Proceedings of The Institute of Acoustics

# SUBSTRUCTURING TECHNIQUES FOR THE VIBRATION ANALYSIS OF LARGE STRUCTURES

J.L. WEARING DEPARTMENT OF MECHANICAL ENGINEERING, UNIVERSITY OF SHEFFIELD
C. PATTERSON DEPARTMENT OF MECHANICAL ENGINEERING, UNIVERSITY OF SHEFFIELD

## Abstract

The analysis of large complex engineering structures for the determination of their natural frequencies and mode shapes requires a large number of degrees of freedom. A substructuring technique for the reduction of the of the number of degrees of freedom is discussed in this paper. A brief discussion of substructuring methods and a description of the technique known as modal synthesis is presented in the paper, followed by results of a typical calculation

## Introduction

Prior to the advent of the digital computer one of the main emphasis in the determination of the natural frequencies of engineering structures was the minimization of the number of hand calculations. This meant that only a few natural frequencies and mode shapes could be considered and in many cases did not provide sufficient information for the engineer.

With ever increasing computing power the natural frequencies of the extremely complex structures, which occur in the many branches of Mechanical and Civil Engineering, can be determined and the finite element method has proved to be extremely effective in this context. It is often necessary, however, in such analysis to consider a large number of degrees of freedom to obtain the detailed displacement information, which is necessary, for the calculations. The direct use of techniques like the finite element method can lead, therefore, to mass and stiffness matrices of large order. The result is long and costly computer calculations yielding in many cases natural frequencies and mode shapes in excess of the number required for practical purposes. It is necessary, therefore, to adopt a technique which can cope with complex structures, be economical in computing costs and produce a sufficient number of natural frequencies.

Various researchers have focussed attention on the problem of the analysis of large structures and the common element in all approaches is to subdivide the structure into a number of substructures and obtain equations of motion for the complete structure from the substructural information.

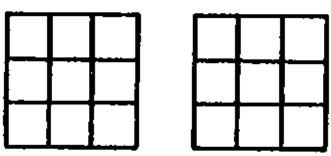

Fig 1. Finite Element and Substructural Subdivision

SUBSTRUCTURING TECHNIQUES FOR THE VIBRATION ANALYSIS OF LARGE STRUCTURES

One of the more widely used approaches in substructuring employs the finite element method and Guyan (1) reduction to obtain stiffness and mass matrices of the complete structure. In this approach, substructures are selected and reduced mass and stiffness matrices are obtained for a limited number of master degrees of freedom using the mass and stiffness matrices of the complete finite element subdivision of the substructure. These reduced substructural mass and stiffness matrices are then combined to give mass and stiffness matrices for the complete structure of a lower order than from the initial finite element model. (See Figure 1.)

Gladwell (2) discussed the approach known as branch mode analysis which replaces the original elastic system by a lumped parameter model. A system of constraints is then imposed in turn on the body only allowing one section to vibrate. The natural frequencies and mode shapes of each branch are determined and the results combined to analyse the complete structure.

The method known as modal synthesis for the vibration analysis of large structures appears to have been proposed initially by Harty (3). In this approach, the structure is subdivided into a number of substructures which are analysed separately to give a set of displacement shapes. With the current widespread availability of computer packages, it is advantageous to analyse the substructure by the finite element method at this stage. The substructural displacement shapes are then used to synthesize a set of generalized coordinates for the analysis of the complete structure which contain at this stage a manageable number of degrees of freedom.

The modal synthesis techniques, therefore, allows large structures to be analysed using relatively few degrees of freedom and thus reduces pressure on computational facilities. A further advantage occurs at the design stage by allowing modifications to substructures without effecting other parts of the structure. This is particularly advantageous in the production of large structures which are often manufactured in stages, at different times or by different contractors.

## The Modal Synthesis Method

In the analysis the structure is divided initially into a number of substructures as shown in figure 1. For each substructure two types of displacements are defined. These are attachment modes and fixed attachment normal modes. Attachment modes are elastic deformations produced by applying, in turn, unit deformations at each modal degree of freedom along boundaries common with other substructures. Fixed attachment normal modes are elastic deformations produced when common boundaries are fixed. Hence the deflection at any point in the substructure is given by

$$\mu = \mu_a + \mu_f \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

and for all points the displacement rector is

$$\underline{\mu} = \underline{\mu_a} + \underline{\mu_f} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

The deflections $\underline{u}$ are the modal deflections of the finite element mesh and these can be related to a set of substructural generalized coordinates $\underline{p}$ by the expression

$$\underline{u} = \underline{\underline{\phi}}\, \underline{p} \qquad \ldots \qquad \ldots \qquad (3)$$

$$\text{or} \quad \underline{u} = \underline{\underline{\phi}}_n \underline{p}_a + \underline{\underline{\phi}}_f \underline{p}_f \qquad \ldots \qquad \ldots \qquad (4)$$

The analysis now requires the derivation of substructural equations of the form

$$(\underline{\underline{k}}_p - \omega_n^2 \underline{\underline{m}}_p)\, \underline{p} = o \qquad \ldots \qquad \ldots \qquad (5)$$

To obtain the generalized mass and stiffness matrices $\underline{\underline{m}}_p$ and $\underline{\underline{k}}_p$ in equation (5) the finite element method is used to determine complete mass and stiffness matrices $\underline{\underline{m}}$ and $\underline{\underline{k}}$ for a completely free substructure. These matrices are partitioned to give for fixed common boundaries mass and stiffness matrices $\underline{\underline{m}}_f$ and $\underline{\underline{k}}_f$ which are used in the expression

$$(\underline{\underline{k}}_f - \omega_n^2 \underline{\underline{m}}_f) \quad \phi_f = o \qquad \ldots \qquad \ldots \qquad (6)$$

to determine the fixed attachment normal modes $\underline{\phi}_f$ of the substructure.

Having obtained the matrix equation (6) for each substructure, they are combined to give a set of equations of the form

$$(\underline{\underline{K}} - \omega_n^2 \underline{\underline{M}})\, \underline{q} = o \qquad \ldots \qquad \ldots \qquad (7)$$

in which $\underline{\underline{K}}$ and $\underline{\underline{M}}$ are the stiffness and mass matrices of the complete structure and $\underline{q}$ is the vector of generalized displacements for the whole structure.

## Results

The method was used to determine the natural frequencies of a steel cantilevered plate having a length of 12 in, a breadth of 6 in, and a thickness of 0.25 in. The results are shown in table 1 and compared with experimental results a full finite element analysis and a Rayleigh-Ritz analysis.

SUBSTRUCTURING TECHNIQUES FOR THE VIBRATION ANALYSIS OF LARGE STRUCTURES

Table 1. Natural Frequencies of Cantilevered Plate

| Mode Shape | Natural Frequencies (Hz) | | | |
|---|---|---|---|---|
| | Experimental | Finite Element | Rayleigh-Ritz | Substructure |
| 1/0 | 58 | 57.5 | 59.5 | 57.5 |
| 2/0 | 359 | 362 | 373 | 375 |
| 3/0 | 1003 | 1035 | 1046 | 1057 |
| 1/1 | 256 | 268 | 252 | 269 |
| 2/1 | 820 | 865 | 831 | 885 |

## Conclusions

The results shown in table 1 using the substructuring technique compare favourably with those obtained by other methods. This means that satisfactory natural frequencies can be calculated for a reduced number of degrees of freedom and with considerably reduced computing costs.

## References


1. R.J. GUYAN 1965, A.I.A.A. Journal 3, 380. Reduction of Stiffness and Mass Matrices.
2. G.M.L. GLADWELL 1964. Journal of Sound and Vibration 1, 41-59, Branch Mode Analysis of Vibrating Systems.
3. W.C. HURTY 1965, A.I.A.A. Journal 3, 678-685. Dynamic Analysis of Structural Systems Using Component Modes.