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A DESKTOP COMPUTER PROGRAM FOR A FLEXTENSIONAL TRANSDUCER

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I. INTRODUCTION

The design and analysis of flextensional transducers can be cumbersome because of the number of complex design equations involved. Although solutions can be obtained through finite element techniques, such an approach can be quite involved and cumbersome itself particularly in the early stages of a design. Our approach has been to use the equivalent circuit developed by Brigham [1] along with a number of simplifying assumptions and additional baffle, array and mode analysis to obtain a compact user oriented computer program. This program is most useful in the initial design of the element or an array of elements.

The program is written in "Rocky Mountain" HP Basic 4.0 for Hewlett-Packard series 200 and 300 desktop technical computers. The program produces numerous graphic outputs for immediate evaluation as well as ABCD parameters for optional use in other array programs in addition to the self contained array interaction program. A discussion of the model along with sample numerical results will be given. The details of the model and program are given in a series of earlier reports [2,3,4,5 and 6].

II. THE ELEMENT MODEL

The current Class IV flextensional transducer is in the form of an elliptical cylinder typically driven by a stack of piezoelectric elements along its major axis as illustrated in Fig. 1. A leverage action is utilized so that small displacements at the ends of the piezoelectric stack produce large displacements over the central surface of the shell with an approximate magnification given by the ratio of the major to minor radii. Although the end motion is out of phase with the central surface, the area and displacements are small so that little interference is caused from the end motion. The high leverage and small end motion is achieved by designs which place the nodes near the end of the transducer.

The major motion of the shell is flexural; and thus, well suited for low frequency operation. In this mode the transducer is small compared to the wavelength of sound and the resulting sound pattern is nearly omnidirectional. Accordingly, an equivalent sphere radiation loading has been used as an approximation for a single element. Also included in the program is a cylindrical loading case for long line arrays of transducers as well as an array interaction loading for planar arrays. Both cases of controlled and uncontrolled velocities are treated.

A sketch of the transducer is shown in Fig. 1. Here a is the semimajor radius, b is the semiminor radius, h is the shell wall thickness, e is the end wall thickness and Lo is the length of the shell. A block diagram of the equivalent circuit for the ceramic stack, the shell and the electrical and radiation load terminals is shown in Fig. 2. In this model we have included the first two even flexural modes (the quadrupole and octopole) and the membrane mode for the shell. These modes are represented by the subscripts 2, 4 and 0 respectively. Here Z stands for an impedance while T represents a transformer. The Z with subscript 3 is for the end pieces while the Z with subscript 1 is the impedance of the ceramic stack. The T1 is the piezoelectric electro-mechanical transformer., C is the blocked capacitance of the stack and R is the electrical dissipation resistor. The mechanical loss has been associated with the quadrupole mode which is the predominate means for acoustic radiation. The higher order modes affect the performance of the quadrupole mode resonance and should be included for acceptable predictions.

III THE BAFFLE MODEL

Since the flextensional transducer is nearly omnidirectional in the vicinity of the quadrupole resonance, baffles are commonly used along with arrays to obtain unidirectional radiation. An approximate model of the baffle based on a virtual image source (see Fig. 3) is used to represent compressional wave reflection and its effect on the self impedance of the element and the far field radiation. The velocity of the image source is determined by the impedance Z of the baffle which is based on a plane wave approximation, compressional model as illustrated in Fig. 4.

The impedance at the baffle is also given by the ratio of the sum of the source and image pressure to the sum of the normal velocities at the baffle. With Zo the normal impedance at the baffle plane for the source alone we can then write the factor

$$G = Vi / Vs = (Z - Zo) / (Z + Zo) \tag{1}$$

where Vi is the velocity of the image and Vs is the velocity of the source. Thus, given the baffle impedance Z from the transmission line model of Fig. 4 we may determine the velocity of the image source. As an approximation, the impedance Zo may be taken as the characteristic impedance of the medium (water).

Under this baffle condition the normalized far field pressure then becomes

$$Pn \simeq 1 + (Vi/Vs) \exp(-ikD \cos \theta) \tag{2}$$

where k is the wavenumber and θ is the angle relative to the line through the source and the image. In the nearfield the total impedance on the source is also

$$Zt = Zs + Zm (Vi/Vs) \tag{3}$$

where Zs is the self radiation impedance of the source and Zm is the mutual radiation impedance between the image and source. In the program the self impedance has been approximated by a small equivalent sphere and the mutual impedance has been approximated by the expression for two small free spheres.

The presence of a baffle can cause an unequal acoustic pressure on the two major radiating surfaces of the transducer resulting in the excitation of the odd modes of vibration. It has been found that the lowest odd mode can occur in the vicinity of the first major even mode and cause considerable reduced performance. Accordingly, we have included this effect in the computer program.

IV. MODEL FOR THE ODD MODE

Because of the symmetry of the design, only the even modes of a free field Class IV flextensional transducer are excited by the piezoelectric ceramic drive stack illustrated in Fig. 1. However, in the presence of a reflecting surface or another transducer the reflected pressure can produce a pressure differential across the radiating surface because of the finite size of the transducer. This net force not only causes the shell to vibrate in odd modes but also causes the entire transducer to move in what may be thought of as a rigid body type mode.

Since the transducer is normally operated in the vicinity of its gravest mode (quadrupole mode), the odd modes that exhibit resonance in the same region are of primary interest. In the first five modes of vibration the even modes are 0, 2 and 4 and the odd are 1 and 3. The frequency of the # 3 shell mode is approximately 2.8 times the frequency of the quadrupole mode and probably out of the range of immediate interest. However, as we shall see, the dipole mode resonance can occur in the vicinity of the quadrupole mode.

In either case the odd modes can not be electrically excited by the ceramic stack unless driven in a way that causes the stack to bend which is unlikely except as a result of imperfection in the material or construction. It is, therefore, believed that this mode is excited as a result of the pressure field reflected by a baffle. Before discussing the excitation of this mode we first consider a simple model for the transducer operating in the #1 or dipole mode.

In this dipole mode we expect the transducer shell to move as an oscillating body with the ceramic stack acting as a beam bending in opposition to this body motion. A finite element model [7] of the British Aerospace transducer [8] in its quiescent state is shown in Fig. 5 while in Fig. 6 we show the transducer undergoing motion in its dipole mode. As seen, the shell moves in a direction in opposition to the beam bending motion. Since the ceramic stack is normally wired for extensional motion only, this mode cannot be excited directly through electrical means unless there is an externally unbalance force induced.

In Fig. 7 we illustrate a simplified model for this mode. An additional mass Mo is also included without further complication. We use Rayleigh's method, suggested by Den Hartog [9] for a similar problem, with the trial function for the displacement

$$Y = Y_o \sin(\pi X/L) - a \qquad (4)$$

Then by requiring the total momentum to vanish we get on integrating over the length of the bar the kinetic and potential energy yielding the mass M and stiffness K. With the stiffness and mass we can then write the Thevenin mechanical impedance of the transducer operating in its dipole mode as

$$Z_d = j\omega M + K/j\omega \, . \qquad (5)$$

The resulting dipole resonance agrees with the results from a finite element analysis with an error less than -8 %.

V. ODD MODE EXCITATION

In the program FLEXT4A the odd (dipole) mode is excited through the pressure generated by the quadrupole mode and reflected by the baffle. In the model the baffle is replaced by image sources as illustrated in Fig. 8. Here for sake of clarity we have numbered each as a separate source and separated the dipole component from the monopole which actually occupy the same location in space. The monopole source does not couple to its own dipole source and vice versa since they are orthogonal. Physically, the monopole mode does not produce a pressure differential over its own surface and the dipole mode does not provide the equal compression over its surface. Thus, the mutual impedances $Z_{13} = Z_{31} = Z_{24} = Z_{42} = 0$.

We can develop the motion of the dipole element #3 from the mutually coupled set of force, velocity equations. With V the velocity of the sources the forces on 1 and 3 are then

$$F_1 = Z_{11} V_1 + Z_{12} V_2 + Z_{14} V_4 \quad (6)$$

$$F_3 = Z_{32} V_2 + Z_{33} V_3 + Z_{34} V_4 \quad (7)$$

At this point we now replace the image sources by the baffle and use the baffle reflection factor $G = (Z - Zo)/(Z + Zo)$ so that $V_2 = G V_1$ and $V_4 = G V_3$ with Z the impedance of the baffle as given in Fig. 4 and Zo the impedance of the medium. Use of the above and reciprocity leads to the simplification

$$F_1 = (Z_{11} + G Z_{12}) V_1 + G Z_{14} V_3 \quad (8)$$

$$F_3 = G Z_{14} V_1 + (Z_{33} + G Z_{34}) V_3 \quad (9)$$

The last equation may be used to evaluate the motion of the dipole relative to the motion of the monopole. If the dipole were driven by a force Fd we would have $F_3 = Fd - Zd V_3$ where Zd is the mechanical impedance of the dipole. However, because of the even symmetry of the electro-mechanical drive, Fd = 0. Thus we get

$$V_3 = -G Z_{14} V_1 / (Zd + Z_{33} + G Z_{34}) \quad (10)$$

and this gives the motion of the dipole mode in terms of the motion of the quadrupole mode which may be used to obtain the total radiation impedance and the far field pressure.

VI. MUTUAL IMPEDANCE FUNCTIONS

Inspection of Eq. (10) reveals that the dipole velocity is a result of the mutual impedance between a monopole and dipole source (Z_{14}) and between two dipole sources (Z_{34}) as well as the mechanical impedance (Zd) and the self radiation impedance of a dipole (Z_{33}).

Evaluation requires a knowledge of the mutual and self impedances. As we have mentioned earlier the quadrupole mode is represented as a monopole radiator because of its small size. For the same reason we believe the radiation from the dipole model may be closely approximated by an oscillating sphere. The expression for the mutual impedance between two small monopoles is also well known and may be written as

$$Z_{12} = Ro\ h_0(kr), \qquad \text{where}\quad Ro = \rho c\ S\ x^2 \tag{11}$$

and where $h_0(kr)$ is the zero order spherical hankel function, S is the area and $x = ka$ where a is the radius of an equivalent sphere.

The mutual interaction between two dipoles or a dipole and a monopole are less well known. It can be shown that the dipole-dipole interaction may be written as

$$Z_{34} = -\ 3\ Rd\ h_1'(kr), \qquad \text{where}\ Rd = \rho c\ S\ x^4 /12 \tag{12}$$

and where the prime (') means derivative with respect to the argument. The real part of the above expression may be reduced to the same expression given by Sherman [10].

It may also be shown that the monopole-dipole interaction may be expressed as

$$Z_{14} = \sqrt{(3\ Rd\ Ro)}\ h_1(kr) \tag{13}$$

A similar expression (but with a different coefficient) has been given by Thompson and Reese [11].

VII. ARRAY INTERACTION ANALYSIS

An important set of parameters for a transducer array analysis are the ABCD parameters which for each mode may be written

$$E = A\ F + B\ V \tag{14}$$

$$I = C\ F + D\ V \tag{15}$$

where E and I are the transducer voltage and current and F and V are the force and velocities for each mode respectively.

The ABCD transducer element modal parameters are calculated in the program FLEXT4A for further array analysis by a separate program if desired. In addition to this, the program FLEXT4A also performs an array analysis for the case of both a controlled velocity condition and an uncontrolled velocity condition under array interaction. In the program the array is assumed planar and there is an option to include a baffle.

In the controlled velocity case the solution is quite simple since all the velocities have the same value. In this case the total average impedance for N elements is simply

$$Z_{av} = \Sigma\Sigma Z_{ij} / N \quad ; \qquad i,j = 1,\ N \tag{16}$$

For the case of variable velocity we need to calculate the force on, say, the j'th transducer and then replace the force by the Thevenin element force F' and Thevenin impedance Z' yielding

$$F'_j = \Sigma Z_{ij}\ V_i + Z'_j\ V_j\ , \qquad i = 1,\ N \tag{17}$$

Since there are N equations the N velocities may be solved. We have used this in the program to determine the array loading for the quadrupole mode which is the main source of acoustic output.

X. RESULTS

We begin first with a comparison in Fig. 9 of the theoretical results from the program FLEXT4A with the measured result for the British Aerospace flextensional transducer [8] which we call BA1. The calculated results for this transducer as a single free element and as a close packed line array of six elements (BA6) are given by the solid lines. As seen, the results compare reasonably well with the measured data given by the X's. The agreement with the cylindrical array case deviates below 1.5 kHz where the array is less than a one-half wavelength long.

In Fig. 10 we illustrate the effect of the two higher order even mode, the octopole and membrane modes, on the fundamental quadrupole mode. As seen, these modes cause a reduction in the resonant frequency at the expense of a reduced output level at resonance. In the program the user may activate the first three even and the odd (dipole) modes by use of Y for Yes or N for No.

The case for a single transducers six inches in front of a nearly perfect rigid baffle is illustrated in Fig. 11 for the conditions with (lower figure) and without (upper figure) the dipole mode activated. There is no effect in the plane of the baffle (90 degrees) since this is in the null direction of the dipole mode for the case of the large radiating surface facing toward the baffle as modeled in Fig. 8. The null at 2.4 kHz for the broadside response is due to the quarter wavelength position of the element in front of the baffle. As seen, the effect due to the dipole resonance unfortunately occurs in the vicinity of the main resonance which is controlled mostly by the quadrupole mode.

In Fig. 12 we show the interactive results for the simple case of two flextensional elements. Here the two elements are spaced one quarter of a wavelength apart and excited with equal voltage magnitude but with an electrical phase difference of 90 degrees. Under this condition we would expect a cardioid type pattern with a pressure increase of 6 dB in one direction due to pressure addition and a perfect null in the opposite direction due to pressure cancellation. However, we see that there is no perfect null. This is a result of the interaction between the two elements and resulting different total radiation loading on each of the elements as indicated by the output data of the figure.

X. SUMMARY AND CONCLUSIONS

We have presented a discussion of the basis and some of the results using the program FLEXT4A for the Class IV flextensional transducer. The model contains the first three even modes and one odd mode. In modeling the odd mode we presumed that it resonates as a dipole mode shell oscillating in opposition to the beam bending of the piezoelectric stack. We also modeled this mode of the transducer on the presumption that this motion is induced by baffle reflections. The program allows the design and evaluation of a single element or a planar array of elements with a baffle condition as an option.

The sample runs showed the effect of the higher order even modes and the strong effect the dipole mode can have on the broadside performance in the direction normal to the baffle. The computations also showed the effect of interaction on the null depth for two phased elements. The comparison with measured results on a British Aerospace transducer showed good agreement.

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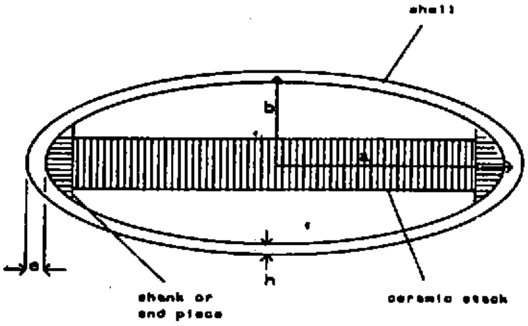
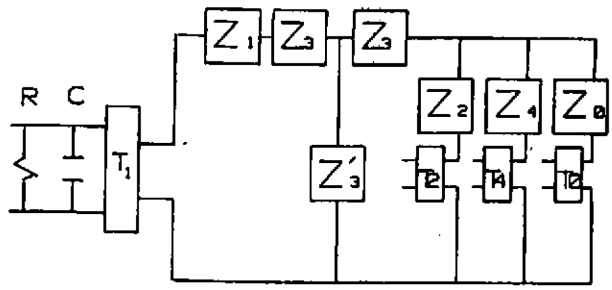


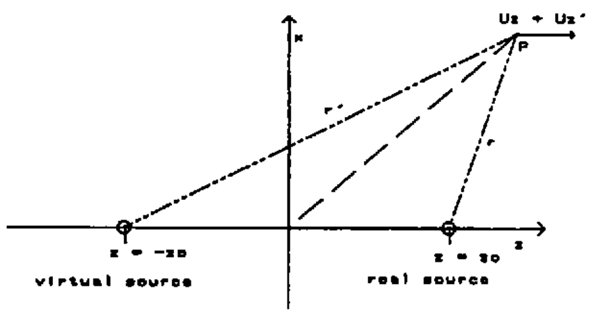
Class IV Flextensional Transducer of length Lo.

Fig. 1

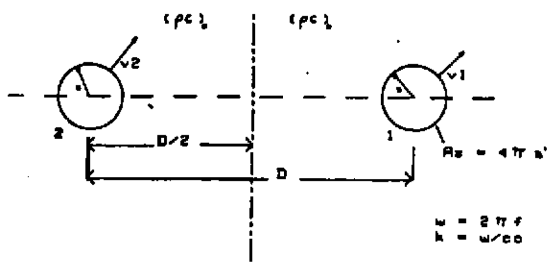


Block diagram of the flextensional transducer circuit.

Fig. 2

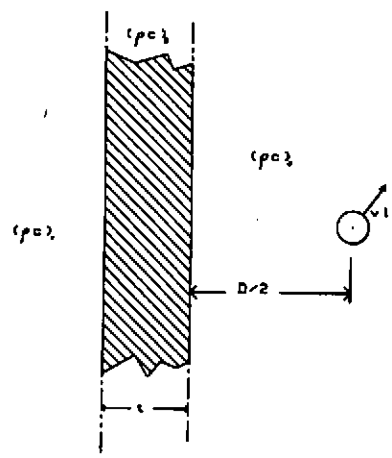


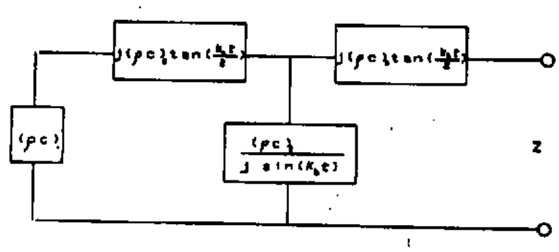
Representation for determining the impedance at P due to the action of the real and virtual sources.



Configuration and parameters for describing the interaction between two small spherical elements.

Fig. 3





Baffle configuration and corresponding plane wave compressional circuit model.

Fig. 4

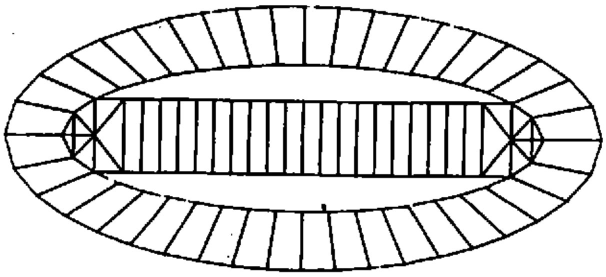


Fig. 5

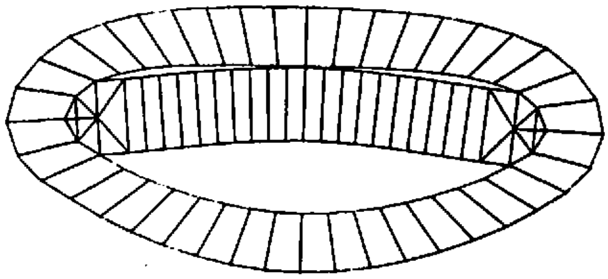
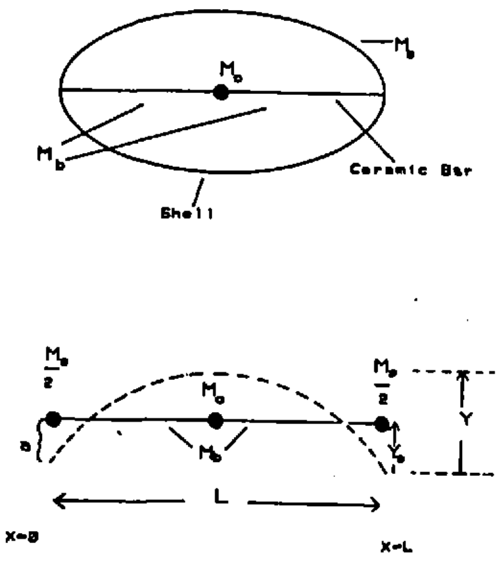
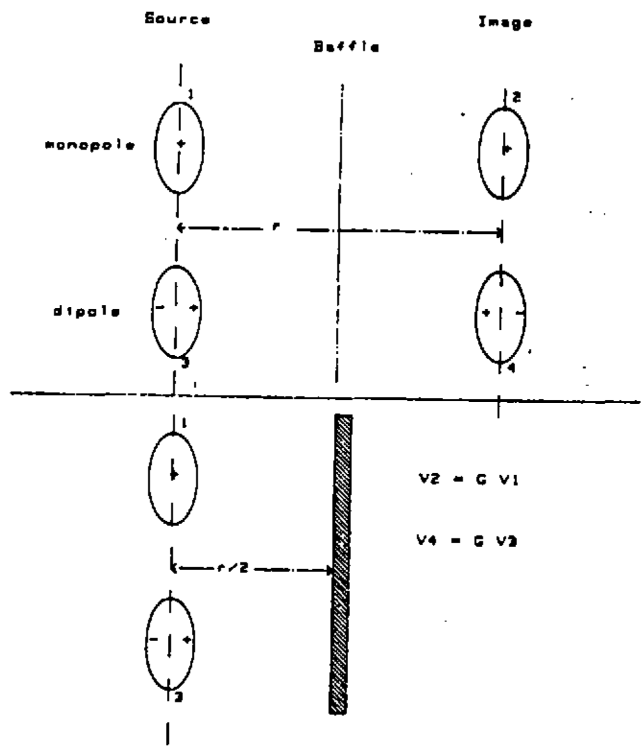


Fig. 6



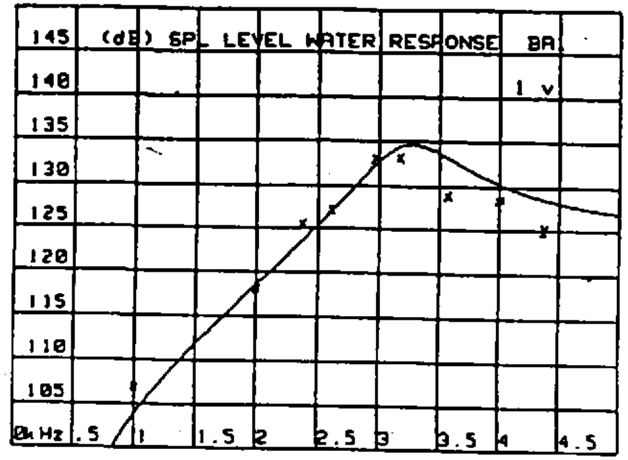
Simple Model for the Dipole Mode

Fig.7

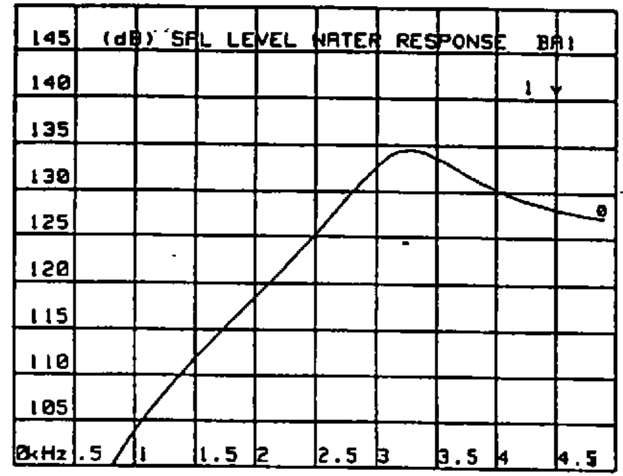


Odd mode excitation model and baffle reflection factor, C.

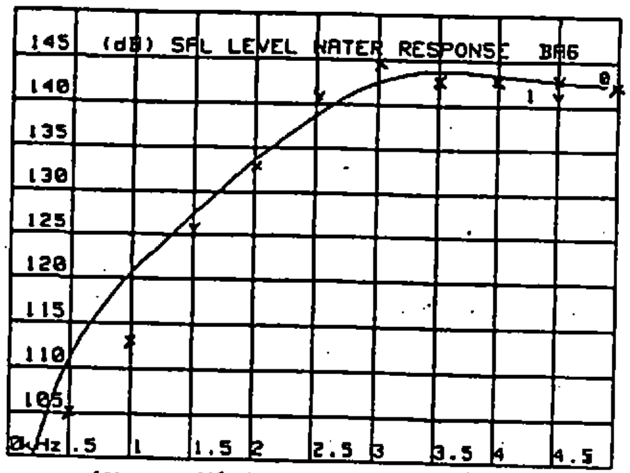
Fig.8



Single Element Calculated (——) and Measured (X X X) TVR

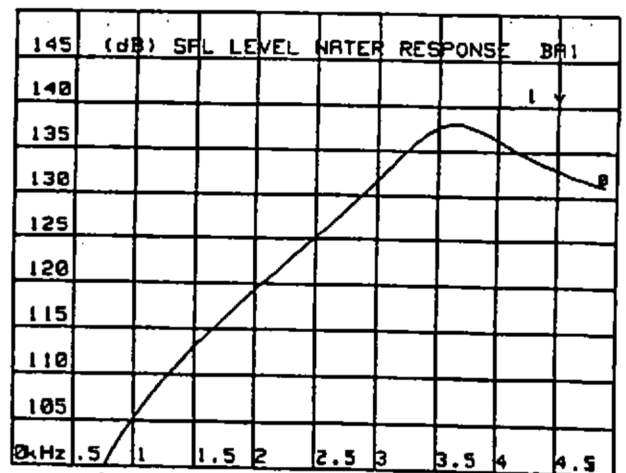


Case BA1 with all modes operative (Y,Y,Y).



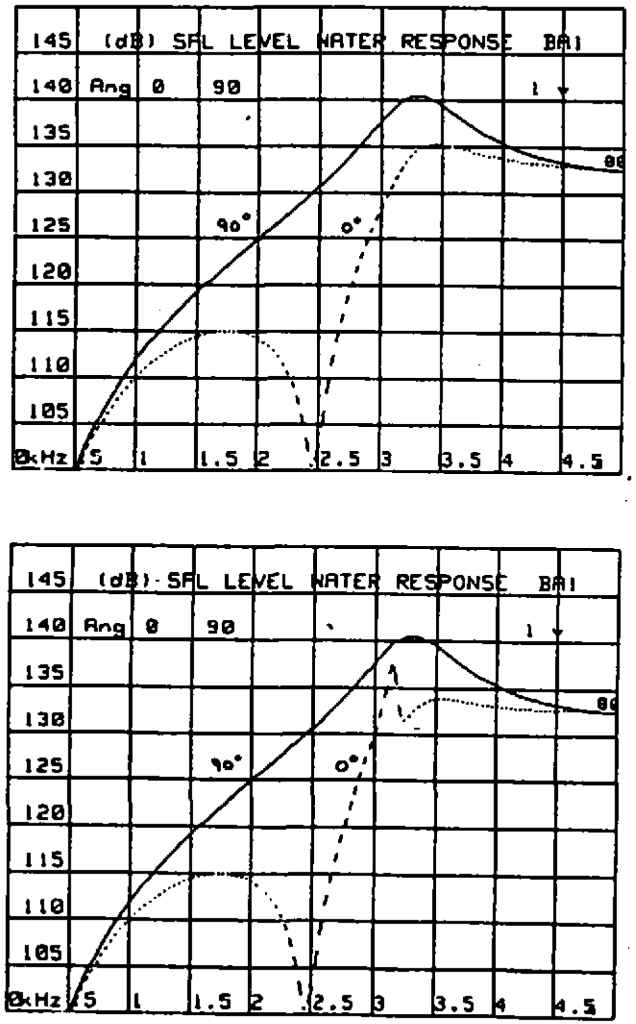
Array case BA6 with DI option activated.

Fig. 9



Case BA1 with only the quadrupole mode (Y,N,N).

Fig. 10

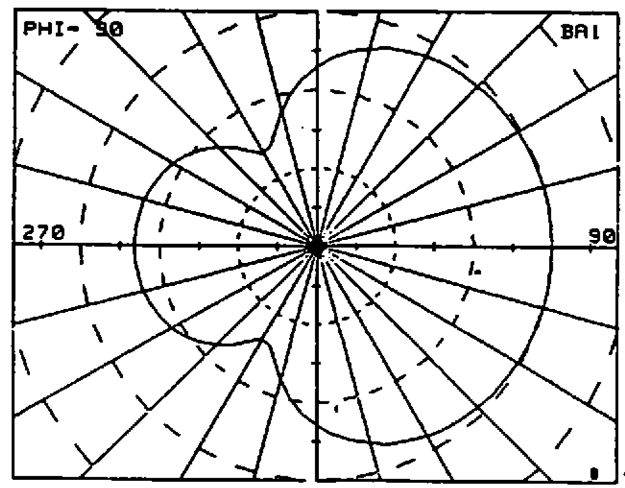


A

B

Transmitting voltage response at 0 and 90° for a single element located six inches from a rigid baffle with (B) and without (A) dipole mode.

Fig. 11



Interactive Input data, transducer file: BA1

Element equivalent spherical radius (m) = .05056
Element Y center-to-center spacing (m) = .113538

Frequency = 3300 Hz

Quad mode ABCD parameters:

- A -		- B -		- C -		- D -	
Real	Imag	Real	Imag	Real	Imag	Real	Imag
2.00E-01	0.00E+00	2.29E+03	-9.31E+03	2.30E-06	1.84E-04	1.36E+01	2.01E+00

Element input voltage, phase, and locations (m) :

EL#	Vin	Phase	X	Y
1	10.00E-01	0.0	0.00	0.00
2	10.00E-01	90.0	0.00	.11

Interactive Output Data, Xducer file: BA1, Frequency = 3300

EL #	VELOCITY		INPUT ELECT IMPEDANCE		RADIATION IMPEDANCE	
	REAL	IMAG	REAL	IMAG	REAL	IMAG
1	12.57E-05	-39.00E-06	43.39E+01	-90.10E+01	12.90E+03	29.07E+03
2	-66.05E-06	11.95E-05	23.61E+01	-71.81E+01	99.30E+02	95.21E+02
AV	29.40E-06	40.27E-06	33.50E+01	-80.96E+01	11.42E+03	19.30E+03
SD	96.26E-06	79.27E-06	98.09E+00	91.44E+00	14.87E+02	97.74E+02
SD(%)	32.74E+01	19.69E+01	29.52E+00	11.29E+00	13.02E+00	50.66E+00

Fig. 12