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# Finite amplitude effects in marine sediments

Jens M. Hovem*

Applied Research Laboratories, The University of Texas at Austin

Abstract

This study attempts to identify the possible sources for nonlinearity in marine sediments and to assess their relative importance. The medium is considered as composed of solid grains in a skeleton frame with fluid filled pores and all the components may have nonlinear stress-strain relations. It is shown that the acoustic parameter of nonlinearity (B/A) can be expressed as a weighted sum of contributions from the pore fluid, the solid, and the frame. Calculations of B/A for sand show that the most important contribution comes from the fluid but also with significant contributions from the bulk and shear moduli of the frame.

## Introduction

Since important applications of nonlinear acoustics involve interaction with the ocean bottom, the nature and degree of the acoustic nonlinearity of marine sediments is a topic of interest. This subject has been treated to some extent by Bjørnø,[1,2] who measured the nonlinearity parameter B/A for saturated sand, using the same technique as normally used for fluids, and also discussed some effects of nonlinearity on propagation.

The common way of measuring the nonlinearity parameter for a fluid is based on a transformation by Beyer,[3] further discussed by Coppens et al.[4] This gives the nonlinearity parameter B/A in the form of sound speed (c) derivatives with respect to pressure (p) and temperature (T) evaluated at the ambient conditions

$$B/A = 2\rho_o c_o \left(\frac{\partial c}{\partial p}\right)_{T,\rho=\rho_o} + \frac{2\beta T c_o}{c_p}\left(\frac{\partial c}{\partial T}\right)_{p,\rho=\rho_o} \qquad (1)$$

where $\rho$ is the density, $\beta$ is the isobaric coefficient of volume expansion, and $c_p$ is the specific heat at constant pressure.

Marine sediments can be considered as porous media composed of solid grains in a skeleton frame and saturated with a fluid. The stress-strain relations of such a composite medium are considerably more complicated than for a fluid. The passage of an acoustic wave both affects the pore pressure on the fluid and grains and changes the effective pressure on the skeleton frame. Thus, all the components will, in general, have nonlinear stress-strain relations and therefore give contributions to the nonlinearity parameter.

*On leave from Electronics Research Laboratory, The University of Trondheim, Trondheim, Norway.

In the present report we will briefly review some of the elastic theory for a homogeneous porous solid and then use this theory to derive an expression for the nonlinearity parameter. Finally, using previously reported values for the elastic properties of saturated sound, we will calculate values for B/A.

In order to simplify the development we will assume that the fluid does not move relative to the solid frame. This assumption is justified in any case for materials of low permeability like clay and silt but can be questioned for high permeability sand where, at least for high frequencies, there might be significant fluid movement relative to the frame.[5,6] The same approach as outlined in this paper can also be used when the fluid movement is taken into account; however, the increased effort does not seem justified at this point. We will also assume that the second part, the temperature part, of Eq. (1) can be neglected when it is compared with the pressure part. This is the case for most fluids. It is also the result of Bjørnø's experiment and is further confirmed by other investigations of sound speed variations with temperature.

## Elastic Properties of Marine Sediment

A porous medium can be considered as mineral grains in a skeleton frame and saturated with a pore fluid.[8] The bulk modulus K of the composite medium will have three components: the bulk modulus of the water $K_w$, the bulk modulus of the solid grains $K_s$, and the bulk modulus $K_f$ of the frame. In addition, there is also a shear modulus $\mu$ for the frame.

Consider a volume V of which $V_s$ is occupied by solid and $V_w$ by water; the porosity is $\phi = V_w/V$. A pressure increase p can be separated into a pore pressure $p_p$ that is effective on the water and on the individual grains and a differential or effective pressure $p_d$ acting on the frame. The relative change in volume can then be expressed as

$$-\frac{\Delta V}{V} = K_s^{-1}\, p_p + K_f^{-1}\, p_d \quad . \tag{2}$$

Equation (2) can be considered as the definition of $K_f$. The change in volume is the sum of the volume changes in the solid $\Delta V_s$ and the water $\Delta V_w$. The change in solid volume $\Delta V_s$ has two components, one due to pore pressure on the grains, the other due to the differential pressure acting on the frame. The total volume change is therefore

$$-\frac{\Delta V}{V} = \phi K_w^{-1}\, p_p + (1-\phi) K_s^{-1}\, p_p + K_f^{-1}\, p_d \quad . \tag{3}$$

With $p = p_p + p_d$, the combination of Eqs. (2) and (3) gives the pore pressure and the differential pressure as

$$p_p = p\,\frac{Q}{Q + K_f} \quad , \tag{4}$$

$$p_d = p\,\frac{K_f}{Q + K_f} \quad , \tag{5}$$

where

$$Q = \frac{K_w \left(K_s - K_f\right)}{\phi \left(K_s - K_w\right)} \quad . \tag{6}$$

The bulk modulus of the composite material is defined by

$$K = \left(-\frac{dV}{V}\right)\left(\frac{1}{p}\right) , \tag{7}$$

and therefore is found to be

$$K = K_s \frac{K_f + Q}{K_s + Q} , \tag{8}$$

and the compressional sound velocity is

$$c = \left[\left(K + \frac{4}{3\mu}\right) \Big/ \rho\right]^{1/2} , \tag{9}$$

where the density $\rho$ is the weighted density of the water $\rho_w$ and the solid $\rho_s$,

$$\rho = \phi\rho_w + (1 - \phi)\rho_s \quad .$$

The Nonlinearity Parameter

We will now derive an expression for the nonlinearity parameter B/A using only the first, pressure dependent part of Eq. (1). Using Eq. (9) this gives

$$B/A = \left(\frac{\partial K}{\partial p} - 1\right) + \frac{4}{3}\left(\frac{\partial \mu}{\partial p} - \frac{G}{K}\right) \quad . \tag{10}$$

As shown, the total pressure can be separated into a pore pressure acting on the fluid and the solid grains, and a differential pressure acting on the frame. Since $K_f$ and $\mu$ are independent of $p_p$, and $K_w$ and $K_s$ are independent of $p_d$, Eq. (10) can be expressed as

$$B/A = W_w \frac{\partial K_w}{\partial p_p} + W_s \frac{\partial K_s}{\partial p_p} + W_f \frac{\partial K_f}{\partial p_p} + W_\mu \frac{\partial \mu}{\partial p_p} - 1 \quad . \tag{11}$$

In Eq. (11) the nonlinearity parameter is expressed as a weighted sum of four contributions from the nonlinear stress-strain relations of the water, the solid grains, and the frame. The weighting factors are found to be:

$$W_w = \left(\frac{Q}{Q+K_f}\right)\left(\frac{K_s}{Q+K_s}\right)^2\left(\frac{\phi Q}{K_w}\right)^2 \frac{1}{\phi} \quad , \tag{12}$$

$$W_s = \frac{Q\left(Q+K_f\right)}{\left(Q+K_s\right)^2} - \frac{QK_s\left(K_s-K_f\right)}{\left(Q+K_s\right)^2\left(K_s-K_f\right)} \quad , \tag{13}$$

$$W_f = \frac{K_f\,K_s^{\,2}}{\left(Q+K_f\right)^2\left(Q+K_s\right)^2} \quad , \tag{14}$$

$$W_\mu = \frac{4}{3}\,\frac{K_f}{Q+K_f} \quad . \tag{15}$$

At this point numbers are needed to estimate the value of the different contributions to B/A. For water, $K_w \approx 2\times10^{10}$ dyn/cm$^2$ and $(\partial K_w/\partial p_p)\approx 6$. For the mineral grains we may use the values for quartz given by Soga,[9] $K_s = 3.8\times10^{11}$ dyn/cm$^2$ and $(\partial K_s/\partial p_p) = 6.4$. Values for $K_f$ and $\mu$ and their derivatives can be determined from measurement of compressional and shear wave velocities in dry sediment as functions of differential pressure. In practice this is difficult to do with silt and clay and it appears that values exist only for sand.[8,10] For estimating the nonlinearity parameter value $K_f \approx \mu = 6\times10^8\times p_d^{\,0.5}$ with $p_d$ in dyn/cm$^2$. With $p_d = 10^6$ dyn/cm$^2$ and a porosity of $\phi = 0.37$, we find the values of Table I for the different terms of Eq. (11).

| | $W_w$ | $W_s$ | $W_f$ | $W_\mu$ | |
|---|---|---|---|---|---|
| | 2.2 | $1.9\times10^{-2}$ | $7.8\times10^{-3}$ | $1.4\times10^{-2}$ | |
| | $\partial K_w/\partial p_p$ | $\partial K_s/\partial p_p$ | $\partial K_f/\partial p_d$ | $\partial \mu/\partial p_d$ | |
| | 6 | 6.4 | 300 | 300 | |
| Product | 13.2 | 0.1 | 2.3 | 4.2 | 19.8 |
| | | | | | Sum |

TABLE I
NUMERICAL VALUES FOR THE TERMS OF EQUATION (11)

This gives B/A≈19 with the most important contribution coming from the pore fluid but also with significant contributions from the bulk and shear moduli of the frame. Repeating the calculation for higher values of $p_d$, which corresponds to higher overburden pressure, gives almost the same value for B/A. This is because the decrease in the derivatives of $K_f$ and $\mu$ is almost balanced by a corresponding increase in the weighting factors $W_f$ and $W_\mu$.

Using Eq. (1) as the basis, Bjørnø found values for B/A for sands of different porosities. He found the temperature part to be insignificant and the pressure part to be dependent on the porosity. It appears, however, that Bjørnø measured sound velocity as a function of pore-pressure only and therefore observed effects of only the two first terms of Eq. (11). Furthermore, since the differential pressure probably was very low and we have found that the contribution from the solid is small, we should expect Bjørnø's result to be given by the first term of Eq. (11), which, with $k_f = \mu = 0$, reduces to

$$B/A = \left(\frac{\phi K_s}{\phi K_s + (1-\phi)K_w}\right)^2 \frac{1}{\phi} \frac{\partial K_w}{\partial p_p} - 1 \,. \tag{16}$$

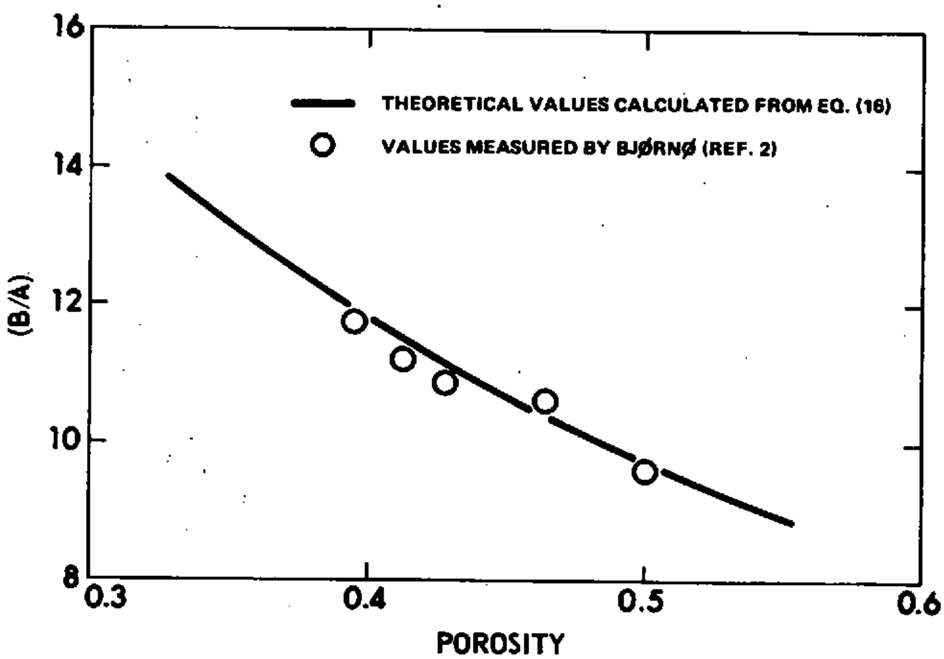


**FIGURE 1**
**NONLINEARITY PARAMETER (B/A) FOR SATURATED SANDS OF DIFFERENT POROSITIES (FLUID PARTS ONLY)**

Figure 1 shows a plot of B/A as a function of porosity, using the previously given values for $K_s$ and $K_w$, together with Bjørnø's experimental results. The very good agreement gives some support to the theory which has been developed here.

## Conclusions and Comments

Assuming a homogeneous porous medium, it has been shown that the acoustic nonlinearity parameter (B/A) can be expressed as a weighted sum of contributions from the nonlinear stress-strain relations of the fluid, the solid, and the frame. For sand we find B/A = 19, with the most important contribution coming from the pore fluid. Even if this value is considerably higher than for water, the importance of sediment nonlinearity to acoustic propagation is believed to be small.[6] On the other hand, it may be that direct measurement of nonlinearity in sediments, for instance, by distortion measurement, could give additional valuable information about the acoustic properties in general.

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