Proceedings of the Institute of Acoustics 'Spectral Analysis and its Use in Underwater Acoustics': Underwater Acoustics Group Conference, Imperial College, London, 29–30 April 1982

USE OF SPECTRAL MATRIX FOR SOURCES INDENTIFICATION

J.L. LACOUME - B. BOUTHEMY - F. GLANGEAUD - C. LATOMBE

CEPHAG - Equipe de Recherche Associée au CNRS
BP 46 - 38402 SAINT MARTIN D'HERES (FRANCE)

ABSTRACT

The spectral matrix rank of the N dimensional signal received on an array of sensors is equal to the number of sources. The spectral matrix rank can be obtained through the number of non zero eigenvalues. The sources can be separate using the spectral matrix eigenvectors. The above properties have been used in order to determine the number of sources and to identify the sources in submarine acoustics and geophysical applications. In actual situations a lot of problems arises :

- Definition of the "best" estimator of the spectral matrix.
- Position of the threshold in order to separate the eigenvalues related to the source from the eigenvalue issuing from the noise.
- Synthesis of the filters separating the sources.

We present the results obtained on submarine acoustic and geophysical signals and we discuss the effects of the estimation on the eigenvalues and the rank of the spectral matrix :

- Relation between the rank of the spectral matrix and the number of degree of freedom of the averaging procedure.
- Modification of the eigenvalues amplitude by the estimation procedure.

We conclude by a presentation of some experimental results of sources separation in actual situations.

\- 0 -

INTRODUCTION

The devices possibilities development has led to very sophisticated processors in N dimensional signal processing. The treatments actually developped use essentially the concepts of linear subspaces theory as theoretical support and the spectral matrix estimator as experimental tool. We will show some applications of these techniques in sources number determination, sources identification and spatial filtering.

1 - THE SPECTRAL MATRIX : DEFINITION AND PROPERTIES

The spectral matrix is used for studying the N components signals :

$$\underline{S}(t) = \begin{pmatrix} S_1(t) \\ \vdots \\ S_N(t) \end{pmatrix}$$

These signals are found in a large number of practical problems. A N component signal can be obtained as the output of an array of N sensors. This situation arises in submarine acoustics (SONAR) electromagnetic ranging (RADAR) remote sensing (earth observation) sismology (vibroacoustic sounding). In some cases the N components are of different physical nature (magnetic component and pressure of the swell are considered in [1]). The N component can also represent the components of a vectorial field [2]. In all these situations with the hypothesis of a random null average N components signal the energetic and correlation properties of the signal are given by the correlation matrix $\underline{\underline{\Gamma}}$:

$$\Gamma_{ik}(t_1,t_2) = E\left[S_i(t_1)\ S_k(t_2)\right] \qquad (1)$$

E : expectation

Generally the signal components are stationary in time and the correlation matrix depends only of the time lag : $\tau = t_1 - t_2$

Then the Fourier Transform (FT) of the correlation matrix gives the spectral matrix $\underline{\underline{\gamma}}(\nu)$ by :

$$\gamma_{ik}(\nu) = FT\left[\Gamma_{ik}(\tau)\right] = \int \Gamma_{ik}(\tau)\ e^{-2j\pi\nu\tau} d\tau \qquad (2)$$

The spectral matrix is interesting when the physical model is linear. The linear hypothesis is generally done (eventually as a first approximation) and the N components signal has then a multivariate linear model. In this model the N components are issued of p random independant excitations called <u>sources</u> transformed by a p input, N output (the signal components) linear filter. The uncertainties arising in all physical system are taken into account by an additive N components noise. This model is particularly well suited to all the propagation situations (submarine acoustics for example) in which one can identify the "physical sources" and can modelize the linear filter using the laws of propagation.

With this model the correlation matrix is :

$$\underline{\underline{\Gamma}}(\tau) = \sum_1^P \underline{\underline{\Gamma}}_i(\tau) + \underline{\underline{\Gamma}}_B(\tau) \qquad (3)$$

where : $\Gamma_i(\tau)$ is the contribution of the source i and $\underline{\underline{\Gamma}}_B(\tau)$ the noise correlation matrix. The spectral matrix is then :

$$\underline{\underline{\gamma}}(\nu) = \sum_1^P \underline{\underline{\gamma}}_i(\nu) + \underline{\underline{\gamma}}_B(\nu) \qquad (4)$$

If $\underline{H_i}(\nu)$ is the transfer function (column matrix of N components) from the source i to the N components signal :

$$\underline{\underline{\gamma}}_i(\nu) = P_i\ \underline{H}_i(\nu).\ \tilde{\underline{H}}_i(\nu) \qquad (5)$$

P_i : power of the source i, $\sim$: transpose complex conjugated

The matrix $\underline{H_i}(\nu)$ will be called the source-vector representation in the physical base (base of the sensors) [9].

With this modelization the spectral matrix alows us [3],[4] :

- to determine the number of sources,
- to identify the source-vectors.

The relations (4) and (5) show that in the noise free case ($\gamma_B = 0$) the number of sources is equal to the rank of spectral matrix. So the spectral matrix rank gives the number of sources when the number of components (N) is greater than the number of sources (p). The spectral matrix rank is given by the number of non zero eigenvalues of the spectral matrix. The calculation of the eigenvalues is time consuming (for large values of N) in order to save computation time other criteria have been proposed [5]. The same method can be applied when the noise is white. In this situation the eigenvalues can be classified in two families :

- a family of minimal equal eigenvalues (associated to the noise),
- a family of non minimal eigenvalues (associated to the sources).

The source vector identification uses the spectral matrix eigenvectors associated to the sources eigenvalues. The linear subset spanned by the p source-vectors and the linear subset spanned by the p source eigenvectors are identical. This leads to two conclusions :

- When one source is present it can be identify without indetermination.
- When there is more than one source it remains indeterminations. In this case, it is necessary to introduce "supplementary informations" in order to do the sources identifications [3]. Some general procedures have been proposed. In linear array processing with equispaced sensors the PRONY-PISARENKO method identifies uniquely point sources [6],[7].

Finally when the sources are identified each source is represented by a vector function of frequency in the linear N dimensional signal space. It is then possible to separate the signals emitted by each source by an appropriate linear transformation (projection).

The source number determination and the source separation by filtering will be illustrated below. It is first necessary to present the estimation method of the spectral matrix.

2 - SPECTRAL MATRIX ESTIMATORS

We observe the N components signals on a duration T and we want to estimate the spectral matrix using this observation.

The classical FOURIER estimator uses the averaged periodogram or correlogram methods [8]. The two procedures are caractherized by the bias and the variance of the estimator. In cross-spectral estimation, in order to reduce the bias, it was shown in [10] that the lag-window must compensate the mean time lag between the signals. The variance is inversely proportional at the number of degrees of freedom :

$$F = T_{in} \cdot B_m$$

where : T_{in} characterize the time duration of the observed signal (integration time) and B_m the bandwith of the analysis (equivalent bandwith).

3 - SOURCE NUMBER DETERMINATION

The principle of the method was stated in (1) : the source number is equal to the number of non minimal eigenvalues. Practically the situation is not so simple. It has been shown in [10] that the rank of the estimated spectral matrix is always smaller than the number of degree od freedom (F) of the estimation. So, in order to estimate p sources it is necessary to have :

$$p < F$$

Let us suppose now that the source number (p) the number of degree of freedom (F) and the number of signal components (N) verify :

a) $p < F < N$

b) $p < N < F$

and that there is an additive noise. It can be shown [10] that, in this situation, the noise associated eigenvalues are not constant as in the theoretical case. In the case a) F-p "noise eigenvalues" are decreasing and the others are null. In the case b) the noise eigenvalues decrease continously.

In order to illustrate this method we present two applications. In the first application we try to detect the number of sources of natural electromagnetic pulsations. This phenomena is in the frequency band (0,5 - 2 Hz) and its origin is a wave particle interaction in the magnetosphere. The signal components are the meridional (H) and latitudinal (D) components of the magnetic field pulsations recorded on three ground stations : So (Sogra in North of Russia), Sv (Sandvall in the Middle of Sweden), Le (Lerwick in the Shetland Iles). The distribution of energy in the time frequency plane for each station is presented in fig. 1. The spectral matrix is calculated in the time interval T4 (the duration is 690 s giving a total time-bandwith product of the order of 170) with a number of degrees of freedom F = 20. The plot of spectral matrix eigenvalues versus frequency (fig. 2) shows two frequency bands. In the lower frequency band two eigenvalues are significative and we can suppose that in this frequency band there are two sources. In the upper frequency band there is only one significative eigenvalue so in this frequency band we can suppose that there is only one source.

The second application is in submarine acoustic. In this experiment a 5 component signal was registered on a linear array of 5 equidistant hydrophones. The two sources were two emiters feeded with large band (3 to 7 kHz) independant white noises and their positions were known. The spectral matrix estimation was made with a number of degrees of freedom F = 50. We can see on fig. 4 that a clear separation between the two first eigenvalues and the following ones can be established for an angular source separations greater than 1° (the Rayleigh resolution power of the antenna is of 5°). It is clear, on these results, that the "white noise" eigenvalues are decreasing.

4 - SOURCE POSITION DETERMINATION

In the submarine acoustic application we have made a source position determination using the PRONY-PISARENKO method. With a linear array of sensor the source vector of a point source is :

$$\underline{v} = \begin{pmatrix} z^1 \\ \vdots \\ z^N \end{pmatrix} \qquad z = e^{j\varphi} \qquad \varphi = 2\pi\frac{a \sin\theta}{\lambda}$$

a : sensors separation, λ : wavelength, θ: source azimuth

Going back to the linear subject interpretation we can say that the vectors sources are orthogonal to the noise associated eigenvectors. The scalar product of the noise eigenvector with $\underline{v}$ leads to an algebric equation whose roots give the azimuths of the sources. The results obtained in successive experiments are shown fig. 5. The sources are separated for $\theta > 1°$ but it remains a bias and a dispersion in the direction measurement.

5 - SEPARATION OF THE SIGNALS EMITTED BY EACH SOURCE BY FILTERING

The principle of the method is the following. If $\underline{v_i}(\nu)$ is the i source vector at the frequency ν the source i excitation is given by the scalar product :

$$e_i(\nu) = \langle v_i(\nu) \mid S(\nu) \rangle$$

The product of this excitation with the source vector gives the i source frequencial component at each sensor and the time components are obtained by inverse Fourier Transform. This method leads to a rejection (not complete) of the other sources and to an enhancement of the signal to noise ratio because all the noise contained in the subspace orthogonal to the vector source is eliminated. The results of this method are shown fig. 3 on the natural magnetic signals presented in 2. The filtered signal exhibits an amplitude modulation characteristic of the geophysical phenomena. This amplitude modulation is not appearant before filtering because the signal components are corrupted by additive noise.

6 - CONCLUSION

The spectral matrix is a very useful and important tool in N components signal processing. We have shown in real situations that, from the spectral matrix estimate it is possible to :

- detect the number of sources
- identify the sources
- separate the signals emitted by each source

This kind of treatments are being developed but a lot of problems stay unsolved. New results can also issue from the development of new spectral matrix estimators using parametric methods like the autoregressive (AR) one.

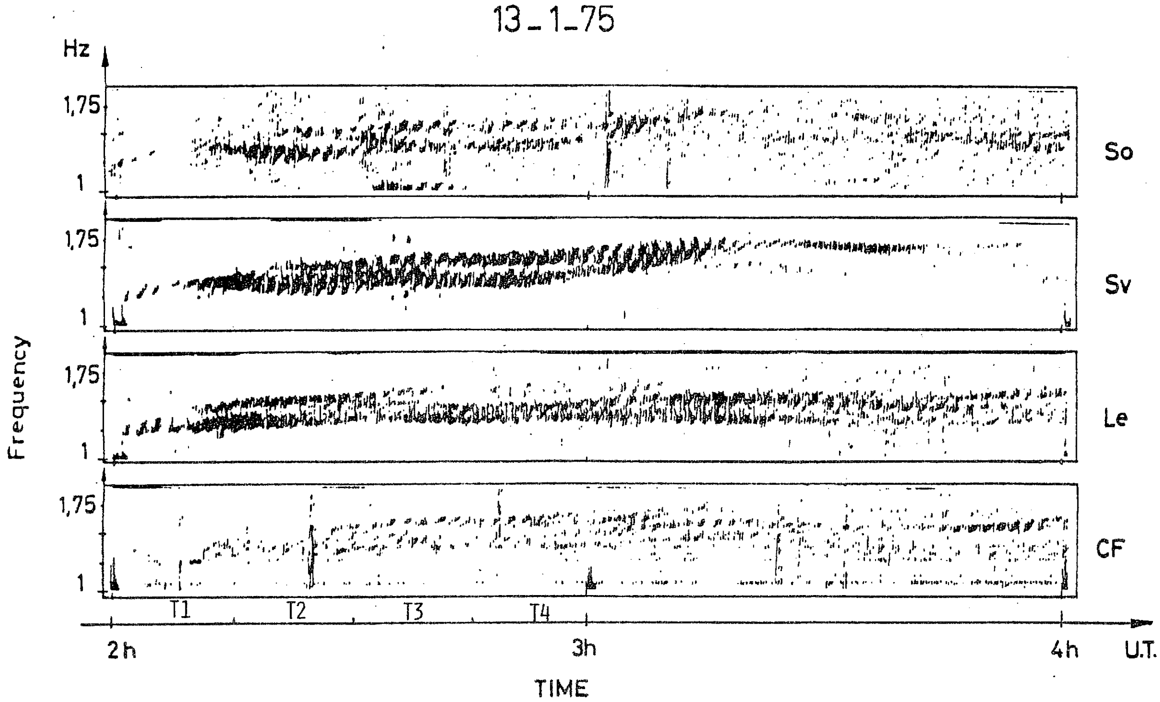

FIGURE 1 : ENERGY DISTRIBUTION OF MAGNETIC PULSATIONS IN TIME-FREQUENCY

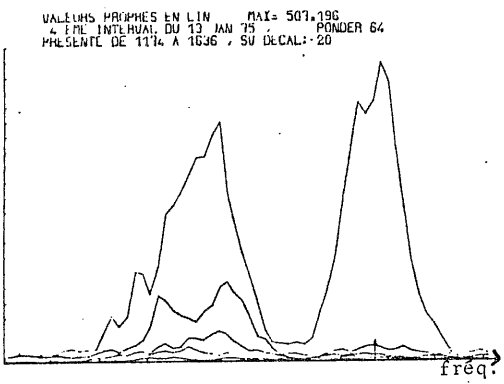

FIGURE 2 : EIGENVALUES OF THE MAGNETIC PULSATIONS SPECTRAL MATRIX

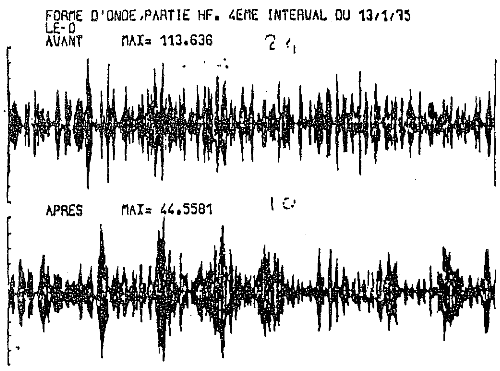

FIGURE 3 : MAGNETIC PULSATION BEFORE (AVANT) AND AFTER (APRÈS) SPATIAL FILTERING

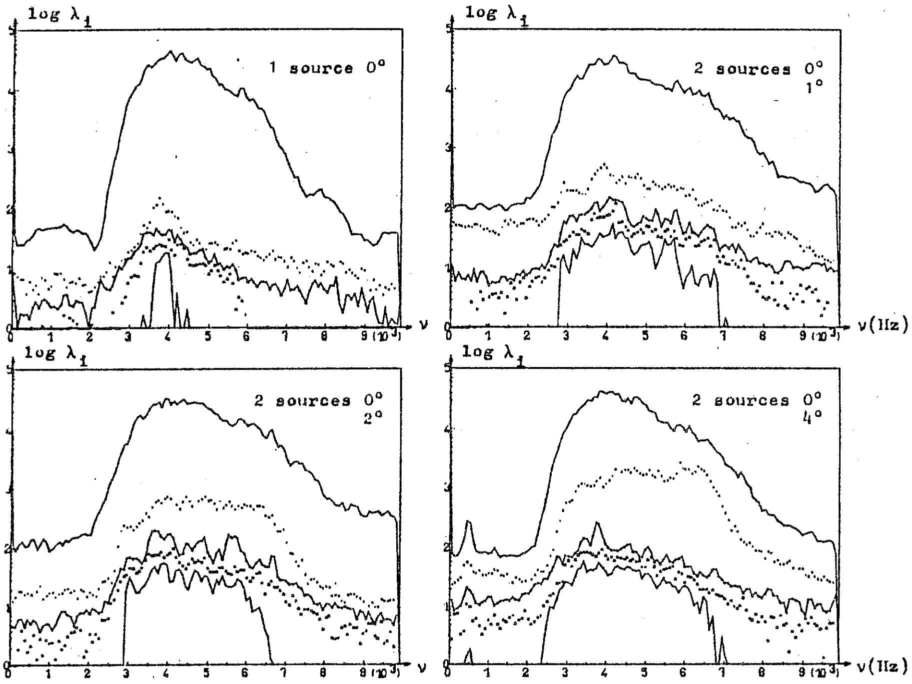

FIGURE 4

EIGENVALUES OF THE SPECTRAL MATRIX IN THE SUBMARINE ACOUSTIC EXPERIMENT

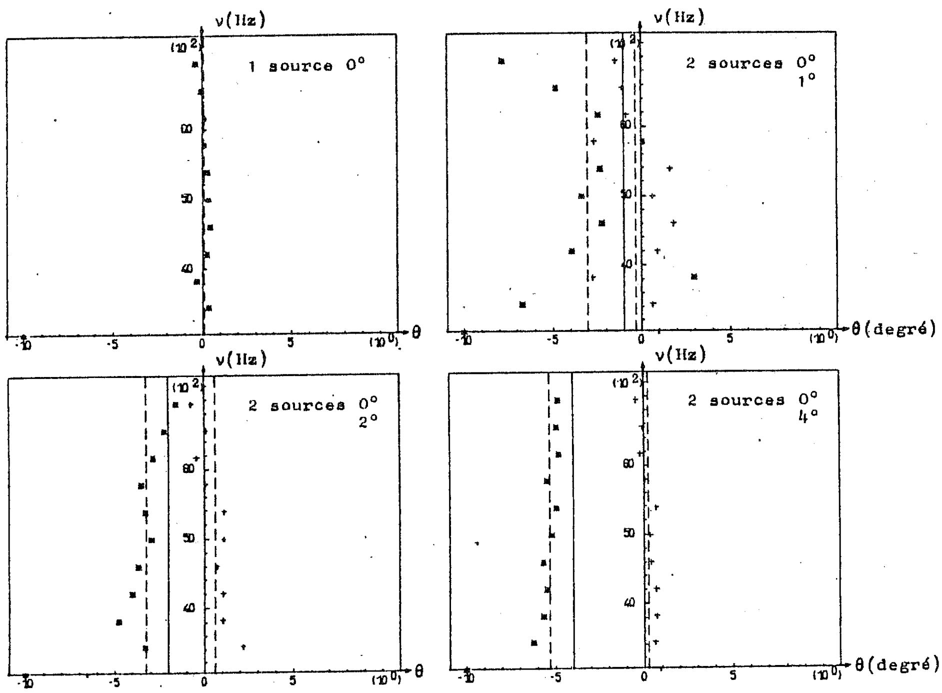

FIGURE 5

SOURCE AZIMUTH (- THEORETICAL, * + MEASURED) IN SUCCESSIVE EXPERIMENTS

REFERENCES

[1] J.L. LACOUME - F. GLANGEAUD - P. LORENZINO - D. BAUDOIS - G. PRETET
Filtrage de signaux multicomposantes utilisant les corrélations intercomposantes.
Colloque GRETSI, Nice, Juin 1981, pp. 319 à 324.

[2] LRO STOREY - F. LEFEUVRE
Analysis of a wave field in a magnetoplasma. The direct problem.
Geophys. J.R. Astr. Soc. 56, p. 255, 1979.

[3] H. MERMOZ
Imagerie corrélation et modèles.
Annales des Télécommunications, T. 31, n° 1-2, 1976.

[4] J.L. LACOUME - B. BOUTHEMY - F. GLANGEAUD - C. LATOMBE - A. SILVENT
Caractérisation par analyse interspectrale du champ d'ondes reçu sur un réseau de capteurs. Applications.
Colloque GRETSI, Nice, 1978, 80/1-80/7.

[5] J.C. SAMSON - J.V. OLSON
Data adaptive polarization filters for multichannel geophysical data.
Geophysics, Vol. 46-10, Oct 1981, pp. 1423-1431.

[6] R. PRONY
Essaï experimental et analytique.
J. Ecole Polytechnique, (2) 24-74, 1795.

[7] V.F. PISARENKO
The retrieval of harmonics from a covariance function.
Geophys. J.R. Astr. Soc. 33, pp. 347-366, 1973.

[8] G.M. JENKINS - D.G. WATTS
Spectral analysis and its applications.
Holden day, N.Y., 1968.

[9] C. TURCAT (LATOMBE)
Problèmes liés à l'estimation d'une matrice interspectrale.
Colloque GRETSI, 1977.

[10] J.L. LACOUME - C. LATOMBE
Analyse interspectrale.
Colloque GRETSI, 1981, pp. 311 à 318.

[11] R.N. Mc DONOUGH
A note on eigenvector decomposition of correlation matrix.
J. Acoust. Soc. Am., 69 (1), July 1980, pp. 165-166.

[12] B. BOUTHEMY
Etude de deux méthodes de caractérisation d'un champ d'ondes.
Thèse de Docteur-Ingénieur, Institut National Polytechnique de Grenoble, Nov. 1981.