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LOW FREQUENCY HIGH POWER UNDERWATER SOUND SOURCE

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1. INTRODUCTION

The task GRAD was presented with was to devise a method of generating high energy underwater sound waves at a frequency of 10Hz. This source had to be cylindrical in form, compact, transmit kilowatts of power and capable of operating independent of depth. After considering a series of possible options (References 1, 2 and 3) we selected a mechanical system based on standard hydraulic components capable of transmitting 10kW. This system, its basic mechanical arrangement and the underlying theory are presented in this paper.

2. THEORY

The basis for any new transducer design must lie with the wave equations which govern the generation and propagation of sound. With this in mind we present the basic wave equations below (see Reference 4 for more details).

From the normal theory of wave equations, three different forms can be derived along with the associated specific acoustic impedance. These forms are summarised below.

The standard equation for a plane wave is:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}$$

From which the specific acoustic impedance z can be calculated:

$$z = \frac{p}{u} = \rho_0 c$$

The cylindrical form of the wave equation is:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial^2 p}{\partial r^2}\right)$$

From which the specific acoustic impedance z can be calculated:

$$z=\frac{p}{u}=\rho_0 c\,\cos\phi$$

Where:

$$\cos\phi=\frac{2kr}{(1+(2kr)^2)^{0.5}}$$

The spherical form of the wave equation is:

$$\frac{\partial^2 p}{\partial t^2}=c^2\left(\frac{\partial^2 p}{\partial r^2}+\frac{2}{r}\frac{\partial p}{\partial r}\right)$$

Which has its own form of specific acoustic impedance:

$$z=\frac{p}{u}=\rho_0 c\,\cos\theta$$

Where:

$$\cos\theta=\frac{kr}{(1+k^2r^2)^{0.5}}$$

A simple analysis shows that the wave length of sound in water at 10Hz is approximately:

$$\lambda=\frac{v}{f}=\frac{1500}{10}=150m$$

Thus any transducer operating at 10Hz is small compared with the wave length, therefore one might expect a spherical wave equation to be the most appropriate. However, the form of the transducer is cylindrical so the particle displacements at the water/transducer boundary will have a cylindrical motion. To examine the effects on particle displacements at the surface of the transducer and thus the motion of the transducer surface a series of calculations were performed. These calculations include the planar form of the wave equation as a comparison. The calculations assume a radiating area of 1.674 m^2, a cylinder of radius 0.2675m and 10kW of power at 10Hz.

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Plane wave calculation based on a planar radiating surface:

$$I=\frac{10^4}{1.674} \quad = \quad 5.973\ x\ 10^3 \quad Wm^{-2}$$

$$p=(2I\rho_0 c)^{0.5} \qquad \rho_0 c\approx 1.5\ x\ 10^6$$

$$p=134 \qquad kPa^{-2}$$

$$p=\rho_0 c u$$

$$u=\frac{p}{\rho_0 c} \quad = \quad 8.925\ msec^{-1}$$

$$\varepsilon=\frac{u}{\omega} \qquad \omega= \quad 2\pi f$$

$$\varepsilon=1.42\ x\ 10^{-3} m$$

Spherical wave calculation based on a sphere of 0.2675m radii:

$$I=\frac{10^4}{4\pi r sup4} \quad = 5.973\ x\ 10^3 \quad Wm^{-2}$$

$$I=\frac{p^2}{2\rho_0 c}$$

$$p=(2I\rho_0 c)^{0.5}$$

$$p=134 \qquad kPa^{-2}$$

$$p=\rho_0 c u\ \cos\theta$$

$$\cos\theta=\frac{kr}{(1+k^2r^2)^{0.5}} \qquad k=2\frac{\pi}{\lambda}$$

$$\cos\theta=11.20\ x\ 10^{-3}$$

$$u=\frac{p}{\rho_0 c\cos\theta} \quad =7.968\ msec^{-1}$$

$$\varepsilon=\frac{u}{\omega} \quad =126\ x\ 10^{-3} m$$

In a similar way the particle displacement can be calculated for a cylindrical wave with a radiating surface of 1.674 m^2:

$$p=134 \quad kPa^{-2}$$

$$\cos\phi=\frac{2kr}{(1+(2kr)^2)^{0.5}}$$

$$\cos\phi=22.4\ x\ 10^{-3}$$

$$u=1.338\ x\ \frac{10^5}{1.5\ 10^6\ x\ 22.40\ 10^{-3}} \quad =3.98 \quad msec^{-1}$$

$$\varepsilon=63.33\ x\ 10^{-3}\ m$$

The three different forms of the wave equation result in three different maximum particle excursions which must be achieved at the surface of the transducer:

Planar	1.42 mm
Cylindrical	63.33 mm
Spherical	126.00 mm

These particle displacements must be generated by the physical movement of the transducer/water interface. However it is not clear to the authors exactly which form of the wave equation is most appropriate. To resolve this problem and to investigate some of the practical aspects of generating high intensity underwater sound, a proof of concept 10kW sound source has been designed.

When high sound levels are generated cavitation is likely to be a problem. Cavitation occurs when the pressure in the wave drops below the vapour pressure of the liquid, i.e. when the low pressure part of the acoustic wave drops below the ambient pressure (hydrostatic plus atmospheric) minus the vapour pressure for the liquid at that temperature.

For example, if the device was to operate at 30m, then the total pressure is approximately 4 Bar, or 4×10^5 Pa. The vapour pressure of water at say 10 degrees centigrade is 1.2277 kPa, so the pressure at which cavitation will occur is:

$$400 - 1.23 - 398.77 \qquad kPa$$

Referring to the peak pressures calculated above it is unlikely that cavitation will occur at 30m.

Non linear effects could further complicate the design. Most simple acoustics assume that water is a linear medium but this is not true. At high source levels the positive half of the pressure wave travels faster than the negative half. This results in a distorted wave form. The distortion increases with distance from the transducer until a shock wave is produced, i.e. the rate of change of pressure in the central part of the wave tends to infinity, a physically unrealisable effect. The result is a saw-tooth wave form and a limit to the size of the far field acoustic wave, effectively making the far field acoustic wave independent of source level at very high powers and thus limiting the maximum source level which can be generate.

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These non linear effects place an upper limit on the source level of practical transducers. The easy way to quantify this is to use the concept of extra attenuation, that is the additional attenuation caused by non linear effects between the transducer and the far field. The extra attenuation can be estimated from the approximation:

$$Extra\ atten' = 20\ \log(\frac{\tau}{4})\ dB$$

$$where{:}\ \tau = \beta\epsilon\frac{k}{\alpha}$$

$$\beta = 1+\frac{B}{2A}$$

$$A = \rho_0(\partial\frac{P}{\partial\rho})_0$$

$$B = \rho_0^2(\partial\frac{P}{d\rho sup2})_0$$

$$\beta = 3.5\ for\ water$$

$$\epsilon = \frac{u_0}{c_0}$$

$$k = \frac{\omega}{c} \qquad = 4.188\ 10^{-2}$$

$$\alpha = 8.7\ 10^{-6}\ m^{-1}\quad at\quad 10\ Hz$$

The extra attenuation is too small to be calculated with precision, torr is approximately 0.12 and the extra attenuation zero.

Having considered the wave equations, cavitation and non linear effects we were confident that a low frequency hydraulically powered sound source is potentially a practical device. The next phase is to design a proof of concept prototype to test the initial calculations.

3. PROOF OF CONCEPT PROTOTYPE

3.1 Design Aims

The aims of a Proof of Concept Prototype are: a) To investigate the accuracy of the theoretical predictions presented above; b) To assess the practicality of generating low frequency sound production underwater using hydraulic coupling; c) To assess whether larger and smaller systems are practical;

3.2 Design Strategy

The displacement of the secondary piston generates the volume change required to extend the membrane and produce the particle displacement. The hydraulic design must ensure that the pressure and volume ratios are matched, i.e. that the primary power source can energises the secondary piston to produce the necessary pressure volume change and peak flow rate. The pressure generated in the secondary cylinder must be sufficient to overcome the natural resistance of the water. These pressures and flow rates are then transformed by the "ratio of the piston's areas" to an equivalent pressure and flow rate on the primary side.

Since the system oscillates about a mean and the required flow rate varies from zero to a peak value throughout a cycle only the RMS power needs to be generated, the peak power requirements being supplied by a hydraulic-accumulator. The accumulator is recharged during the low power portion of the cycle.

The sound producing cycle can be accurately controlled by incorporating a linear voltage displacement transducer within the hydraulic ram, the output from the ram being used as a feedback mechanism for a proportional, integral and differential control loop. A varying set point, in the form of a sine wave (or any arbitrary wave form within the slew rate of the system) is applied to the control loop. Experience with similar systems has demonstrated that the piston can be controlled to within 0.1% of the required wave form with a constant phase lag of approximately 5^0 throughout the cycle.

The hydraulic system can readily be designed to operate independent of depth, by simply compensating the low pressure side of the system to be a little above ambient pressure. This ensures that hydraulic pressure is generated relative to ambient pressure.

The volume change caused by the movement of the secondary cylinder must be accommodated within the transducer. This is perhaps the most difficult aspect of the mechanical design. The only convenient method is to use some form of gas reservoir, the mean pressure of the gas within this reservoir being held close to the ambient pressure and allowed to adiabatically compress and decompress as the piston cycles.

3.3 Description of Sound Producing Mechanism

The sound source is, in effect, a pressure transformer which converts high pressure low flow rate hydraulics into low pressure high volume hydraulics by using a simple differential piston (Figure 1). High pressure hydraulics are readily available in commercial hydraulic systems and the proof of concept prototype has been designed around commercially available parts.

The sound waves will be generated by the oscillation of the surface of a cylindrical membrane. The movement of these surfaces is generated by pressure differences between the inside of the cylinder and the surrounding sea water. These pressure differences are generated in the following way.

The primary piston is moved back and forward by a controlled flow of fluid from the hydraulic pump via a high precision servo valve. The pressure of this fluid (P1) at any given time exerts a force on the rod of the primary cylinder, which is coupled to the secondary cylinder. The secondary cylinder moves and creates a pressure (P2) in the fluid. Because the two pistons are linked by a rigid rod, the force experienced by the two pistons must be identical. Thus, if friction is ignored, the following equation is true:

$$P2=\frac{Area\ of\ piston\ 1\ x\ P2}{Area\ of\ piston\ 2}$$

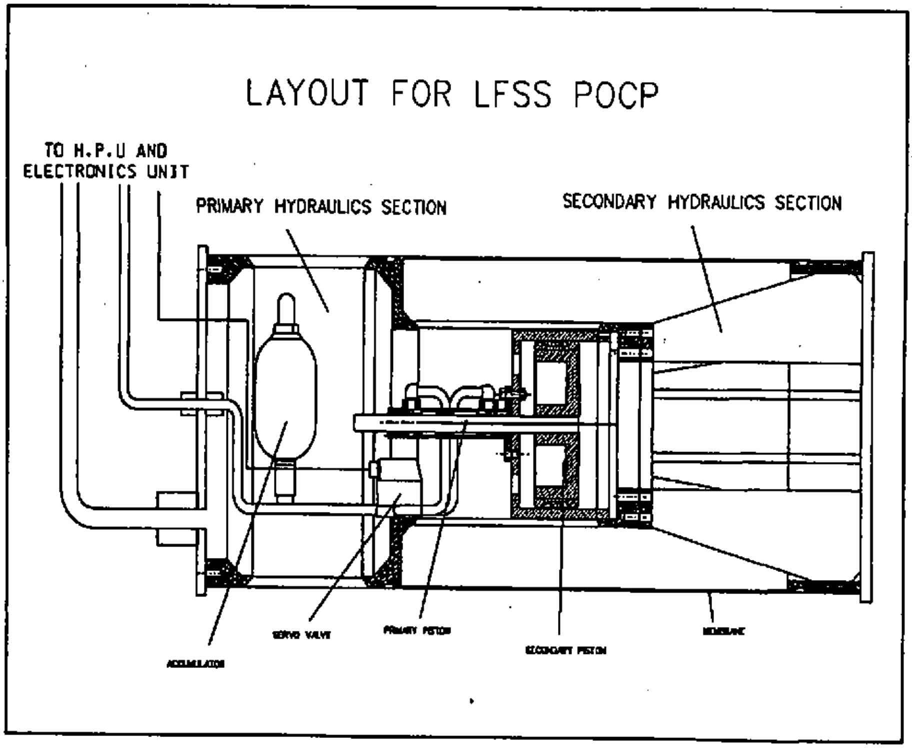

Figure 1 - Lay Out for Low Frequency Sound Source
Proof of Concept Prototype

The pressure P2 also acts directly on the flexible membrane, therefore P2 cannot be greater than the resistive force of the surrounding water, plus an amount necessary to stretch the membrane, plus an amount which is converted into kinetic energy in moving the fluid.

The force on the secondary piston creates a pressure change in the secondary fluid, also the movement of the piston causes a displacement of fluid into the annular cylinder of which the outer wall is the membrane. The final effect is to cause a displacement of the membrane in a controlled fashion, which creates sound waves in the surrounding water.

This design is for a 220 dB//1μPa sound source. This will produce a sound source of approximately 8kW output, which is large enough to determine performance of a hydraulically powered sound source without being unmanageable or dangerous during testing.

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The expected overall efficiency is about 40%. This is due to losses in the hydraulic power unit, the servo valve and the sound producing mechanism.

4. CONCLUSION

The low frequency underwater sound source described in this paper is in the authors' opinion a simple and cost effective method of generating high amplitude sound waves at low frequencies. The design is based on the pressure transforming properties of two linked cylinders with different piston areas. This mechanical arrangement allows high pressure low flow rate commercial hydraulic systems to be used to drive the low pressure high flow rate hydraulics which are well matched to the generation of acoustic waves. The system has been designed to avoid both cavitation and non linear effects. It has also been designed to operate independent of depth by compensating the primary power generation to ambient pressure and by matching the gas reservoir to the displaced volume and allowing the pressure to oscillate about the ambient pressure. A proof of concept prototype has been designed to test the theoretical and practical aspects of working at high power and low frequency.

5. SYMBOLS

ω-*Angular frequency*	*radian secsup*-1	*c-Velocity of sound*	*m* sec^{-1}
f-Frequency	sec^{-1}	*t-Time*	sec
u-Particle velocity	*m* sec^{-1}	*I-Intensity*	*watts* m^{-1}
z-Specific acoustic impedance	*Rayls*	λ-*Wavelength*	*m*
ρ-*Density*	*kg* m^{-3}	*e-Particle displacement*	*m*

6. REFERENCES

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