$. The ordering of the path pairs used to index the elements of $\mathbf{H}$ is obtained by concatenating the $N-1$ rows in the upper triangle of this square matrix.

The matrix equation for $\Delta\mathbf{D}_N$ is valid provided $\Delta\mathbf{p}_c$ is the independent variable. Unfortunately, the equation is not, in general, valid when $\Delta\mathbf{D}_N$ is the independent variable since the matrix equation represents an overspecified system. If $\Delta\mathbf{D}_N$ is chosen arbitrarily, it is extremely unlikely that the system of equations will be consistent. It is, however, possible to find a $\Delta\mathbf{p}_c$ that minimizes the magnitude of the difference vector $\Delta\mathbf{D}_N - \mathbf{A}_N\Delta\mathbf{p}_c$. In other words, it is possible to find a $\Delta\mathbf{p}_c$ that gives a least square error fit to $\Delta\mathbf{D}_N$. Such a $\Delta\mathbf{p}_c$, denoted $\Delta\hat{\mathbf{p}}_c$, is the least mean square estimation error, and is given by

$$\Delta\hat{\mathbf{p}}_c = [\mathbf{A}_N^T\mathbf{A}_N]^{-1}\mathbf{A}_N^T\Delta\hat{\mathbf{D}}_N, \tag{9}$$

where $\Delta\hat{\mathbf{D}}_N$ is a $N(2N-1)$ by 1 vector containing the errors in the time difference of arrival measurements.

There are situations, for example when the TDOA measurements have different variances, where it is advantageous to find the source position that minimizes the weighted difference $\|\mathbf{W}_N(\Delta\mathbf{D}_N - \mathbf{A}_N\Delta\mathbf{p}_c)\|$, where $\mathbf{W}_N$ is a $N(2N-1)$ by $N(2N-1)$ weighting matrix. The source position that gives the least square error in the weighted TDOAs is

$$\Delta\hat{\mathbf{p}}_c = \mathbf{B}_N\mathbf{W}_N\Delta\hat{\mathbf{D}}_N, \tag{10}$$

where

$$\mathbf{B}_N = [\mathbf{A}_N^T\mathbf{W}_N^T\mathbf{W}_N\mathbf{A}_N]^{-1}\mathbf{A}_N^T\mathbf{W}_N^T. \tag{11}$$

3. LOCALIZATION ERROR DUE TO OCEAN NOISE

There are two components of localization error. One component is due to background acoustic noise, the other component is due to sensor position error. In this section only the error due to the background noise is considered. The sensors are assumed to be perfectly positioned.

Ocean noise causes error in the time difference of arrival measurements which propagate through the equations for the hyperboloids and cause error in localization. The TDOA measurement error vector is denoted

$$\Delta\hat{\mathbf{D}}_N = \hat{\mathbf{D}}_N - \mathbf{D}_N. \tag{12}$$

Let $\hat{\mathbf{p}}_c$ be the source position that minimizes the magnitude of the difference vector $\mathbf{W}_N(\hat{\mathbf{D}}_N - \mathbf{A}_N\hat{\mathbf{p}}_c)$. Then, using (10), the covariance of $\Delta\hat{\mathbf{p}}_c$ given the actual source position is given by

$$\begin{aligned} \mathrm{cov}[\Delta\hat{\mathbf{p}}_c|\mathbf{p}_c] &= \mathbf{B}_N \mathrm{E}[\mathbf{W}_N\Delta\hat{\mathbf{D}}_N\Delta\hat{\mathbf{D}}_N^T\mathbf{W}_N^T\big|\,\mathbf{p}_c]\mathbf{B}_N^T \\ &= \mathbf{B}_N\mathbf{W}_N\mathrm{cov}[\Delta\hat{\mathbf{D}}_N\big|\,\mathbf{p}_c]\mathbf{W}_N^T\mathbf{B}_N^T, \end{aligned} \tag{13}$$

where the matrix $\mathrm{cov}[\Delta\hat{\mathbf{D}}_N\big|\,\mathbf{p}_c]$ is the covariance matrix for $\Delta\hat{\mathbf{D}}$ given $\mathbf{p}_c$.

The effects of ocean noise on the measurements of TDOAs in a multipath environment has been investigated by Daku[12]. He derived the following expression for the variance of error of TDOAs from time average correlations of length T:

$$\begin{aligned} \mathrm{var}[\hat{D}_{jk}] \approx\ & \frac{\frac{1}{2\pi T}\int_{-\infty}^{+\infty}\omega^2|H_o(\omega)|^4 S_{11}(\omega)S_{22}(\omega)d\omega}{\left(\frac{1}{2\pi}\int_{-\infty}^{+\infty}\omega^2|H_o(\omega)|^2 S_{12}(\omega)e^{j\omega D_{jk}^b}d\omega\right)^2} \\ & - \frac{\frac{1}{2\pi T}\int_{-\infty}^{+\infty}\omega^2|H_o(\omega)|^4 S_{12}(-\omega)S_{21}(\omega)e^{-j\omega 2D_{jk}^b}d\omega}{\left(\frac{1}{2\pi}\int_{-\infty}^{+\infty}\omega^2|H_o(\omega)|^2 S_{12}(\omega)e^{j\omega D_{jk}^b}d\omega\right)^2} \end{aligned} \tag{14}$$

where, $j = s_n$ or d_n , $k = s_n$ or d_n, and $j \neq k$ and ${D_{jk}}^b$ is the time difference of arrival obtained from perfect (infinite time) auto- and cross-correlation plots. The superscript b is used to indicate that even the perfect correlation plots produce a bias in the time difference of arrivals. This bias is caused by the multipath interference. The quantity $\|H_o(\omega)\|$ is the magnitude of the frequency response of optimizing filters. $S_{11}(\omega)$ and $S_{22}(\omega)$ are the spectral densities of the multipath signals received at the first and second sensor respectively and $S_{12}(\omega)$ is the cross-power spectral density of the two multipath signals.

The trace of the covariance matrix $\mathrm{cov}[\Delta\hat{p}_c | p_c]$ gives the variance of Euclidean localization error. Comparing the Euclidean variances of systems based on different numbers of sensors is very difficult as the expressions for the covariance matrices are mathematically complex and there are a large number of variables. However, a comparison of two specific sensor systems serves the useful purpose of demonstrating that significant improvement in performance can be realized by going from a two to a three sensor system.

A sensor system based on two sensors is used in the comparison. The sensors are separated by 1000 meters and are both submerged to a depth of 200 meters. In terms of an $(x\ y\ z)$ coordinate system the sensors are located at (-500, 0, 200) and (500, 0, 200), where z is the depth. The other sensor system used in the comparison is based on three sensors. Two of the sensors are in the same position as those in the two sensor system. The third sensor is located at (0, 866, 200). This places the three sensors on the corners of an equilateral triangle with 1000 meter sides. The sensor position and source track under consideration are illustrated in Figure 1.

For the purposes of this comparison the ratio of the source level to the ocean noise intensity level is assumed to be 80 dB and the bandwidth of signal is assumed to be 400 Hz.

The Euclidean variances of the localization error for the two and three sensor systems are illustrated in Figure 2. The variances are shown with and without a weighting matrix. The weighting matrix used is the inverse of the covariance matrix of the time difference of arrivals. There are peaks in the curves when the weighting is not applied. These peaks are caused by very noisy time difference of arrival estimates for the direct-direct and surface bounce-surface bounce paths. These peaks result when the source position is such that the cross-correlation of the received signals has broad, somewhat flat peaks whose positions are not sharply defined and very prone to noise.

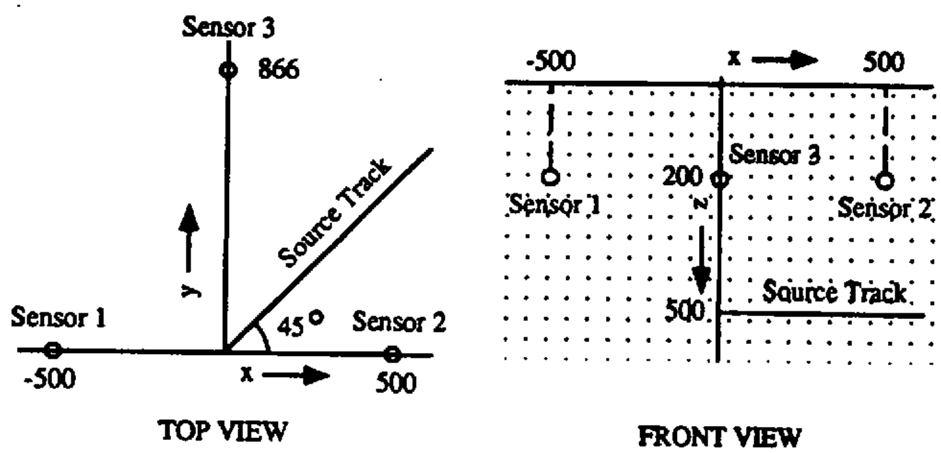

Figure 1: Illustration of the two and three sensor systems and the source track used in the comparison. The two sensor system consists of sensors 1 and 2 only.

4. LOCALIZATION ERROR DUE TO SENSOR PERTURBATIONS

In this section, only the sensor perturbations are considered and the ocean noise is assumed to be zero. Under ideal conditions, the n^{th} sensor is positioned at $\mathbf{p}_{d_n} = [x_{d_n}\ y_{d_n}\ z_{d_n}]^T$. Because of waves and ocean currents, the actual sensor locations derivate from these positions. The actual position of the n^{th} sensor is denoted $\dot{\mathbf{p}}_{d_n} = [\dot{x}_{d_n}\ \dot{y}_{d_n}\ \dot{z}_{d_n}]^T = [x_{d_n} + \Delta\dot{x}_{d_n}\ \ y_{d_n} + \Delta\dot{y}_{d_n}\ \ z_{d_n} + \Delta\dot{z}_{d_n}]^T$. The random error in the position of n^{th} sensor is denoted $\Delta\dot{p}_{d_n} = (\Delta\dot{x}_{d_n}\ \Delta\dot{y}_{d_n}\ \Delta\dot{z}_{d_n})$. The average error is assumed to be zero.

The problem is to determine the effect of the random sensor position error on localization. The source position is calculated from estimated time difference of arrivals using the ideal sensor positions. If a sensor is displaced from its ideal position, then it will receive the source signal either sooner or later than assumed, depending on whether the sensor displacement is toward or away from the source. This will affect the time difference of arrivals between the displaced sensor and the other sensors. The problem becomes one of relating sensor perturbations to time difference of

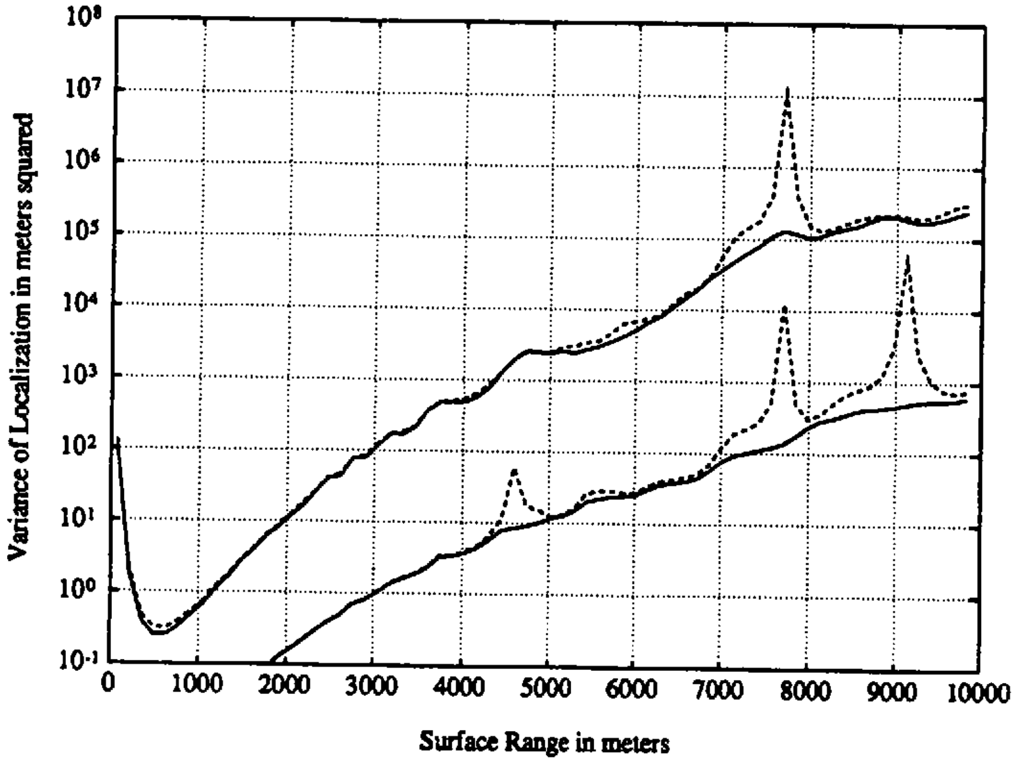

Figure 2: Weighted and unweighted least mean square localization variances for systems based on two and three submerged sensors. The solid curves are the weighted variances. The top two curves represent the two sensor system and the bottom two curves represent the three sensor system. The source level to noise intensity level is 80dB and the bandwidth of the source is 400 Hz. The surface is assumed to be a perfect reflector.

arrival errors and then using the equation developed in the previous section to find the covariance of localization error as a function of the covariance of sensor perturbations.

The time difference of arrivals are related to sensor positions through the equation of the hyperboloids given by (3). Only the N vertical components, i.e. $\Delta\dot{z}_1$, $\Delta\dot{z}_2$, ..., and $\Delta\dot{z}_N$, are considered and hence, (2) and (3) can be rewritten as

$$\dot{D}_{jk} = \frac{1}{c}\left[\sqrt{(x_c - x_j)^2 + (y_c - y_j)^2 + (z_c - (z_j - \Delta\dot{z}_j))^2} - \sqrt{(x_c - x_k)^2 + (y_c - y_k)^2 + (z_c - (z_k - \Delta\dot{z}_k))^2}\right]. \quad (15)$$

Assuming the perturbations are much smaller than the sensor depth, i.e. $\Delta\dot{z}_j \ll z_j$, then the change in delay due to the change in sensor positions can be obtained from a linear approximation to the above equation. Linearizing with a Taylor series about the unperturbed sensor positions and source position $\mathbf{p_c}$ produces

$$\Delta\dot{D}_{jk} = \dot{D}_{jk} - D_{jk} = \frac{\partial h_{jk}}{\partial z_1}\Delta\dot{z}_1 + \frac{\partial h_{jk}}{\partial z_2}\Delta\dot{z}_2 + ... + \frac{\partial h_{jk}}{\partial z_N}\Delta\dot{z}_N, \quad (16)$$

where D_{jk} is the time difference of arrival in the absence of sensor perturbations for a source located at $\mathbf{p}_c$, and $\Delta\dot{D}_{jk}$ is the error in the time difference of arrival between paths j and k caused by the sensor perturbations. These equations can be put into the matrix form

$$\Delta\dot{\mathbf{D}} = \dot{\mathbf{A}}_{\mathbf{N}}\Delta\dot{\mathbf{z}}, \quad (17)$$

where $\Delta\dot{\mathbf{D}}$ is a $N(2N-1)$ by 1 vector, $\Delta\dot{\mathbf{z}}$ is a N by 1 vector and $\dot{\mathbf{A}}_N$ is a $N(2N-1)$ by N matrix. They are defined as

$$\Delta\dot{\mathbf{D}} = \dot{\mathbf{D}} - \mathbf{D}, \quad (18)$$

$$\dot{\mathbf{A}}_N = \left[\frac{\partial \mathbf{H}}{\partial z_1}\ \frac{\partial \mathbf{H}}{\partial z_2} \cdots \frac{\partial \mathbf{H}}{\partial z_N}\right]\Bigg|_{\mathbf{p}_c, \mathbf{p}_{d1}, \mathbf{p}_{d2}, ..., \mathbf{p}_{dN}}, \quad (19)$$

with $\mathbf{H}$ defined by (8) and

$$\Delta\dot{\mathbf{z}} = [\Delta\dot{z}_1\ \Delta\dot{z}_2\ ...\Delta\dot{z}_N\ ]^T. \quad (20)$$

The variance of the time difference of arrivals due to the vertical sensor perturbations for a given source position is given by

$$\mathrm{cov}[\Delta\dot{\mathbf{D}}_z|\mathbf{p}_c] = \dot{\mathbf{A}}_N \mathrm{cov}[\Delta\dot{\mathbf{z}}]\,\dot{\mathbf{A}}_N^T. \quad (21)$$

It is pointed out that $\mathrm{cov}[\Delta\dot{\mathbf{D}}_z|\mathbf{p}_c]$ still depends on $\mathbf{p}_c$ through $\dot{\mathbf{A}}_N$. Assuming the sensors are sufficiently separated, $\Delta\dot{z}_n$ are independent of each other, and $\mathrm{cov}[\Delta\dot{\mathbf{z}}]$ will be a diagonal matrix with diagonal elements $\sigma^2_{z_n}$.

The expression for location error covariance of the least square error (no weighting) source position is given by

$$\begin{aligned}\mathrm{cov}[\Delta\dot{\mathbf{p}}_c|\mathbf{p}_c] &= [\mathbf{A}_N^T\mathbf{A}_N]^{-1}\mathbf{A}_N^T\mathrm{cov}[\Delta\dot{\mathbf{D}}|\mathbf{p}_c]\mathbf{A}_N[\mathbf{A}_N^T\mathbf{A}_N]^{-1},\\ &= [\mathbf{A}_N^T\mathbf{A}_N]^{-1}\mathbf{A}_N^T\dot{\mathbf{A}}_N\mathrm{cov}[\Delta\dot{\mathbf{z}}]\dot{\mathbf{A}}_N^T\mathbf{A}_N[\mathbf{A}_N^T\mathbf{A}_N]^{-1},\end{aligned} \quad (22)$$

where $\Delta \dot{\mathbf{p}}_c$ is the error in source position due to sensor perturbations.

The performances of the two and three sensor systems described in the previous section are again compared. In this comparison, ocean noise does not exist and the only source of error is sensor perturbations in the vertical direction. A plot of the Euclidean variances given by equation (22) is shown in Figure 3. The covariance matrices of the vertical sensor perturbations are taken to be identity matrices, i.e. variances of 1m^2. The variances of Euclidean error shown in Figure 3 are relatively smooth functions of surface range and do not have sharp peaks like the unweighted curves in Figure 2.

5. CONCLUSIONS

Expressions for the variance of localization error for a system of N submerged sensors has been derived for a least mean square error estimator. The expressions include the effect of background ocean noise as well as perturbations in the vertical positions of the sensors. The nature of these expressions is such that it is difficult to make general comparisons. However, a comparison of

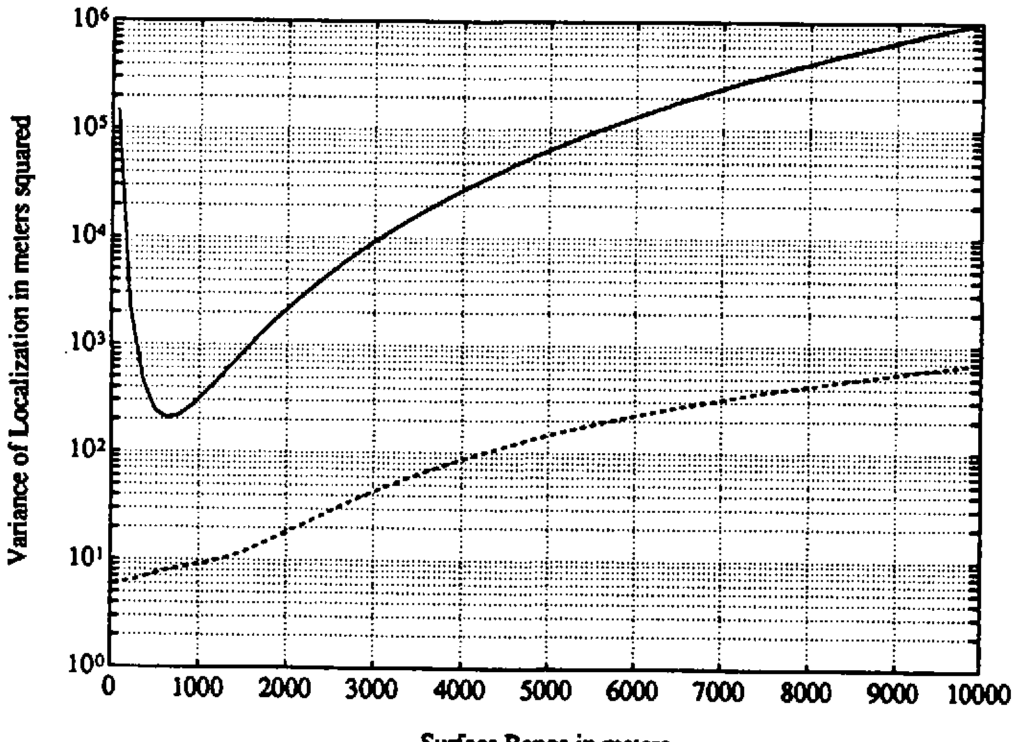

Figure 3: Localization variances for systems based on two and three submerged sensors that have vertical perturbations with variances of 1.0 meters squared. The solid curve represents the two sensor system and the dashed curve represents the three sensor system.

specific systems based on two and three sensors suggests that the variance of localization error due to both sensor perturbations and background ocean noise can be reduced by a few orders of magnitude by adding a third sensor to a two sensor system. The comparison also suggests that most of the localization error will be caused by sensor perturbations when the source is near the sensor system, even with perturbations of only moderate variances.

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