Proceedings of the Institute of Acoustics

ACOUSTIC DESIGN OF REFLECTORS IN AUDITORIA

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1. INTRODUCTION

The use of sound reflectors is an old remedy in room acoustics. The canopy above the pulpit in churches is one example, although it is often too small to be fully effective. An attempt to establish a quantitative design criteria on a scientific basis was made in 1953 by Cremer [1] in relation to the 'Herkulessaal' in Munich. However, the difference in the acoustic behaviour of panel arrays and single panels was not considered, and in this and many other cases the result was the design of relatively few, large reflectors. A different design criterion for panel arrays has been used recently for the Danish Radio Concert Hall in Copenhagen [2].

Today easy access to computer power means that rigorous solutions to complicated diffraction and reflection phenomena should not be a problem. For the architect and the acoustic consultant, however, it is more useful to have some simple design criteria and a basic understanding of the physical behaviour. Such simple methods will be emphasized in the present paper.

2. REASONS FOR THE USE OF REFLECTORS

Several reasons for the use of reflectors can be listed. The position of the reflectors and frequency range to be considered depends on the purpose.

2.1 Support of a (weak) sound source

This can be reflectors above or behind a speaker. Sometimes concave reflectors can be very efficient, especially in the open air. Orchestra shells for open air concerts is another example.

2.2 Improvement of balance between groups in an orchestra

In the design of orchestra shells this is one of the most important criteria.

2.3 Improvement of ensemble on an orchestra platform

In large concert halls with a high ceiling it can be necessary to use suspended reflectors above the orchestra. The height above the platform should typically be 7 to 10 m. There is a risk of coloration, i.e. an unwanted change of timbre, if the reflector hight is too low. Sometimes tilted reflectors on the walls surrounding the orchestra are sufficient.

2.4 Blocking echo paths
A very high ceiling can create an echo in the front part of the audience area. This can be avoided by installing reflectors with an excess path length $a_1 + a_2 - a_0 < 17$ m. The distances are defined in Fig. 1.

2.5 Increase of level and clarity at remote seats
Reflectors at the back wall, on balcony fronts etc. can turn useful reflections to remote seats. This is similar to 2.1 but here the reflectors are close to the receiver instead of the source.

2.6 Creation of lateral reflections
The importance of early lateral reflections in concert halls sometimes leads to the use of reflectors along the side walls. To be useful the excess path length should be $a_1 + a_2 - a_0 < 27$ m.

2.7 Creation of diffuse reflections
Often diffuse reflections are preferred to specular reflections. The spread of reflected energy means an attenuation of the geometrical reflection without additional absorption. Small panels, convex curved surfaces or specially designed diffusing surfaces may be used. Methods for the design of diffusing surfaces by use of highly irregular forms will not be discussed here.

3. ATTENUATION OF SOUND REFLECTIONS

The level of a sound reflection may be defined relative to the level of the direct sound from a point source. The attenuation of the reflection can be divided into different physical effects, which will be treated separately in the following. These attenuations are supposed to be additive, so the level difference may be written:

$$\Delta L = L_{refl} - L_{dir} = \Delta L_{dist} + \Delta L_{abs} + \Delta L_{diffr} + \Delta L_{curv} \tag{1}$$

In general $\Delta L < 0$ dB due to the excess attenuation of the reflection.

3.1 Distance
With distances a_0, a_1 and a_2 as defined in Fig. 1 the attenuation due to geometrical spread of energy from a point source is:

$$\Delta L_{dist} = 20 \log \frac{a_0}{a_1 + a_2} \tag{2}$$

3.2 Material of Reflector
If the sound absorption coefficient of the reflecting surface is α the proportion of the incident energy being reflected is $(1 - \alpha)$ or:

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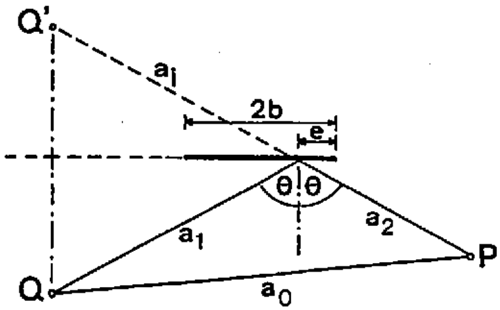


Figur 1. Sound reflection from a panel with width $2b$. Q is source, P is receiver and Q' is the image source.

$$\Delta L_{abs} = 10 \log(1 - \alpha) \tag{3}$$

The attenuation is negligible if $\alpha < 0.10$. Reflectors are usually made from hard materials like wood, plexiglass or metal. For a plate of a dense material the absorption may be set equal to the transmission, which is related to the mass per unit area m as originally shown by Cremer [1]. From this the following can be derived:

$$\Delta L_{abs} = -10 \log \left[1 + \left(\frac{\rho c}{\pi f m \cos\theta} \right)^2 \right] \tag{4}$$

where f is the frequency, θ the angle of incidence and $\rho c \approx 415$ kg $m^{-2}s^{-1}$ is the characteristic impedance of air. Usually a mass of at least 10 kg m^{-2} should be sufficient to make this attenuation term negligible.

3.3 Size of Reflector

The dimensions of a reflector must be seen in relation to the distances to source and receiver, see Fig. 1. A useful parameter in the following is the characteristic distance a^*, which is defined by the relation:

$$a^* = \frac{2a_1 a_2}{a_1 + a_2} \tag{5}$$

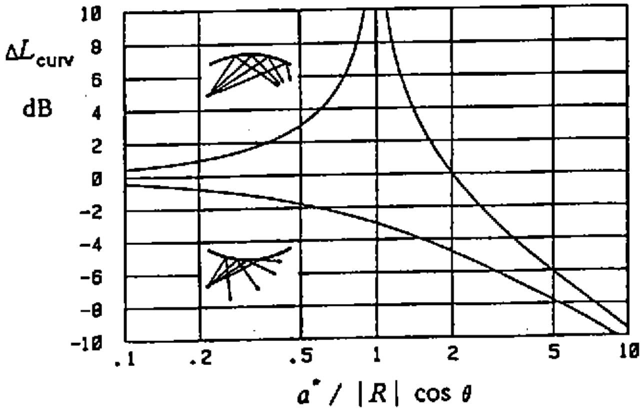
It follows that if the distances from the reflection point to source and receiver are approximately equal, then $a^* \approx a_1 \approx a_2$, but if $a_1 << a_2$ then $a^* \approx 2a_1$. The dimensions of a freely suspended reflector must be compared to the characteristic distance and to the wave length $\lambda = c/f$. As a simple design guide for a reflector with area S, diffraction losses can be considered negligible above the limiting frequency:

$$f_g = \frac{c\ a^*}{2\ S\ \cos\theta} \tag{6}$$

Below the limiting frequency the reflection is attenuated with a rate of 6 dB per octave. Thus:

$$\Delta L_{diffr} \approx \begin{cases} 0\ \text{dB} & \text{for} \quad f \geq f_g \\ 20\log(f\ /\ f_g) & \text{for} \quad f < f_g \end{cases} \tag{7}$$

The shape of the reflector is of minor importance as long as its length and width do not differ very much, say more than a factor of 4. More details are given in section 4.



Figur 2. Attenuation due to the curvature of a concave or convex cylinder.

3.4 Curvature of Reflector

A simple geometrical consideration [3] leads to the following approximate term for the reflection from a cylinder with radius of curvature R:

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$$\Delta L_{curv} = -10\log\left|1 + \frac{a^*}{R\cos\theta}\right| \tag{8}$$

A convex surface has $R > 0$, whereas a concave surface has $R < 0$. In the latter case energy is concentrated by the reflection, and a focusing effect appears if $R = -a^*/\cos\theta$. Equation (8) is illustrated in Fig. 2 for a convex and a concave cylinder.
In the case of a doubly curved surface with two radii of curvature the attenuation term (8) should be used twice, applying the appropriate projections into the two normal planes of the surface.

For a geometrical construction of the reflected rays it may be useful to find the distance from the reflection point to the image source, a_i on Fig. 1.

$$a_i = \left(\frac{1}{a_1} + \frac{2}{R\cos\theta}\right)^{-1} \tag{9}$$

4. SINGLE REFLECTORS

4.1 The Exact Solution
The Kirchhoff-Fresnel approximation to the diffraction problem shown in Fig. 1 leads to the following reflection factor:

$$K_1 = \frac{1}{2}\left[(C(\nu_1)+C(\nu_2))^2 + (S(\nu_1)+S(\nu_2))^2\right] \tag{10}$$

where the variables are:

$$\nu_1 = \frac{2}{\sqrt{\lambda a^*}}e\cos\theta \quad , \qquad \nu_2 = \frac{2}{\sqrt{\lambda a^*}}(2b-e)\cos\theta \tag{11}$$

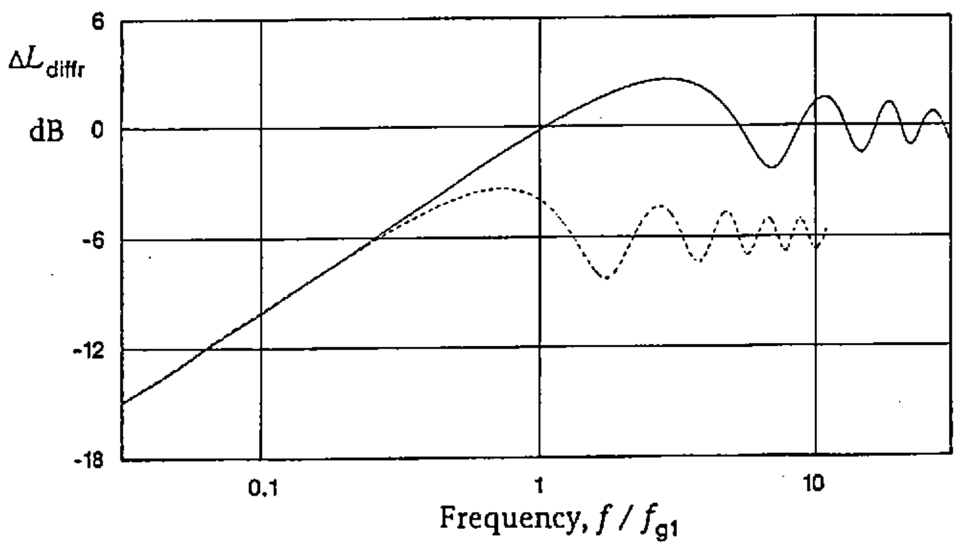
In (10) the functions C and S are the Fresnel integrals:

$$C(\nu) = \int_0^\nu \cos\left(\frac{\pi}{2}z^2\right)dz \quad , \qquad S(\nu) = \int_0^\nu \sin\left(\frac{\pi}{2}z^2\right)dz \tag{12}$$

4.2 The Orthogonality Principle
The diffraction from a rectangular panel with two edges perpendicular to the direction from source to receiver can be described by two independent factors:

$$\Delta L_{diffr} = 10\log(K_1 K_2) \tag{13}$$

K_1 and K_2 are reflection factors for two infinite strips corresponding to two orthogonal sections through the reflecting surface. Section 1 is supposed to contain the source and receiver points and K_1 is a function of the projection of the panel width $2b \cdot \cos\theta$, see Fig. 1. K_2 is a similar function of the panel length $2l$ in the perpendicular direction.



Figur 3. Attenuation due to diffraction from a long strip (exact solution). f_{g1} is the limiting frequency. ___ reflection at the centre, ... reflection at the edge.

4.3 Approximation for a Long Strip

A long reflecting strip is a 1-dimensional reflector and $K_2 = 1$ in (13). The reflection factor K_1 as a function of frequency is shown in Fig. 3 for two special positions of the geometrical reflection point, either in the centre of the strip ($e = b$) or at the edge ($e = 0$). From the exact solution shown it is seen that if the reflection point is at the centre of the strip, the reflection is fully effective above a limiting frequency f_{g1} (the subscript 1 indicating section 1):

$$f_{g1} = \frac{ca^*}{2\,(2b\cos\theta)^2} \tag{14}$$

Below this limiting frequency the reflection is attenuated, and a good approximation for the reflection factor is:

$$K_1 = \begin{cases} 1 & \text{for} \quad f \geq f_{g1} \\ f\,/\,f_{g1} & \text{for} \quad f < f_{g1} \end{cases} \tag{15}$$

This is valid if the reflection point is at the centre of the strip.

4.4 Reflection near the Edge of a Long Strip

The distance of the reflection point from the edge is called e, see Fig. 1. The influence of the position of the reflection point was studied in some detail in [4], and some of the following results are taken from that reference. If the reflection point is exactly at the edge ($e = 0$), the asymptotic attenuation at high frequencies is 6 dB ($K_1 = 1/4$) and the crossover frequency will be two octaves lower than the limiting frequency given by eq. (14). In the general case $0 < e < b$ the reflection will be less than fully effective below a limiting frequency f_{p1}, which depends on e:

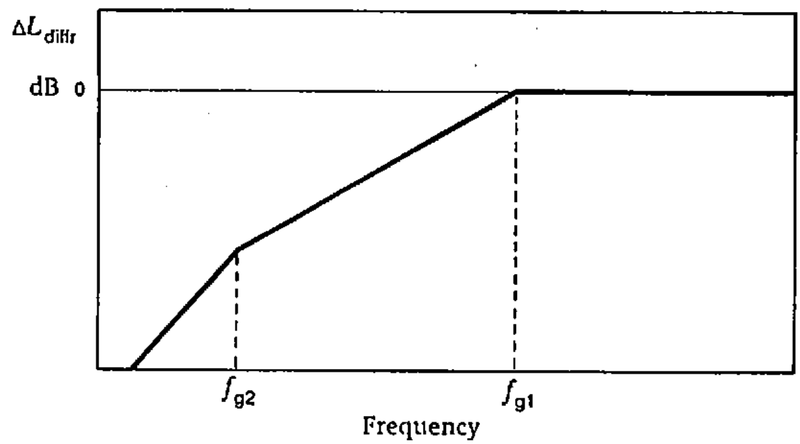
$$f_{p1} = \frac{ca^*}{2\,(2e\cos\theta)^2} \tag{16}$$

The reflection factor in this case is approximately:

$$K_1 \approx \frac{1}{4} + \frac{3}{4}\sqrt{\frac{f}{f_{p1}}} \qquad \text{for} \quad f < f_{p1} \tag{17}$$

Note that $f_{p1} \leq f_{g1}$ and that $f_{p1} \rightarrow \infty$ for $e \rightarrow 0$. At low frequencies, $f \leq f_{g1}/4$, (15) is valid, but in the range $f_{g1}/4 < f < f_{g1}$ the lowest value from (15) and (17) should be used for the reflection factor. The transition frequency cannot be calculated from a simple function. However, from (17) it appears that the attenuation at frequencies below f_{p1} increases very slowly, and for practical use it may be more realistic to define a 'cutoff frequency' at which the attenuation reaches 3 dB. From (17) this cutoff frequency is found to be $f_{p1}/9$.

A special application of (17) is for a long reflecting wall with a free edge. Such walls are sometimes used close to the orchestra podium, typically if there are seats for a choir or an extra audience behind the orchestra. The important parameter that determines the attenuation of a reflection towards the main audience is the distance e of the reflection point from the edge. As an example, if 3 dB attenuation is acceptable at 50 Hz then the limiting frequency f_{p1} is 450 Hz, and from (16) the necessary distance e to the edge can be found. If $a^* = 10$ m and $\theta = 0°$ we get $e \approx 1.0$ m.



Figur 4. Approximate attenuation from a rectangular panel with a large ratio between length and the projection of width. The slopes are 6, 3 and 0 dB per octave.

4.5 Approximation for a Rectangular Panel

This is a 2-dimensional reflector and according to the orthogonality principle two reflection factors must be calculated. K_1 is related to the projection of width $2b \cdot \cos\theta$, and is the same as for the long strip above. K_2 is determined from the length of the panel $2l$, and the corresponding limiting frequency is:

$$f_{g2} = \frac{c a^*}{2(2l)^2} \tag{18}$$

Using the approximate reflection factor (15) for both dimensions will give a combined reflection factor for the panel with a general form as shown in Fig. 4 for the case $f_{g2} << f_{g1}$. The slope of the curve is 6 dB per octave at very low frequencies and 3 dB per octave between the two limiting frequencies. If l and $b \cdot \cos\theta$ do not differ too much (say less than a factor of 4) the resulting limiting frequency above which the panel can be considered effectively reflecting, i.e. $K_1 \cdot K_2 \approx 1$, is given by:

$$f_g = \sqrt{f_{g1} f_{g2}} = \frac{c a^*}{2 S \cos\theta} \tag{19}$$

This will be recognised as eq. (6), and (7) is obtained if (15) is inserted in (13) for both reflection factors. So, usually the combined limiting frequency (19) will be sufficient for the design of single reflectors. The shape of the panel does not need to be rectangular, the approximate equations above may be applied for other shapes as well, if length and width do not differ too much.

5. REFLECTOR ARRAYS

5.1 The Useful Frequency Range

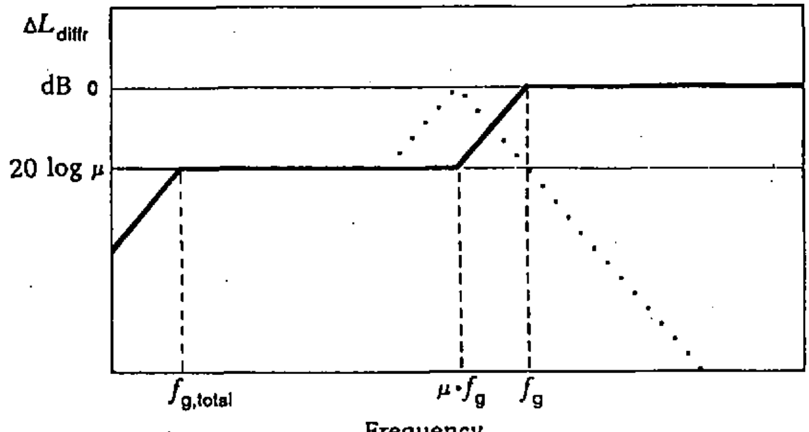
If S_{total} is the total area of the reflector array, a lower limiting frequency similar to (19) can be calculated:

$$f_{g,total} = \frac{c\,a^*}{2S_{total}\cos\theta} \tag{20}$$

The array can be considered useful above this frequency up to the limiting frequency f_g related to the area of the individual panels in the array. Between these two limiting frequencies the reflector array has an average attenuation of the reflection equal to:

$$\Delta L_{diffr} = 20\log\mu \qquad f_{g,total} \le f \le f_g \tag{21}$$

where μ is the relative density of the reflector array, defined as the ratio between the area of all reflectors and the total area covered by the array, S_{total}. Thus within the useful frequency range, a reflector array with μ = 0.25 will give an average reflection level 12 dB lower than a single reflector with area S_{total}.



Figur 5. Approximate attenuation due to diffraction from a reflector array with relative density μ. ___ reflection point on a panel, ... reflection point in gap between panels.

The principle of a reflector array may be illustrated as in Fig. 5. The limiting frequency f_g for the individual panels in the array is the important design parameter, but here as an upper frequency limit. The same parameter was used as a lower limiting frequency in the design of single reflectors.

The reason for this is that at higher frequencies the reflections are strongly dependent on whether the geometrical reflection point is on one of the individual panels or not. Thus, for an average of source and receiver positions the array is not reliable at high frequencies. The curves for typical worst and best positions of the reflection point are shown on Fig. 5. Above the upper limiting frequency f_g the attenuation will vary between the two curves depending on the position of the reflection point.

Measurements or calculations from an exact solution to the diffraction problem of a panel array will show a lot of irregularities in the frequency response, but the average level of the reflection level will follow the simple curves in Fig. 5. The irregularities are due to the interference between many diffracted sound paths and they can be 25-30 dB, but typically with very narrow frequency dips. This is similar to the irregularities known from the frequency response in any reverberant room, which is due to interference between a large number of modes. In relation to speech or music these irregularities will not be detected by the human ear.

5.2 A Design Guide

The design of a reflector array can proceed in several steps. First the relative density is decided from the desired attenuation. Typical values are μ = 0.25 - 0.5. Secondly the size of panels is decided through consideration of the upper limiting frequency (19). Many and relatively small panels should be preferred in a reflector array in order to get a high upper limiting frequency. In the remodelling project described in [2], f_g was changed from about 600 Hz to about 1200 Hz, and this was considered an improvement. The mass and material of the panels is considered with reference to eq. (4).

The shape of the panels in a reflector array is of minor importance; they may be triangular, circular or many other shapes. The 'doughnuts' in the concert hall of the Sydney Opera House is an example of a shape very different from the rectangular panel; the latter being better understood, however. Finally, it may be considered to apply a convex curvature to the panels, which may contribute to avoiding missing reflections from the gaps between panels. So, this may be equivalent to an upward shift of the upper limiting frequency (19). In other words, a convex curvature can compensate for the use of relatively large panels in a reflector array.

6. REFERENCES

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