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VIBRATION POWER TRANSMISSION IN PIPES AT LOW FREQUENCIES

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1. INTRODUCTION

A typical machine installation consists of a source of vibration mounted on resilient isolators attached to a flexible substructure. There are usually other vibration transmission paths, such as pipework, which act as short circuit elements across the isolators. Thus there are a variety of mechanisms by which vibration is transmitted from the source to the point of interest on the substructure. When attempting to control the vibration transmitted from the machine into and through the structure upon which is is monted, it is desirable to be able to identify and quantify these vibration transmission paths. Knowledge of transmission path characteristics enables procedures to be carried out, for example, to reduce vibration levels at points remote from the source, perhaps with the objective of reducing unwanted radiation of sound.

One method for obtaining transmission path information is to use the concept of power flow or vibrational power transmission. This technique allows the contributions from the various wave types propagating through the structure to be compared quantitatively. By identifying not only which section of the structure is a major transmission path, but in what manner the power is transmitted, it is possible to apply the correct vibration control technique. Thus vibrational power transmission measurements allow the direction of propagation to be determined and a magnitude to be assigned to each path.

Most structures carry more than one wave type, especially when discontinuities occur in the system. To determine the complete distribution of power in the structure, methods must be devised to measure the power associated with all the travelling waves in the system. By using the time domain concepts developed by Pavic [1] it is possible to measure the transmitted vibrational power of a flexural travelling wave in a beam to within half a wavelength of a discontinuity. Similarly, frequency domain techniques have been devised by Verheij [2] to measure the power associated with flexural, longitudinal and torsional travelling waves in beams, rods and pipes.

2. THEORY OF VIBRATIONAL POWER TRANSMISSION

2.1 Flexural Waves in a Uniform Beam

Consider a uniform beam carrying a flexural wave only. Assuming that the point of interest is in the far field of any discontinuity or source and that the flexural wave motion may be described using Euler-Bernoulli beam theory, then if

$$W = A_f \sin(\omega t - k_f x)$$

POWER TRANSMISSION IN PIPES

$$\text{Shear force, 'S', on a section of the Beam} = E\,I\,\frac{\partial^3 w}{\partial x^3}$$

$$\text{Bending Moment, M, on a Section of the Beam} = E\,I\,\frac{\partial^2 w}{\partial x^2}$$

Thus instantaneous rate of working $X = S\,\frac{\partial w}{\partial t} - M\frac{\partial^2 w}{\partial x\,\partial t}$

$\therefore$ Time Averaged Power $<P>_f = \frac{1}{T}\int_0^T X\,dt$

and

$$<P>_f = EI\,\omega k_f^3\,A_f^2 \qquad (1)$$

2.2 Longtudinal Waves in a Uniform Rod
Consider a uniform rod carrying a longitudinal wave only.

If $U = A_l \sin(\omega t - k_l x)$

then instantaneous rate of working $X = -\,EA\frac{\partial u}{\partial x}\;\frac{\partial u}{\partial t}$

$\therefore$ Time Averaged Power $<P>_l = \frac{1}{T}\int_0^T x\,dt$

$$<P>_l = \frac{1}{2}E\,A\,\omega\,k_l\,A_l^{\,2} \qquad (2)$$

3. MEASUREMENT OF VIBRATIONAL POWER IN A BEAM

Equations (1) and (2) show that transmitted power is proportional to the square of the propagating wave amplitude. When a single transducer is attached to a structure, its output is related to the sum of all the waves ie. travelling and standing, at the point of interest. Thus it is necessary to configure the expressions for measurement of power transmission in such a manner that any standing wave contributions are cancelled out.

3.1 Measurement of Flexural Power

Previous work [1] on flexural power transmission overcame the problem of erroneous contributions from standing waves by utilising two measurement positions. In the far field, it may be shown that the power due to the shear force on an element of a beam equals the power due to the bending moment on the element. It was shown that if two accelerometers are mounted a distance Δ apart, the time averaged transmitted flexural power is given by

$$<P>_f = \frac{\sqrt{E I \rho A}}{\Delta \omega^2} <(a_2 - a_1)(a_2(q) + a_1(q)> \qquad (3)$$

Where suffixes 1 and 2 refer to the two accelerometer positions and the symbol 'q' indicates a signal whose phase has been shifted by $-0.5\,\pi$

3.2 Measurement of Longitudinal Power

In order to obtain equation (3), it may be shown that

$$<(a_2 - a_1)(a_2(q) + a_1(q))> = a^2 \sin \varnothing \qquad (4)$$

where a is the acceleration of a travelling wave only (standing wave contributions are negated) and ∅ is the phase angle between accelerations a_1 and a_2.

Thus applying (4) to (2), gives the following expression for the measurement of longitudinal power in the presence of standing waves.

$$<P>_l = \frac{1}{2} \frac{E A k_l}{\omega^3 \sin \varnothing} <(a_2 - a_1)(a_2(q) + a_1(q))> \qquad (5)$$

4. PIPES CARRYING FLEXURAL AND LONGITUDINAL WAVES SIMULTANEOUSLY

Consider both a flexural wave and longitudinal wave which propagate along a uniform pipe vibrating at low frequencies, such that it may be considered as a beam. If the cross-section at any position along the pipe is examined its displacement at that point will be the sum of the displacement of the two wave types. It may be considered that a longitudinal wave causes a uniform expansion or contraction of the pipecross-section, whilst a flexural wave causes a lateral displacment of the pipe cross-section. These properties may be utilised to separate the various wave types. At any two positions A and B that are diametrically opposite each other on the pipe cross-section (figure 1), the measured amplitudes will be

POWER TRANSMISSION IN PIPES

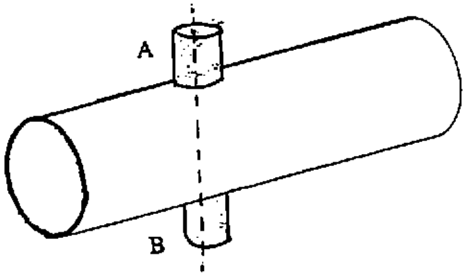

Figure 1: Transducer Positions

$$\text{At A} \qquad A_A = A_l + \alpha \; A_f \qquad (6)$$

$$\text{At B} \qquad A_B = A_l - \alpha \; A_f \qquad (7)$$

α indicates a certain proportion of the true flexural wave amplitude; it accounts for measurements made out of the true plane of propagation of the flexural wave. Thus if measurements were made on a pipe that only vibrates in flexure in one plane, then $\alpha = 1$.

By combining the above two expressions, the amplitude of the longitudinal wave is given by

$$A_l = \frac{1}{2}(A_A + A_B) \qquad (8)$$

Thus if four transducers were mounted on a pipe as shown in Figure 2.

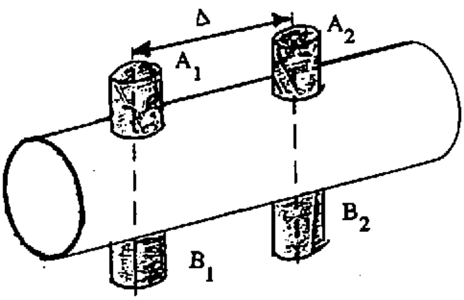

Figure 2: Transducer Configuration for Longitudinal Power Measurement

POWER TRANSMISSION IN PIPES

Applying equation (8) to the signals from A_1 and B_1 would yield the expression a_1 in equation (5) and applying (8) to the outputs of A_2 and B_2would yield a_2 in (5). Thus time averaged longitudinal power in a pipe carrying both flexural and longitudinal waves may be measured using four transducers.

To obtain the amplitude of the flexural wave, consider three transducer positions on the pipe circumferance (Figure 3).

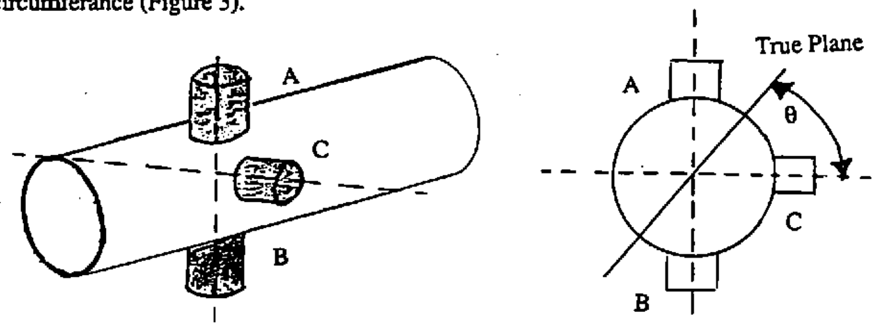

Figure 3: Transducer Positions

At A $\quad A_A = A_l + \cos\theta\, A_f$ (9)

At B $\quad A_B = A_l - \cos\theta\, A_f$ (10)

At C $\quad A_C = A_l + \sin\theta\, A_f$ (11)

From equations (10) and (11)

$$A_A - A_C = A_f\,(\cos\theta - \sin\theta) \qquad (12)$$

and from (9) and (10)

$$A_f = \frac{A_A - A_B}{2\cos\theta} \qquad (13)$$

and (13) in (12) gives

$$\mathrm{Tan}\theta = \left[\frac{A_A + A_B - 2A_C}{A_B - A_A}\right] \qquad (14)$$

Therefore in order to measure flexural power on a pipe carrying both flexural and longitudinal waves, six transducers are required as shown in Figure 4

POWER TRANSMISSION IN PIPES

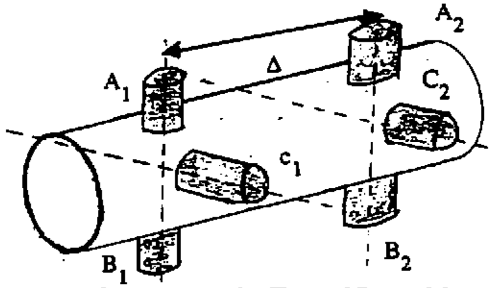

Figure 4: Transducer Configuration for Flexural Power Measurement

The application of equation (13) and (14) to the output of transducers A_1 B_1 and C_1 yields the a_1 term in equation (3). Similar application of (13) and (14) to A_2 B_2 and C_2 results in the a_2 term. If the plane of the transmitted flexural wave is known, $\theta = 0$ and equation (13) reduces to

$$A_f = \frac{1}{2}(A_A - A_B)$$

This allows the transducer configuration shown in Figure 2 to be used for flexural power measurements.

5. PIPE CARRYING ONLY A FLEXURAL WAVE

If the pipe only carried a flexural wave whose true plane of propagation was unknown, the resultant transmitted power could be measured using four transducers. The configuration is shown in Figure 5.

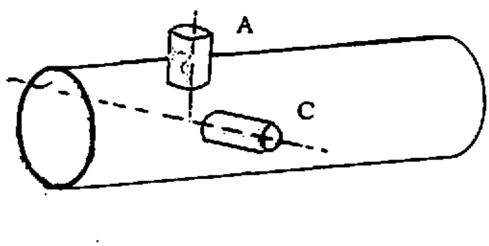

Figure 5: Transducer Configuration

POWER TRANSMISSION IN PIPES

Now $A_A = \cos\theta \, Af$

$A_C = \sin\theta \, Af$

Thus $$A_f = \frac{A_A}{\cos\theta} \qquad (15)$$

where $$\mathrm{Tan}\theta = \frac{A_A}{A_C} \qquad (16)$$

Equations (15) and (16) can be applied in the same manner as equations (13) and (14) to determine transmitted flexural power.

6. CONCLUSIONS

Expressions are presented for measuring time averaged transmitted vibrational power due to either flexural or longitudinal waves. These expressions may be used on time domain data taken from pipes excited at low frequencies carrying both wave types. Measurements of vibrational power allow transmission paths to be identified and hence the correct vibration control procedure to be chosen and applied.

7. REFERENCES

[1] G PAVIC, "Measurement of Structure-borne Wave Intensity, Part 1: Formulation of the Methods', Journal of Sound & Vibration 49 p221-230 (1976)
[2] J W VERHEIJ, 'Cross Spectral Density Methods for Measuring Structure-borne Power Flow in Beams and Pipes', Journal of Sound and Vibration 70 p133-139 1980

8. NOTATION

A	cross-sectional area
A_f	amplitude of flexural wave
A_l	amplitude of longitudinal wave
a	acceleration of travelling wave
a_1	measured acceleration at point 1
a_2	measured acceleration at point 2
E	modulus of Elasticity
I	second moment of area
K_f	flexural wave number
K_l	longitudinal wave number

M	bending moment
$<P>_f$	time averaged flexural power
$<P>_l$	time averaged longitudinal power
S	shear force
T	time period
t	time
U	longitudinal displacement
W	flexural displacement
X	instanteneous rate of work
x	distance
α	proportion of flexural wave amplitude
Δ	accelerometer spacing
θ	angle between propagation and measurement plane
ρ	density
ω	frequency