PARABOLIC-COHERENCE FUNCTION PROPAGATION MODELS IN OCEAN ACOUSTICS

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ABSTRACT

Primary quantities for estimating the performance of acoustic aperture systems are the total power in the received signal and the directional resolution of this power. This paper summarizes some work done on developing propagation models to predict these quantities for ocean acoustic experiments. The models can incorporate diffraction and refraction effects as well as a stochastic volume scatter. The incorporation of a randomly rough boundary interaction is planned.

INTRODUCTION

The propagation models to be discussed are formulated in terms of an acoustical field measure termed a directional spectral density. Denoted by $\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})$ ($\{\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})\}$ when discussing stochastic fields), this measure provides a resolution of the (averaged) power flux through an elemental aperture positioned at $\underset{\sim}{x}$ into a continuum of directions represented by $\underset{\sim}{\theta}$.

The relationships between this description of the acoustic field with others that are more familiar are of interest. A discussion is available [1]. We note here that a classical geometric description is obtained as a limit in which the continuous spectrum, measured by $\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})$, is replaced by a discrete spectrum, measured by the acoustic intensity $I(\underset{\sim}{x})$ and a single ray path direction (a finite number of ray paths if one allows for a multipath structure). We note further that $\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})$ can also be related to a plane wave decomposition of the acoustic field itself. In drawing a precise mathematical equivalence between the $\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})$ measure and the plane wave decomposition, it is necessary to introduce an averaging. In discussing deterministic fields the averaging is over either a region of $\underset{\sim}{x}$ space (a finite aperture) or a region of $\underset{\sim}{\theta}$ space (a cone of directions) with the extent of the averaging windows being governed by an uncertainty principle. In discussing stochastic fields that can be termed locally homogeneous, the averaging can be interpreted as an ensemble averaging.

Referring to the work of Clarke [2] is helpful in placing our models in context. Clarke formulates his ideas in terms of a plane wave decomposition of the acoustic field. Thus, as the above paragraph implies, there is a strong similarity between some of his ideas and some of ours. Two important distinctions, however, can be drawn. One is that $\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})$ cannot be precisely equated to the directional resolution of the acoustic field itself but of the power flux in the acoustic field. Secondly, in discussing stochastic fields, a statistical averaging is implied in our measure $\{\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})\}$. Thus $\{\Gamma(\underset{\sim}{x},\underset{\sim}{\theta})\}$ is a deterministic field variable; Clarke's plane wave decomposition, when applied to stochastic acoustic fields, is a stochastic field variable.

THE PROPAGATION MODEL

A propagation model for predicting the evolution of $\{\tilde{\Gamma}(x,\theta)\}$ in a deterministic but possibly inhomogeneous medium is presented in References [3, 4]. The model is described by the statement that $\{\tilde{\Gamma}(x,\theta)\}$ is a constant along ray paths. Thus, the value of $\{\tilde{\Gamma}\}$ for a specific x and specific θ is given by that associated with the ray path that passes through the point of x in the direction θ. To apply the model we assume that $\{\tilde{\Gamma}\}$ values are known for all points on a given "source" plane and for all directions. Ray paths are then launched from each point on the source plane in all directions; and values of $\{\tilde{\Gamma}\}$ are assigned to each ray path so launched, according to this initial data. For points down range, values of $\{\tilde{\Gamma}(x,\theta)\}$ are determined as indicated above.

Several comments concerning the model bear repeating. First, it is a mathematically derived result with its basis in a parabolic wave theory. Examples presented in References [3, 4] demonstrate the incorporation of both diffraction effects, as measured by beam spreading, and interference effects. Secondly, the model is equally applied to deterministic and to stochastic experiments; the only difference being one of interpretation. Since we have so far excluded a stochastic volume scatter, the source of any randomness must arise from the nature of the acoustic source or possibly the prior interaction with a rough boundary surface. Finally, the model is well suited to numerical implementation [5].

RANDOM VOLUME SCATTER

Random volume scatter is incorporated in the propagation model by the addition of scattering terms that describe the rate at which acoustic energy, measured by $\{\tilde{\Gamma}(x,\theta)\}$ values, is scattered from each ray path and is then redistributed among the remaining ray paths. We write the following equation [6]

$$\frac{d\{\tilde{\Gamma}(x,\theta)\}}{d\rho_\theta} = -\frac{\bar{k}^4}{16\pi^2}\left[\bar{\sigma}(\theta)\{\tilde{\Gamma}(x,\theta)\} - \int \tilde{\sigma}\left[\bar{k}(\theta-\theta')\right]\{\tilde{\Gamma}(x,\theta')\}\,d\theta'\right] \tag{1}$$

The derivative with respect to ρ_θ is to be interpreted as a substantive derivative taken with respect to distance over a curved (in general) ray path. The function $\tilde{\sigma}[\bar{k}(\theta-\theta')]$ is the Fourier transform of the correlation function defined on the randomly varying refractive index, and $\bar{\sigma}(\theta)$ is the integral of $\tilde{\sigma}[\bar{k}(\theta-\theta')]$ taken over θ'. The integral in Eq. (1) is taken over all directions, θ'. The first term on the right-hand side gives the rate at which energy is scattered from the θ ray path; the second term gives the rate at which energy is scattered to the θ ray path, i.e., from the remaining ray paths. We note that Eq. (1) predicts an energy conservant scattering.

One can properly interpret the redistribution of the power scattered from one ray path to the remaining ray paths as a spatial filtering, and the filter function as a locally scattered beam pattern. Thus,

$\tilde{\sigma}[\tilde{L}(\underset{\sim}{\theta} - \underset{\sim}{\theta}')]$ gives the distribution, in $\underset{\sim}{\theta}$, of scattered acoustic intensity for a plane wave incident in the direction of $\underset{\sim}{\theta}'$. Since it can be obtained from a single scatter calculation, this interpretation reduces the determination of $\tilde{\sigma}[\tilde{L}(\underset{\sim}{\theta} - \underset{\sim}{\theta}')]$ to a classical problem (cf. [7]). The result that $\tilde{\sigma}$ is related to the spatial correlation of the randomly varying refractive index field as a Fourier transform pair is well known. The simplicity of the derivation of Eq. (1) as outlined is interesting in that it has been largely overlooked. The reason for this apparently is that most calculations of the two point statistics of the stochastic field are formulated in terms of the signal coherence function, which is a Fourier transform of $\{\tilde{\Gamma}(\underset{\sim}{x}, \underset{\sim}{\theta})\}$

A scattering model that has been derived by a large number of researchers (the earliest apparently by Beran [8] and Tatarskii [9]) can be shown to be special cases of Eq. (1). For an isotropic scattering mechanism one need only assume that the differences between $\underset{\sim}{\theta}, \underset{\sim}{\theta}'$ directions are small enough to replace the sine of the angle by the angle itself and the models are the same. (In the earliest versions referenced the models did not allow for an inhomogeneous background medium, but the required extension was easily accomplished [10].) Application of the Beran-Tatarskii model to an anisotropic scattering medium (the ocean) requires an additional approximation, one that restricts the degree of anisotropy. The calculations will be given in the verbal report.

With the quasi-isotropic Beran-Tatarskii model suitable for one class of ocean acoustic experiments, a second class is more properly described by a highly anisotropic model presented by the present author and Beran [11, 12]. This second model is also contained in Eq. (1), and is obtained by making a set of additional approximations that are different from those that lead to the Beran-Tatarskii model. We note that while the quasi-isotropic and highly-anisotropic approximations lead to models that can be applied analytically in special cases, they do not simplify the numerical computations required to analyze a realistic experiment. In our present work we are writing a general purpose program based on Eq. (1) with no assumption as to the degree of anisotropy of the scattering mechanism.

RANDOM BOUNDARY INTERACTION

We can discuss [13] randomly rough boundary interaction models that are compatible with the propagation and volume scatter models. Compatibility refers to formulating the boundary interaction model in terms of $\{\tilde{\Gamma}(\underset{\sim}{x}, \underset{\sim}{\theta})\}$. A phenomenological model in the form of a linear filter model might be written

$$\{\tilde{\Gamma}_R(\underset{\sim}{x}, \underset{\sim}{\theta})\} = \int \mathcal{A}(\underset{\sim}{\theta}, \underset{\sim}{\theta}'; \underset{\sim}{x}) \{\tilde{\Gamma}_I(\underset{\sim}{x}, \underset{\sim}{\theta}')\} \, d\underset{\sim}{\theta}' \qquad (2)$$

where $\mathcal{A}(\underset{\sim}{\theta}, \underset{\sim}{\theta}'; \underset{\sim}{x})$ is to be determined from experimental data. It gives the distribution, in $\underset{\sim}{\theta}$, of the reflected acoustic intensity for a plane wave incident in the direction of $\underset{\sim}{\theta}'$. To "derive" Eq. (2) requires two assumptions, a statistical independence assumption between the variations of the rough boundary and those of the incoming signal, and an assumption that the boundary interaction is local.

To obtain analytically an expression for $\mathcal{A}(\theta, \theta'; x)$, which we might apply to a wide range of experiments, is very difficult. For slightly rough surfaces we can make use of perturbation theory to derive one expression; for high-frequency experiments we can make use of Kirchhoff scattering theory to derive another. While the recent literature contains several references reporting on theoretical studies to relax either of these restrictions, the prospects for incorporating this work in prediction models for realistic sea tests would not appear to be too bright.

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