

THE SPATIAL VARIABILITY OF RANDOM NOISE IN A REVERBERANT SPACE

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The spatial variability of a frequency band of random noise in a reverberant space is a contributor to the uncertainty of the measurement of sound insulation. This paper looks at the theory for predicting the spatial variance of a frequency band of random noise in a reverberant space and compares the theory with experimental measurements. In the mid-frequency range, the relative spatial variance is well predicted by the well-known formula, $3 \ln(10)/(B \cdot T_{60})$, where B is the statistical bandwidth of the random noise and T_{60} is the reverberation time of the room. This formula under predicts the experimental results at low frequencies in all reverberant rooms. This paper gives a formula, derived from a formula for the pure tone spatial relative covariance in a reverberant space, which predicts the measured results surprisingly well. It depends on the modal overlap of the reverberant space. The formula given above also underestimates the measured spatial variance in larger rooms at high frequencies, and the measured spatial variance is greater when the sound source is more directional. Lubman's theory, which depends on the directivity of the sound source, gives a better estimate of the spatial variance in large rooms at high frequencies. Interestingly, this increase in spatial variance at high frequencies in large rooms does not occur when the sound field is decaying. This is indication that the properties of the sound source affect the steady state variance at high frequencies in large rooms. Another interesting observation is that a rotating diffuser reduced the spatial variance of third octave band reverberation time measurements but had almost no effect on the spatial variance of steady state third octave band random noise levels in a reverberant space. The paper also gives the history of the development of the theory for predicting the spatial variance.

Keywords: spatial variability, random noise, reverberant spaces

1. Introduction

A theoretical formula for the spatial variance of the steady state squared sound pressure in a reverberant space excited by a frequency band of random noise was derived by Kurtović [1] in 1961. Andres [2] in 1965/1966, Lubman [3] in 1968, and Schröder [4] in 1969 derived similar theoretical formulae. Unfortunately, the common version of these formulae underestimates the experimental results in the low and high frequency ranges. This is surprising, because in 1977 Davy [5-8] used the common version of these formulae to successfully predict the spatial variance of the decaying sound field after the sound source had been turned off and hence to successfully predict the spatial variance of the reverberation time except at low frequencies. Also puzzling was the 1978 observation of Bartel and Magrab [9] that a rotating diffuser reduced the spatial variance of third

octave band reverberation time measurements but had almost no effect on the spatial variance of steady state third octave band random noise levels in a reverberant space.

Andres' [2] theory actually included a factor which increased the spatial variance of the steady state squared sound pressure in a reverberant space excited a frequency band of random noise if the decay rates of the modes were different. Unfortunately he was unable to evaluate this factor theoretically. From experiment, he concluded that this factor was 2. Andres also discovered that the sound pressure level in the reverberant sound field decreased with increasing distance from the source. His experimental spatial variance of sound pressure level was actually that about the linear regression of sound pressure level with increasing distance from the sound source. The experimental results which lay outside his confidence limits he attributed to his sound source becoming directional at high frequencies.

Lubman [3] also commented "that measured values were usually higher than predicted, but rarely off by more than 50%" and that "the apparent discrepancy between predicted and measured variances is 64%, which is fairly typical of the many experimental results to date". In 1974, Lubman [10] presented a high frequency model based on the residual effect of the direct field on the spatial variance of the squared sound pressure on a straight radial line from the sound source. This model was designed for semi-reverberant fields and agreed well with Lubman's experimental measurements [10]. However the linear regression with distance used by Andres would appear to remove the extra variance that Lubman's high frequency model predicts. Lubman [10] also produced a low frequency model which multiplies the bandwidth of the random noise by the noise modal overlap when the noise modal overlap is less than one. Work by Lyon in 1969 [11], Davy in 1981 [12] and Weaver in 1989 [13] extended the theory to low frequencies. Schröder [4] obtained good agreement between his formula and a Monte Carlo simulation.

In 2003 Chiles and Barron [14-17] found that the spatial variance of octave band steady state reverberant sound fields in scale models was often larger than the common theoretical formula.

2. Theory

Let $H(\omega)$ be the frequency response function of a band pass filter where ω rad/s is the angular frequency. The "statistical" or "equivalent" bandwidth B_s rad/s is

$$B_s = \frac{\left[\int_0^\infty |H(\omega)|^2 d\omega \right]^2}{\int_0^\infty |H(\omega)|^4 d\omega} \text{ rad/s.} \quad (1)$$

Most fractional octave band pass filters used in acoustics are n th order Butterworth filters which are designed to have a noise or effective bandwidth equal to their nominal bandwidths. In 1987, Davy and Dunn [18] showed that the ratio of the statistical or equivalent bandwidth to the noise or effective bandwidth of an n th order Butterworth filter is $\frac{2n}{2n-1}$. For the most commonly used order of $n = 3$, the statistical bandwidth is 20% greater than the noise or effective bandwidth.

Let γ 1/s be the damping rate of the amplitude of the modes of the reverberant space, which is assumed to be constant in this paper. If T_{60} s is the reverberation time of the reverberant space, it can be shown that (Davy [12])

$$\gamma = \frac{3 \ln 10}{T_{60}} \text{ 1/s.} \quad (2)$$

The statistical bandwidth of a mode of the reverberant space is $B_{ms} = 2\pi\gamma$ rad/s (Schröder [4], Davy [12]).

The statistical bandwidth can also be expressed in frequency rather than angular frequency, where f Hz is the frequency and f as a subscript denotes that the bandwidth is in hertz.

$$B_{sf} = \frac{B_s}{2\pi} = \frac{\left[\int_0^\infty |H(f)|^2 df \right]^2}{\int_0^\infty |H(f)|^4 df} \text{ Hz.} \quad (3)$$

Thus, the statistical bandwidth in hertz of a mode of the reverberant space is $B_{msf} = \gamma \text{ Hz}$.

In 1965/1966 Andres [2] and in 1969 Schröder [4] showed that the spatial variance of the steady state squared sound pressure in a reverberant space excited by a frequency band of random noise is a function of the ratio Z of the statistical bandwidth of the random noise to the statistical bandwidth of the modes of the reverberant space.

$$Z = \frac{B_s}{B_{ms}} = \frac{B_s}{2\pi\gamma} = \frac{B_{sf}}{B_{msf}} = \frac{B_{sf}T_{60}}{3\ln 10}. \quad (4)$$

The relative variance is the ratio of the variance to square of the mean value. In 1968 Lubman [3] and in 1969 Schröder [4] derived a formula for the relative spatial variance r of the steady state squared sound pressure in a reverberant space excited by an ideal rectangular frequency band of random noise. Work by Lyon [11], Davy [12], Weaver [13], Lobkis et al. [19] and Langley and Cotoni [20] extended this formula to cases of low modal overlap.

$$r = \frac{1}{Z} \left(1 + \frac{K}{M_s} \right) F(\pi Z). \quad (5)$$

$$F(\theta) = \frac{2 \arctan \theta}{\pi} - \frac{\ln(1 + \theta^2)}{\pi \theta}. \quad (6)$$

$$K = \left[\frac{\langle p_m^4(\mathbf{x}) \rangle}{\langle p_m^2(\mathbf{x}) \rangle^2} \right]^2 - 3C(M_s). \quad (7)$$

where M_s is the statistical modal overlap which is the product of the modal density with the statistical modal bandwidth, $p_m(x)$ is a modal spatial function and the brackets $\langle \rangle$ denote the average value over positions x in the room and over modes in the frequency range of interest. The modal density n in modes per hertz may be approximated by (Davy [12])

$$n = \frac{4\pi f^2 V}{c^3} + \frac{3\pi f V^{2/3}}{c^2} + \frac{3V^{1/3}}{2c} \text{ s,} \quad (8)$$

where $V \text{ m}^3$ is the volume of the reverberant space and $c \text{ m/s}$ is the speed of sound.

$$\frac{\langle p_m^4(\mathbf{x}) \rangle}{\langle p_m^2(\mathbf{x}) \rangle^2} = \frac{1}{n} \left(\frac{27}{8} \frac{4\pi f^2 V}{c^3} + \frac{9}{8} \frac{3\pi f V^{2/3}}{c^2} + \frac{3}{8} \frac{3V^{1/3}}{2c} \right). \quad (9)$$

$$M_s = n\gamma = \frac{3n \ln 10}{T_{60}}. \quad (10)$$

$$\begin{aligned} 3C(M_s) = & \left(\frac{M_s}{4} + 2 - \frac{5}{4M_s} \right) + e^{-2M_s} \left(\frac{M_s}{4} + \frac{1}{2} + \frac{5}{4M_s} \right) \\ & - E_1(M_s) e^{-M_s} \left(\frac{M_s^2}{4} + \frac{3M_s}{4} + \frac{5}{2} + \frac{5}{2M_s} \right) \\ & - E_1(M_s) e^{M_s} \left(\frac{M_s^2}{4} - \frac{3M_s}{4} + \frac{5}{2} - \frac{5}{2M_s} \right) \end{aligned} \quad (11)$$

E_1 is the exponential integral.

When Z tends to zero, $F(\pi Z)$ tends to Z , and the relative spatial variance r tends to

$$r = 1 + \frac{K}{M_s}. \quad (12)$$

When Z tends to infinity, $F(\pi Z)$ tends to one, and the relative spatial variance r asymptotes to (Kurtović 1961 [1], Andres 1965/1966 [2])

$$r = \frac{1}{Z} \left(1 + \frac{K}{M_s} \right). \quad (13)$$

The spatial variance v dB² of the steady state sound pressure level in a reverberant space excited by a frequency band of random noise is approximately (Andres 1965/1966 [2])

$$v = \left(\frac{10}{\ln 10} \right)^2 r = (10 \log_{10} e)^2 r \text{ dB}^2 \quad (14)$$

for large values of Z (small values of r). Craik [21] has given a version of this equation which applies for large values of r .

At high frequencies in large rooms, Eq. (5) under estimates the measured variance. Lubman [10] has derived a formula for the variance of the sound pressure squared of a band of random noise in a reverberation room along a segment of a radial line from the acoustical centre of the sound source between distances r_1 and r_2 from the acoustical centre of the sound source. The directivity factor of the sound source in the direction along the radial line is Q . The ratio D of the mean direct squared sound pressure to the mean reverberant squared sound pressure along the radial line segment between r_1 and r_2 is

$$D = \frac{3QV}{2\pi \log_{10}(e) c T_{60} r_1 r_2} \quad (15)$$

where c is the speed of sound and V is the volume of the reverberant room. The relative variance r' along the line segment is

$$r' = \frac{(\rho - 1)^2 D^2}{3\rho(1 + D)^2} + \frac{r}{(1 + D)^2} \quad (16)$$

where r is given by Eq. (5) and

$$\rho = \frac{r_2}{r_1}. \quad (17)$$

Eq. (16) will be used to predict the spatial variance in this paper even when the measurement points are not on a radial line from the acoustical centre of the sound source. Q will be set equal the maximum directivity of the sound source over all directions and will be assumed constant over frequency because it usually only has a significant effect at high frequencies.

3. Comparison with experiment

In this paper, new theory refers to the use of Eqs. (5), (16) and (14) and old theory refers to the use of Eq. (5) without the first bracketed term and equation (14). In the first lot of four figures, the measurements were made along a radial straight line from the sound source. In these cases, experiment refers to the standard deviation of the sound pressure levels about the straight line of best fit as a function of distance along the measurement line and uncorrected refers to the standard deviation of the sound pressure levels. In the second lot of four figures, experiment refers to the standard deviation of the sound pressure levels because the measurement positions were randomly distributed in the room rather than placed along a straight radial line from the sound source. 90 % confidence limits for the experiment results are given in all figures. In the first four figures the number of independent points is limited to the number of half wavelengths along the measurement line. In the second lot of four figures, the number of independent points is limited to the room volume divided by the volume of a cube whose sides are half a wavelength.

In the first four figures, the measurements were made in a 200 m³ reverberation room at 16 microphone positions spaced at 0.25 m along a radial straight line at distances from 2 to 5.75 m from the sound source [22]. In the first two figures a dodecahedron sound source was used in order to have a more omnidirectional sound source. It was not completely omnidirectional at the high frequencies and a directivity factor of 4 was assumed.

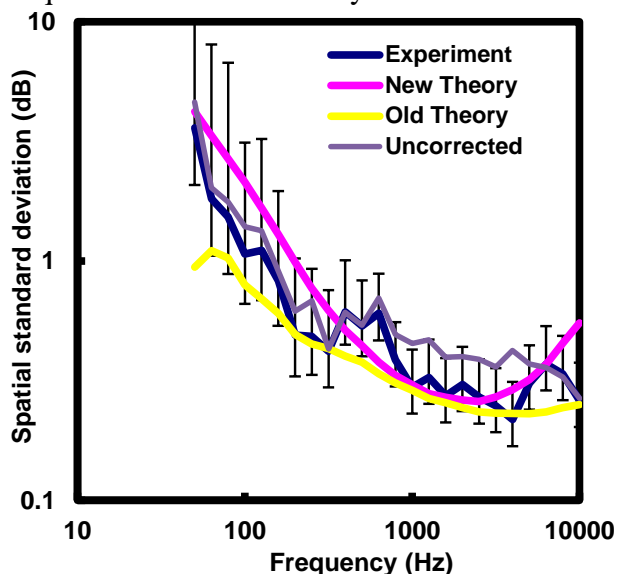


Figure 1. Spatial standard deviation of third octave bands of random noise in a 200 m³ reverberation room without added sound absorbing material and an omnidirectional sound source. 16 microphone positions with a spacing of 0.25 m on a radial line from the sound source, 2 to 5.75 m from the source.

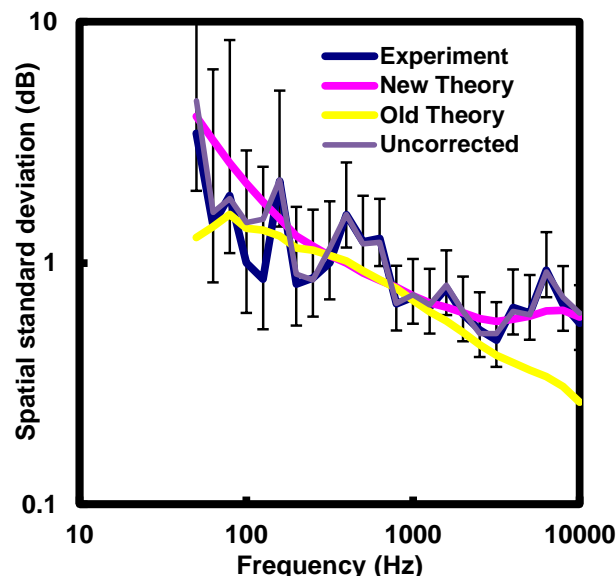


Figure 2. Spatial standard deviation of third octave bands of random noise in a 200 m³ reverberation room with added sound absorbing material and an omnidirectional sound source. 16 microphone positions with a spacing of 0.25 m on a radial line from the sound source, 2 to 5.75 m from the source.

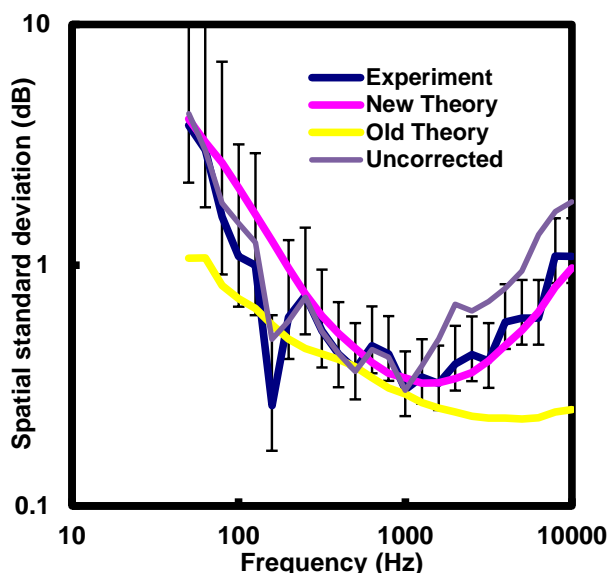


Figure 3. Spatial standard deviation of third octave bands of random noise in a 200 m³ reverberation room without added sound absorbing material and a directional sound source. 16 microphone positions with a spacing of 0.25 m on the sound source axis, 2 to 5.75 m from the source.

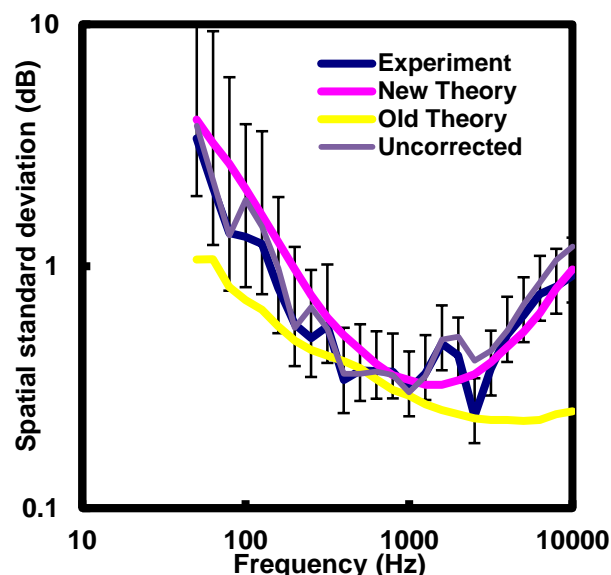


Figure 4. Spatial standard deviation of third octave bands of random noise in a 200 m³ reverberation room without added sound absorbing material and a directional sound source. 16 microphone positions with a spacing of 0.25 m on a radial line at 90° to sound source axis, 2 to 5.75 m from the source.

Figure 1 shows the case when no sound absorbing material was added to the reverberation room. Figure 2 shows the increase in the spatial standard deviation that occurs when sound absorbing

material is added to the reverberation room. The difference between the experiment and new theory results and the old theory results at the highest frequency is greater with the added sound absorption. Interestingly, the difference between the experiment results and the uncorrected results was greater in the case without added sound absorption.

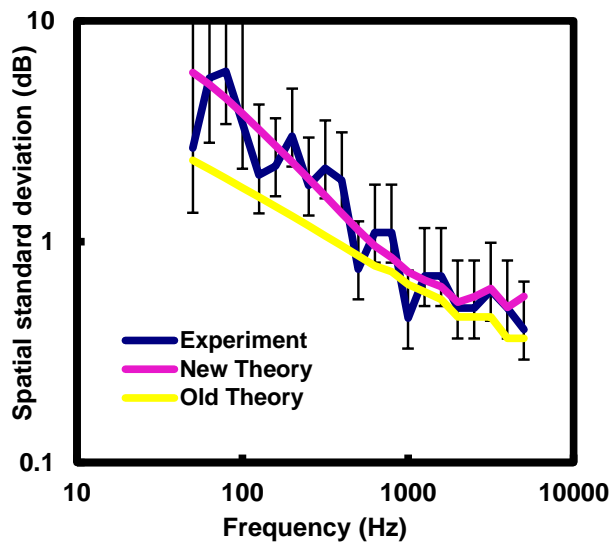


Figure 5. Spatial standard deviation of third octave bands of random noise in an 11.1 m³ room. 10 randomly distributed microphone positions.

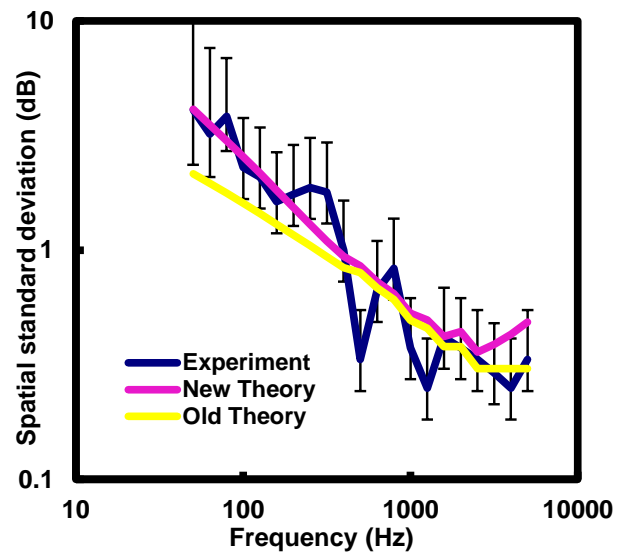


Figure 6. Spatial standard deviation of third octave bands of random noise in a 64.8 m³ room. 10 randomly distributed microphone positions.

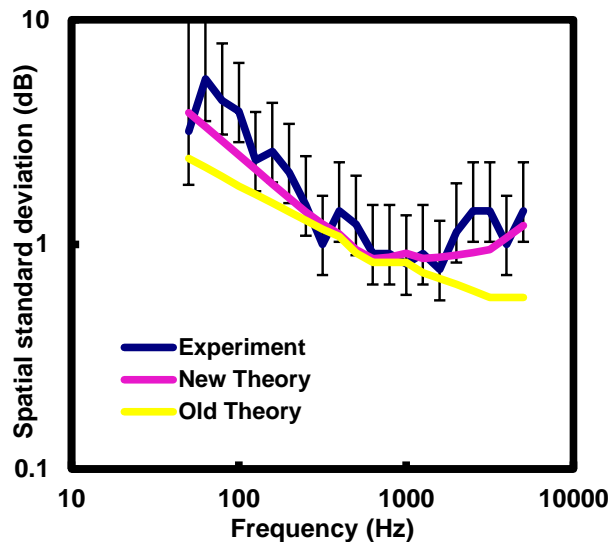


Figure 7. Spatial standard deviation of third octave bands of random noise in a 69.2 m³ room. 10 randomly distributed microphone positions.

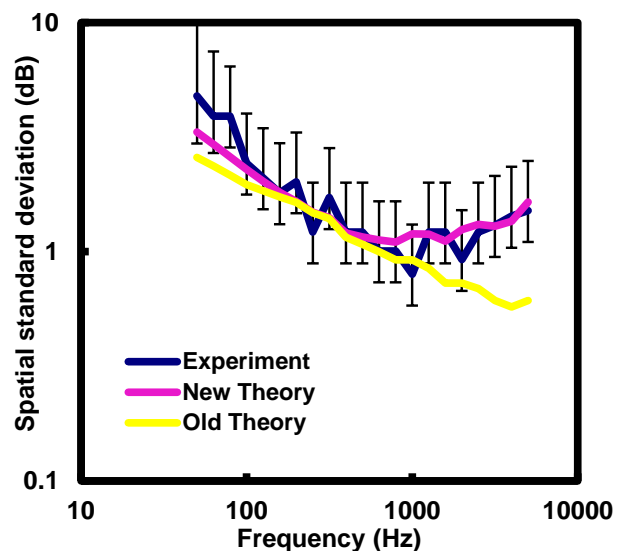


Figure 8. Spatial standard deviation of third octave bands of random noise in a 102.0 m³ room. 10 randomly distributed microphone positions.

In the next two figures a more directional sound source was used. This source consisted of a low frequency loudspeaker in a box with a high frequency horn loudspeaker on top of the box. Figure 3 shows the case when the microphone positions were on the axis of the sound source. Figure 4 shows the case when the microphones were on a line at right angles to the axis of the sound source. Surprisingly there is little difference between the two graphs and a directivity factor of 7 was used for both cases. There is a bigger difference between the experiment and the new theory results and the old theory results at high frequencies in both cases than in the cases with the omnidirectional sound source. The difference between the experiment results and the uncorrected results was greater in the on axis case.

Davy [23] had previously put forward a high frequency correction which depended on the mean free path length as an alternative to Lubman's correction given by Eqns. (15) to (17). Unfortunately this alternative correction had an empirical constant which varied widely with room and situation. This fact and the variability of the high frequency correction with the directivity of the source and the amount of sound absorption in the room has convinced the author that Lubman's correction is more likely to be correct. Lubman's correction only applies to microphone positions on a radial straight line from the source centre. However it will also be used for randomly distributed microphone positions in this paper. One of the reasons for this is that, unexpectedly, there was little difference between the on axis case and the case with the microphones on a line at right angles to the axis of the sound source.

Another reason for adopting Lubman's high frequency correction was the important experimental observation of Chiles and Barron [16, 17] that the spatial variance of the sound pressure levels during the middle stages of a reverberant decay was much closer to that predicted by the old theory. This observation also explains why Davy [5-8] was able to use the common theoretical formula to successfully predict the spatial variance of the decaying sound field after the sound source had been turned off and hence successfully predict the spatial variance of the reverberation time except at low frequencies.

Figures 5 to 8 [24] show the effect of the room volume increasing from 11.1 to 102 m³. 10 randomly distributed microphone positions were used. The directivity factor of the sound source was assumed to be 4. The maximum measurement distance from the sound source was assumed to be the cube root of the room volume. The minimum measurement distance from the sound source was assumed to be 1.3 m, except for the case of the smallest room where it was assumed to be 0.6 m. The reverberation times were back calculated from Olesen's theoretical results [24] and checked by reference to the reverberation times at 100, 1000 and 3150 Hz given in Table 4.1 of Olesen [24]. The difference at high frequencies between the experiment values and the old theory values increases from almost no difference at 11.1 m³ to a significant difference at 102 m³ (and 200 m³). This explains why Hopkins and Turner [25] found almost no difference at high frequencies with two rooms with volumes of 33.6 and 38.7 m³, while Davy [23] and Lubman [10] found significant differences at high frequencies with rooms with volumes of 607 and 715 m³.

Of course most field measurements of sound insulation are made with rooms with small volumes where the low frequency corrections are more important. Lubman [10], Craik [21] and Olesen [24] have given empirical approximations in the low frequency range. Although these empirical approximations work well [10, 24, 25], the new theory has the advantage of being theoretically correct in this frequency range.

4. Conclusion

The new theory presented in this paper for the spatial standard deviation of a frequency band of random noise in a reverberation room agrees reasonably well with the experimental values. This is because of a theoretical low frequency correction which is a function of the statistical modal overlap and a high frequency correction which is a function of the directivity of the sound source. The high frequency correction is very small in small rooms and increases with increasing room volume.

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