STABILITY CRITERIA FOR PLATFORMS FOR MULTI-BEAM SIDESCAN SONARS

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1. INTRODUCTION

To acquire useful sonar images, the sonar platform must be sufficiently stable. Precisely how stable depends on the image processing technique, on sonar parameters such as range and beamwidth, and on the scale of detail sought in the images. In particular, quite different stability criteria apply to the waterfall plot generated in real time and to processed images.

In a waterfall plot, the sonar image is displayed on the shipboard recorder with each trace parallel to the previous trace. No processing is done except possibly water-column removal (which corrects for towfish heave). If image processing is used, distortions which may be present in the waterfall plot can be corrected, so more platform motion can be tolerated. If each trace is tagged with the towfish attitude and position (TAP) data for the instant at which it was acquired, the traces can later be overlaid on a geographic reference frame, with averaging as necessary. This geocoding technique could compensate for towfish motions which would result in waterfall images containing effects such as duplicated returns, which stretches images. Other processing techniques, for example correction for towfish sway by adjusting the range at which the return is displayed, are also in use. Geocoding uses all the TAP coordinates. Even with geocoding, though, the stability of the towfish must satisfy some criteria, while obtaining useful images for immediate display requires more demanding criteria.

The criteria appropriate to each level of image processing are displayed in Table 1. A key concept is that the criteria are based on towfish motion in a short period, not the overall size of yaw or pitch, for example. If the attitude is nearly constant over the time required to image an object, the only effect is that the location of the object may not be where the operator believes it to be if he assumes yaw and pitch to be zero. If attitude changes significantly while imaging an object, the image will be distorted because neighbouring returns (for single-beam sonars) or blocks (for multi-beam sonars) will not align properly. In some cases, such as the target on the outside of a yaw, and with somewhat larger rates of change, there may be gaps in coverage. These distortions will be removable by geocoding if the attitude changes slowly enough during the round-trip travel of the sound energy. If changes are more rapid than that, there will not be enough overlap between the transmitted and received beams and insufficient energy will be received.

TABLE 1

Stability Criteria

Criterion	Towfish Motion Allowed	Criteria Chosen to Ensure	Useful Image Requires Image Processing
Strongest	Least	Useful Image with Acceptable Distortion	No
		Useful Image without Coverage Gaps	No
Weakest	Most	Adequate Energy Return	Yes

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The analysis in this paper is directed primarily to high-resolution sidescan sonars on towfish. While the concepts are generally applicable, four considerations are particular to this case:

- Sidescan sonars have broad vertical beams, which makes towfish roll less important than other motions. Roll affects images through the vertical beam pattern, but this is not considered here.
- In a sonar system which employs beamforming, real-time correction for attitude changes is possible by altering the time or phase delays in accordance with measured attitude. The stability criterion for such a sonar would contain no constraints on attitude except those imposed by the limitations of the correction system. The analysis given below still applies to such a sonar, because there remain constraints on velocity and transverse displacement, but in applying the equations below to such a system, all attitude variations should be set to zero. On a towfish, however, dynamic beam steering is less useful than one might expect because the orientation of the array allows only yaw variations to be corrected directly. Also, in a high-resolution system, only modest yaw angles can be accommodated because of grating sidelobes.
- The beamforming process itself imposes a theoretical stability limit in that the difference in arrival times at the ends of the array, for a sonar return from some angle, must not be significantly affected by motion of the array. For high resolution sonars of a size suitable for a towfish, it can be shown that these criteria are far less demanding than those described above.
- The velocity of the towfish must not produce a Doppler shift which places the returning energy outside the bandwidth of the receiving filters. Because very short pulses are used in high resolution sonar imagery, the bandwidths are large enough that this is rarely a constraint.

2. CRITERION FOR GEOCODED IMAGES

If geocoding is used to remove distortions, the stability criterion is that there be adequate overlap between the transmitted and received beams. This amounts to a criterion for towfish motion during the round trip of the ping. If the same transducer is used for transmission and receipt, the criterion depends only on beam pattern and range. In multi-beam systems, the offset angle between the transmit beam and each receive beam must be considered.

2.1 Geometry of Towfish Motion

A towfish moving along the x axis at a mean velocity v can be considered to be stationary, with the sea bottom moving at mean speed -v. In that coordinate system, the x-coordinate of the velocity of the centre of the footprint of the beam at range r is -v - dx/dt, where x is the displacement parallel to the axis due to yaw, pitch and surge. The angle between the transmitted beam and the axis of the receive beam at time of receipt is the net displacement divided by the range. Maintaining an overlap of at least the fraction f between the beam footprints at the instants of broadcast and receipt imposes a limit on that angle, namely:

$$\int_{-r/c}^{r/c} \mathrm{MAX}\left[\,\left|\frac{dx}{dt} + v\right|\,\right] dt \; < \; r\,\theta_{max}(f, \theta_o) \qquad (1)$$

where MAX means to select m (defined below) to maximize the integrand. The time between the sonar pulse being transmitted and received is 2r/c for range r and sound speed c. The limiting angle θ_{max} is a function of the criterion for acceptable fractional decrease in the intensity in a receive beam, f, and the initial offset angle of that beam, θ_o. Note that because of the large vertical extent of the side-scan beam, overlap perpendicular to the towfish axis is assured, and we are concerned only with overlap parallel to the towfish.

The relationships in a summary (1) of the geometry of image formation by a towfish can be used to put this criterion into a useful form. The distance in the image plane parallel to the towfish course and due to yaw and pitch alone is given in Eq. 6 of Reference 1:

$$x = -m \sin\alpha \left[r^2 - h^2 \sec^2\gamma\right]^{1/2} + h \cos\alpha \tan\gamma \qquad (2)$$

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where h is the towfish altitude, α and γ are the towfish yaw and pitch respectively, and m is +1 for the port sonar channel and -1 for the starboard channel.

High-resolution sonar imagery is characterized by short ranges and thus round-trip transit times of a fraction of a second. Since wave-driven motions have periods of several seconds, it is reasonable to assume constant rates of change of towfish yaw, pitch, and surge during a round trip. Thus

$$x = x(0) + \frac{2r}{c}\frac{dx}{dt} = x(0) + \frac{2r}{c}\left[\frac{\partial x}{\partial \alpha}\frac{d\alpha}{dt} + \frac{\partial x}{\partial \gamma}\frac{d\gamma}{dt} + \frac{\partial x}{\partial t}\right] \qquad (3)$$

where all the derivatives are evaluated at t=0 and the last term is the surge. Equation 2 applies in a coordinate frame which is yawed and pitched from the towfish by the angles α and γ, but since we are considering here only the motion of the beam over a short time, we will choose the orientation of the frame so that $\alpha = \gamma = 0$ at t=0, and therefore x(0) = 0. The partial derivatives have simple forms under these conditions:

$$\frac{\partial x}{\partial \alpha} = -m\left[r^2 - h^2\right]^{1/2} \qquad \frac{\partial x}{\partial \gamma} = h \qquad (4)$$

The criterion, Equation 1, can then be written (since MAX(|a+mb|) = |a| + |b|)

$$\left[r^2 - h^2\right]^{1/2}\left|\frac{d\alpha}{dt}\right| + \left|h\frac{d\gamma}{dt} + v + \frac{\partial x}{\partial t}\right| < \frac{c}{2}\,\theta_{max}(f,\theta_o) \qquad (5)$$

2.2 Maximum Acceptable Angle, $\theta_{max}(f,\theta_o)$

The limiting angle $\theta_{max}(f,\theta_o)$ is that angle between the axes of the transmit and receive beams which reduces the echo intensity to the minimum acceptable fraction, f, of its value with axes coincident. It depends also on any constant angular offset between these axes, θ_o. As an example, Figure 1 shows a transmit beam pattern with kL=100 (L is aperture, λ is wavelength, and $k=2\pi/\lambda$) and a receive beam pattern with kL=400, offset by 0.03 radians. The echo signal would be the integral of the product beam pattern and would be some fraction, f, of the signal with zero shift. In this work, the integrals have been calculated over the larger main lobe and three sidelobes on each side.

Figure 2 is a plot of the overlap fraction against shift angle/FWHM (Full Width at Half Maximum) for a line source used for transmit and receive. Because the independent variable has been divided by FWHM, the plots are almost independent of kL. To use the plot, one first selects the minimum acceptable overlap fraction. A popular choice would be 0.5. For this choice, the maximum acceptable shift angle is then found to be 0.67 to 0.69 of FWHM, depending on the value of kL.

If a separate array is used for receive, another variable must be considered, namely the ratio of the lengths of the receive array and the transmit transducer, L_R/L_T. Figure 3 shows the overlap fraction, with the independent variable being the offset angle divided by the sum of the FWHM of the transmit and receive beams. The longer the receive array compared to the transmitter, the larger the offset angle which can be tolerated before the overlap falls to unacceptable levels. With $L_R/L_T > 5$ or 6, reasonable overlap is ensured for any offset angle. In fact, though, this is an unacceptable design because the separation between grating lobes is less than the width of the transmit beam, as shown in Figure 4.

Figure 3 contains the data necessary for obtaining stability criteria for towfish carrying multi-beam sonars. The perpendicular lines illustrate the process. Each receive beam is characterized by a static offset angle from the array normal (a series of calculations will be needed if this angle is a function of range). For any beam, divide this angle by the sum of the FWHMs to find the initial abscissa and then its intersection with the curve for the relevant value of L_R/L_T. Draw a vertical line of length corresponding to the acceptable decrease in overlap. The length of the horizontal line from this point back to the relevant curve gives the maximum acceptable dynamic offset angle, $\theta_{max}(f,\theta_o)/\Sigma(\text{FWHMs})$.

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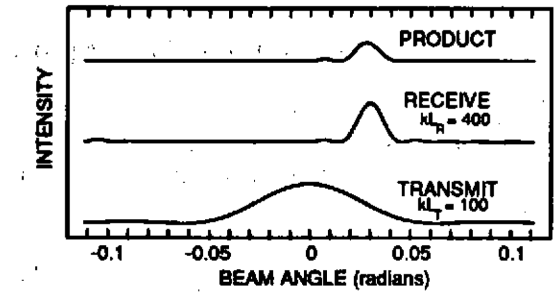

Fig. 1. Beam patterns of a transmit line source with kL=100 and a 10-element receive array with kL=400, shifted by 0.03 radians.

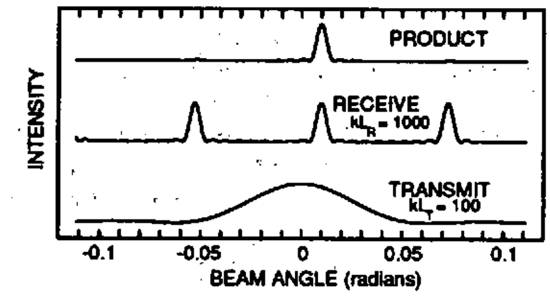

Fig. 4. Beam patterns of a transmit line source with kL=100 and a 10-element receive array with kL=1000, shifted by 0.01 radians. At least one grating lobe overlaps the transmit beam for any offset angle.

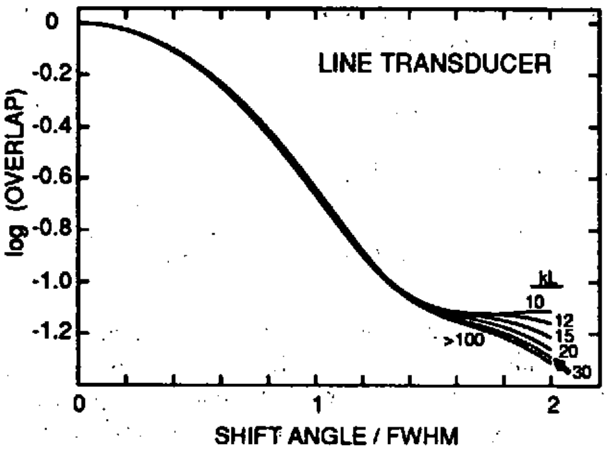

Fig. 2. Overlap fraction as a function of shift angle/FWHM for a line transducer used for transmit and receive. For any kL>100, the plot overlays that shown for kL=100.

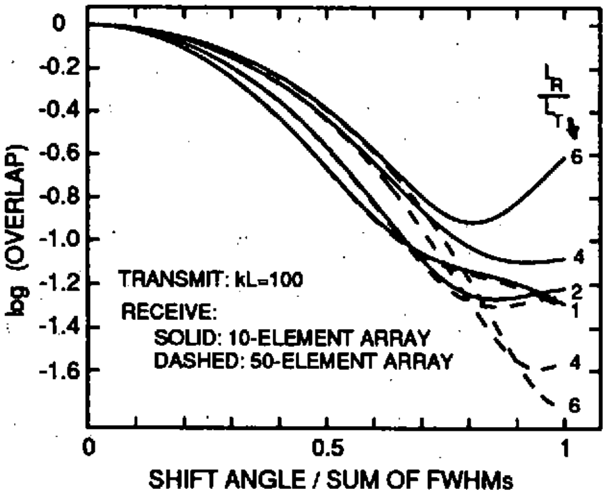

Fig. 5. Overlap fraction as a function of shift angle divided by the sum of the FWHMs for a transmit line source of length L_T and 10- and 50-element receive arrays of length L_R.

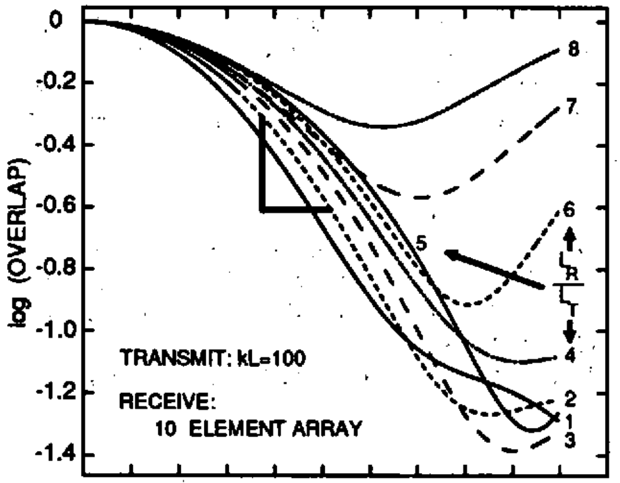

Fig. 3. Overlap fraction as a function of shift angle divided by the sum of the FWHMs for a transmit line source of length L_T and a 10-element receive array of length L_R. The perpendicular lines indicate a method of using the plotted data, as explained in the text.

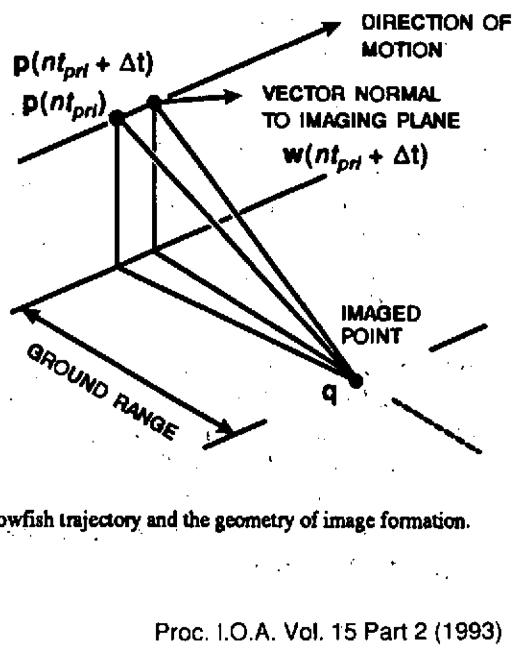

Fig. 6. Towfish trajectory and the geometry of image formation.

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TABLE 2

Maximum Shift Angles for a 50% Decrease in Overlap for Receive Beams which Abut and Fully Occupy the Transmit Beam

Number of Beams[1]	Beam Number[2]	Overlap Fraction No shift	Overlap Fraction Shifted	Angle/Σ(FWHMs) No shift	Angle/Σ(FWHMs) Shifted	$\theta_{max}(f,\theta_0)/\Sigma$(FWHMs)
4	3	.964	.483	.100	.431	.331
	4	.710	.359	.300	.508	.208
5	3	1.00	.501	0.0	.458	.458
	4	.911	.457	.167	.487	.320
	5	.700	.351	.333	.557	.224
6	4	.969	.486	.071	.340	.269
	5	.751	.377	.214	.395	.181
	6	.450	.226	.357	.492	.135

1. Number of receive beams = L_R/L_T. Beams spaced by FWHM.
2. Beams are numbered from left to right, e.g. beam 3 is the centre beam of 5.

As examples, consider receive arrays whose beams fully occupy the transmit beam on a FWHM basis, that is the receive beams are spaced by their FWHM, and the ratio of the transmit FWHM to the receive FWHM is equal to the number of beams. Table 2 contains the maximum shift angle, divided by the sum of the FWHMs, which the towfish could undergo during a ping, $\theta_{max}(f,\theta_0)/\Sigma$(FWHMs), such that the beam overlap decreased by at most 50%. This value and Eq. 5 give limits to changes in towfish attitude for receive beams distributed as in this example.

The calculations above were done for an array with 10 elements. Figure 5 shows that even with 50 elements the results differ but little over most of the range of interest. None of the values in the Table would be different with a 50-element array.

3. CRITERION FOR UNCORRECTED IMAGERY

In an uncorrected image the sonar return is displayed on the shipboard recorder with each trace parallel to the previous trace. The criterion of Section 2 ensures only that traces will be received and that their intensity variations will not be excessive. For the image of a target to be recognizable, more demanding criteria must be satisfied, unless the target is within a beam block, that is, within a set of beams received simultaneously. Changes in towfish yaw, for example, while imaging a target, cause under- or over-sampling, and thus compression or stretching of the uncorrected image. Coverage gaps may be generated. Analysis of these effects is best done with a different approach.

3.1 Locations of Beam Blocks

Every sample in the uncorrected sonar imagery corresponds to a point on the ocean floor which lies at the intersection of the following geometric forms:

a. A plane which contains the peak of the azimuthal pattern of the receive transducer array at the reception instant (transducer centre plane).
b. An ellipsoid with foci which coincide with the position of the projector at the time of transmission of the sonar pulse and the centre of the receive transducer array at the reception instant.
c. A model of the imaged surface (i.e. ocean floor).

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Ideally, the model of the imaged surface would be a representation of the actual shape of the ocean floor referenced to a standard ellipsoidal model of the earth. However, in the absence of information about the actual shape of the ocean floor, it is reasonable for our purposes to represent the imaged surface by a plane. The following equations may be used to determine the location of an imaged point when the position and attitude of the towfish and the round-trip time of the sonar energy are known.

$$\left\{ \mathbf{q} - \mathbf{p}(nt_{pri} + \Delta t) \right\} \cdot \mathbf{w}(nt_{pri} + \Delta t) = 0 \qquad (6)$$

$$\left| \mathbf{q} - \mathbf{p}(nt_{pri} + \Delta t) \right| + \left| \mathbf{q} - \mathbf{p}(nt_{pri}) \right| = c\,\Delta t \qquad (7)$$

$$\mathbf{s} \cdot \mathbf{q} = a \qquad (8)$$

where $\mathbf{p}(t)$ is the position vector of the towfish and $\mathbf{w}(t)$ is the unit vector which is normal to the plane which contains the peak of the azimuthal pattern of the receive transducer array at time t. This unit vector is determined by towfish attitude. The period between pulses is t_{pri}, and thus nt_{pri} is the time of transmission of the n^{th} pulse. Δt is the time between transmission and reception of the sonar energy reflected from a point with position vector **q** and c is the speed of propagation of the sound energy in water. Figure 6 shows these vectors.

Equation 8 is used to model the imaged surface. Here the surface is assumed to be a plane, in any desired orientation, in which case s is a vector of coefficients and a is a scalar. When the ocean floor is represented by a horizontal plane at depth d, this equation reduces to $q_z = d$.

In order to determine the position of the imaged point from the available measurements, it is necessary to express **p** and **w** in a consistent coordinate frame. It is convenient to make use of a coordinate frame centred on the position of the projector at t = 0 with the X-axis aligned with the nominal platform track (i.e. the along-track direction), the Y-axis pointing to the right of the platform (i.e. the across-track direction), and the Z-axis pointing down to complete a right-handed coordinate system. This coordinate frame is referred to as the track-referenced coordinate frame. **w** may be determined by expressing the unit vector normal to the transducer centre plane in the track-referenced coordinate frame by making use of the (assumed known) beam steering (azimuth and elevation) angles and platform attitude (pitch, roll and yaw) angles.

The position of the imaged point may be determined by simultaneously solving Equations 6-8. Iterative methods may be used to provide the required solutions. There are two solutions (corresponding to each side of the platform) and a means of selecting the appropriate solution must be provided. Equations 6-8 enable the position of the imaged point to be determined from the beam steering angles, the position and attitude of the towfish and the round-trip time of the sonar energy. The procedure may be repeated as required to locate the boundaries of the beam blocks on the ocean floor and forms the basis of the simulations. A simple dynamic model of the motion of the platform is used to generate simulated position and attitude measurements.

The beams of a multibeam sonar may be formed in such a way as to produce a rectangular grid of resolution cells within a beam block. The azimuth beam steering angle is continually adjusted with range in such a way as to correctly form the outer beams. A five-beam multibeam sonar of this type has been simulated. The system parameters have been chosen in such a way that there is approximately one beam width of overlap from beam block to beam block when the platform travels at its nominal velocity with zero pitch, roll and yaw. The coverage of the system (in ground coordinates assuming a flat ocean floor) under these nominal conditions is illustrated in Figure 7 which shows the outline of the beam blocks calculated by the method described above. The skewing of the beam blocks which results from the movement of the platform during the time taken to collect the data which constitutes each block can be clearly seen in the figure.

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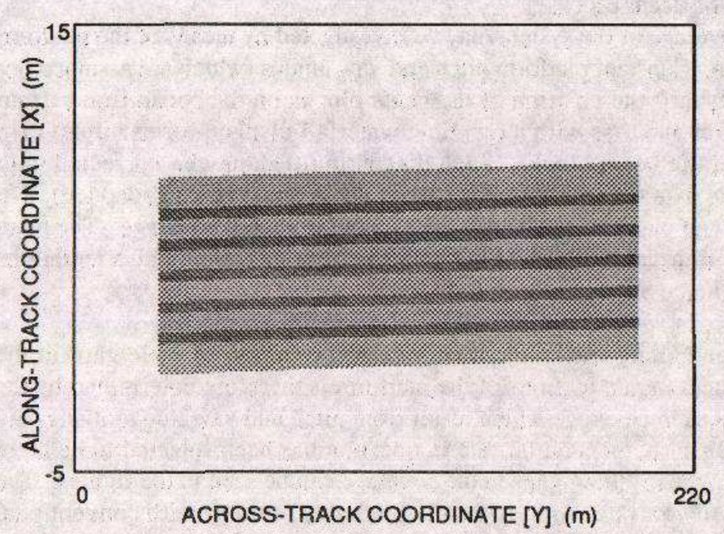

Fig. 7. Sketch of the beam blocks on the ocean floor in track-referenced coordinates with the towfish travelling at nominal velocity with zero pitch, roll, and yaw. Dark shading indicates areas covered by two beam blocks.

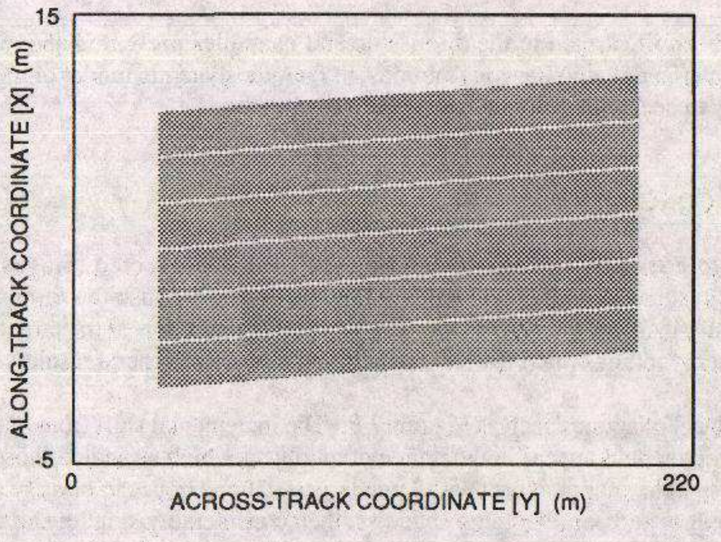

Fig. 8. Effect of pitch rate on the location of the beam blocks on the ocean floor.

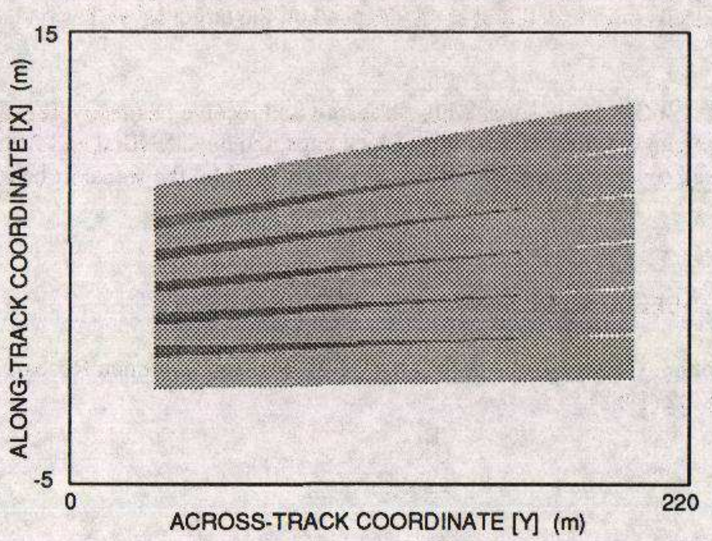

Fig. 9. Effect of yaw rate on the location of the beam blocks on the ocean floor.

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3.2 Effects of Vehicle Motions on the Locations of Beam Blocks
The effect of various platform motions on the coverage of the system may be investigated by means of the platform motion model which forms part of the simulation. Constant platform pitch and yaw angles (which are assumed not to be corrected in any way by the beamformer) shift the position of the beam blocks on the ocean floor. If no corrections are applied to the imagery (as is often the case with a real time waterfall display, for example) then discontinuities may occur at the boundaries of the beam blocks. Such discontinuities may be corrected with knowledge of the platform position and attitude data (as would be the case if the imagery were geocoded). No significant loss of information results from constant platform pitch and yaw except for altered coverage. The main consequence of platform roll is to displace the transducer illumination patterns from their nominal orientations which will result in a change (usually reduction) in the Signal to Noise Ratio.

Pitch and yaw rate motions and across-track and along-track velocities can lead to unrecoverable gaps in the coverage of the sonar. The tolerance of the system to rate motions of the platform is therefore determined by the extent of the overlap of the beam blocks. The gaps in coverage which result from pitch and yaw rate motions may be seen in Figures 8 and 9 respectively. In each case, the angular rate in question has been selected to be large enough to exceed the tolerance of the system and the resulting gaps in the coverage can be seen in the figures. The imaging system will usually be more sensitive to yaw rate motions than to pitch rate motions with conventional imaging geometries.

The effect of individual platform motions has been illustrated in the discussion and examples presented above. However, it is straightforward to demonstrate the effect of simultaneous motions, in fact, the dynamic model of the platform motion may be as elaborate as necessary to deal with the anticipated platform motions.

4. CONCLUSIONS

Stability criteria for processed images would normally be less demanding than for uncorrected images. Comparisons without consideration of specific systems and targets is not possible, however, because the former depends only on the beam footprint while the latter includes the size, shape, and acceptable distortion of the target. Only for processed images is there a criterion, energy return, which leads to analytic and numerical general results.

For uncorrected images, simulations give beam-block diagrams such as Figures 7-9. The incremental shift from one beam block to its neighbour is the distortion which would appear in the image of an object which straddled those blocks. Gaps between blocks would give unacceptable images except with large targets. If one hopes to classify a target as man-made by the presence of straight edges and parallelogram shadows, then considerably smaller shifts would be unacceptable. Thus these criteria are indeed complicated in that they depend on the target as well as on the sonar.

As an example, consider a very high-resolution (0.1^{o}) five-beam sonar whose transmit and receive beams overlap as in the example in Table 2. For motions which just satisfy criteria for no inter-block gaps, the outside (first and fifth) beams would be reduced by almost 50% compared to smooth motion. If this is not acceptable, the transmit beam should be broadened.

5. REFERENCES

1. "Coordinates as Determined by Side Scan Sonar - Theory and Application", J.M. Preston, Defence Research Establishment Pacific Technical Memorandum 88-02 (1988).