SELF-FOCUSED ULTRASONIC IMAGING RECONSTRUCTION TECHNIQUES

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INTRODUCTION

Imaging systems in the near field, based on ultrasonic arrays, can readily be achieved if a knowledge of the object plane is assumed. This prescribed distance allows focusing based on computational or electronic processing to be carried out. When no prior knowledge of the position of the object plane is available the field intensity across a number of different planes can be computed by back propagation of the recorded wavefront. A search can then be made for a focused image on the basis of maximum rate of change of the image, which is assumed to be associated with a well defined object [1].

A mathematical solution to the problem of self-focusing has been reported (Sepehr et al 1982 [2]) which enables reconstruction of the image of an object to be obtained solely from a measurement of the phase and amplitude of the wavefront in a plane at an unknown distance from the object. This mathematical solution relies for its justification on the application of the Fresnel integral and assumes uniform temporal phase in the object plane.

A second analysis of the system based on the relationship between the spatial frequency spectrums in the object and sampling planes has been developed. This justifies the original algorithm for the limited case of uniform temporal phase in the object plane, and confirms the inherent nature of the π radians spatial phase ambiguity introduced by the self-focusing algorithm.

SELF-FOCUSING IMAGE RECONSTRUCTION

For self-focused reconstruction an equation is required which enables the object wave to be computed without knowledge of the distance between the object and sampling planes. The pressure waves at the sampling and object planes, p(x,y,z) and p(x,y,o), can be related using the Fresnel integral with the obliquity factor omitted by:

$$p(x,y,z) = \frac{1}{j\lambda} \iint_{-\infty}^{\infty} p(x,y,o) \frac{\exp(jkr)}{r} \, dx \, dy \tag{1}$$

where $r = \sqrt{z^2 + (x-u)^2 + (x-v)^2}$ and $k = \frac{2\pi}{\lambda}$

The object wave p(x,y,o) is in general a complex quantity, but in order to obtain a mathematical solution to the self-focused reconstruction problem it is necessary for the object wave to be a real quantity. This implies a uniform phase distribution across the object plane. With this limitation equation (1) may be expanded into the real part p_1 and imaginary part p_2 as:

$$p_1(x,y,z) = \iint_{-\infty}^{\infty} p(u,v,o) \frac{\cos\left(k\sqrt{z^2 + (x-u)^2 + (y-v)^2} - \frac{\pi}{2}\right)}{\lambda\sqrt{z^2 + (x-u)^2 + (y-v)^2}} \, du \, dv \tag{2}$$

and

$$p_2(x,y,z) = \iint_{-\infty}^{\infty} p(u,v,o)\, \frac{\sin\left(k\sqrt{z^2 + (x-u)^2 + (y-v)^2} - \frac{\pi}{2}\right)}{\lambda\sqrt{z^2 + (x-u)^2 + (y-v)^2}}\, du\, dv \qquad (3)$$

where u and v represent the variables x, y, in the object plane.

Both the equations may be recognised as two dimensional convolution integrals and written as:

$$p_1(x,y,z) = p(x,y,o) * c(x,y) \qquad (4)$$

$$p_2(x,y,z) = p(x,y,o) * s(x,y) \qquad (5)$$

where

$$c(x,y) = \frac{\cos\left(k\sqrt{z^2 + x^2 + y^2} - \frac{\pi}{2}\right)}{\lambda\sqrt{z^2 + x^2 + y^2}} \qquad (6)$$

and

$$s(x,y) = \frac{\sin\left(k\sqrt{z^2 + x^2 + y^2} - \frac{\pi}{2}\right)}{\lambda\sqrt{z^2 + x^2 + y^2}} \qquad (7)$$

Equations (4) and (5) may now be transformed into the spatial frequency domain. Thus if $P_1(f_x,f_y)$ and $P_2(f_x,f_y)$ represent the Fourier transform of p_1 and p_2, and $P_o(f_x,f_y)$ represents the Fourier transform of the object wave, the diffraction equations in the spatial frequency domain will be:

$$P_1(f_x,f_y) = P_o(f_x,f_y)\; C(f_x,f_y) \qquad (8)$$

$$P_2(f_x,f_y) = P_o(f_x,f_y)\; S(f_x,f_y) \qquad (9)$$

where $C(f_x,f_y)$ and $S(f_x,f_y)$ are the Fourier transforms of the cosine and sine propagation terms. The sum of the squares of equation (8) and (9) gives:

$$P_1(f_x,f_y)^2 + P_2(f_x,f_y)^2 = P_o(f_x,f_y)^2 \left[S(f_x,f_y)^2 + C(f_x,f_y)^2 \right] \qquad (10)$$

It has been shown, Sepehr [3], that at low spatial frequencies the Fourier sum $\left[S(f_x,f_y)^2 + C(f_x,f_y)^2 \right]$ is unity. Thus $P_o(f_x,f_y)$ is given approximately by

$$P_o(f_x,f_y)^2 = P_1(f_x,f_y)^2 + P_2(f_x,f_y)^2 \qquad (11)$$

The object wave will then be given by:

$$p(x,y,o) = F^{-1}\left[P_1(f_x,f_y)^2 + P_2(f_x,f_y)^2 \right]^{\frac{1}{2}} \qquad (12)$$

Equation (12) may be regarded as a mathematical solution to the self-focused reconstruction problem within the given limitations. Thus the following algorithm may be used for the self-focused reconstruction of ultrasonic images.

(a) Obtain the complex wavefront $p_s(u,v,z)$ at an arbitrary sampling plane.

(b) Calculate the real part p_1 and imaginary part p_2 of the wavefront.

(c) Perform Fourier transformations on p_1 and p_2 to obtain the Fourier functions $P_1(f_x,f_y)$ and $P_2(f_x,f_y)$.

(d) Calculate the sum of the squares of $P_1(f_x,f_y)$ and $P_2(f_x,f_y)$ to obtain the square object wave $\left[P_o(f_x,f_y)\right]^2$.

(e) Calculate the square root of this sum to obtain $P_o(f_x,f_y)$.

(f) Take the inverse Fourier transform of $P_o(f_x,f_y)$ to obtain the object wave $p(x,y,o)$.

(g) Calculate the square of the modulus of $p(x,y,o)$ to obtain the image.

It should be noted that step (e) introduces a phase ambiguity of π radians.

The method has been tested using a computer simulation and by experimental measurement.

To simulate the reconstruction process on the computer, a simple object, letter H, with an arm size of 8 mm was chosen (see Figure 1(a)). The diffraction pattern was calculated using the Fresnel integral over a 6 x 6 cm^2 observation plane positioned at a distance of 10 cm in front of the object over 3600 complex data points. Figure 1(b) shows the magnitude of the computed field. In this calculation all points in the object plane were in phase and the wavelength of 1·5 mm was assumed which corresponds to the wavelength of a 1 MHz wave frequency in water. The simulated data was then processed using the discrete Fourier transform according to the self-focused reconstruction algorithm outlined above. The resulting computed image is shown in Figure 2.

The simulation has been extended to indicate the likely effect of non-uniform phase distribution in the object plane. A good degree of tolerance has been shown giving recognisable reconstruction providing that the extent of non-uniform phase distribution does not exceed that with which the sign ambiguity program can cope.

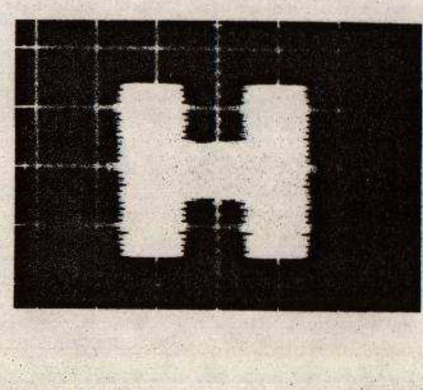

Figure 1(a)
Amplitude distribution of object function.

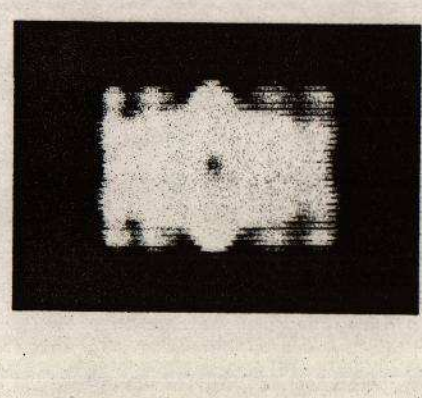

Figure 1(b)
Computed diffraction field.

Figure 2. Reconstructed image.

For the experimental measurements the transmitter, test object and the sampling probe were all suspended in a water filled standard glass fibre water-storage tank with approximate dimensions: 120 cm x 60 cm x 60 cm. For all tests through transmission was used. The transmitter unit (3 x 3 mm) being cut from a 1 MHz thickness mode resonance PZT-5 transducer disc, and driven by a gated sinewave burst of 20 cycles duration with an amplitude of 60 volts in order to improve the signal to noise ratio of the received signal.

The propagated wave on reaching the measurement plane was sampled by a single manually scanned probe to perform point-by-point sampling, over 60 x 60 sampling points spaced 1 mm apart.

The instantaneous value of the pressure signal at each sample point was recorded twice with a time interval of a quarter period to specify the signal.

Several test objects of different shape were used, Figure 3(a) is a photograph of one of the objects used, an aluminium plate 2 mm thick in which 7 mm holes were drilled, so that the test piece represented a high contrast object. The magnitude of the recorded field in the sampling plane for this object is shown in Figure 3(b). The reconstructed image using the self-focusing algorithm is shown in Figure 3(c).

For comparison the same data, together with the range, were used to produce a reconstructed image by backward propagation, the result is shown in Figure 3(d).

An alternative analysis can be carried out by relating the spatial frequencies in the sampling plane directly to those in the object plane.

The Fourier transform relating spatial frequencies to spatial variation in the object plane is given by:

$$P_o(f_x,f_y) = \iint_{-\alpha}^{\alpha} p(x,y,o) \exp(-j2\pi(f_x x + f_y y))\, dx\, dy \quad (13)$$

The inverse transform expresses p(x,y,o) as a continuum of complex exponentials of the form:

$$\exp(j2\pi(f_x x + f_y y))$$

i.e.

$$p(x,y,o) = \iint_{-\alpha}^{\alpha} P(f_x,f_y) \exp\,(j2\pi(f_x x + f_y y))\, df_x\, df_y \qquad (14)$$

where f_x and f_y are positive or negative real variables.

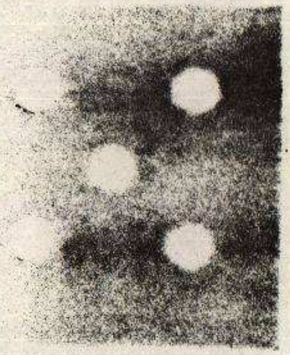

Figure 3(a). Test Object

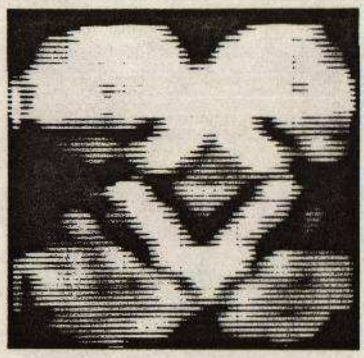

Figure 3(b). Diffraction field

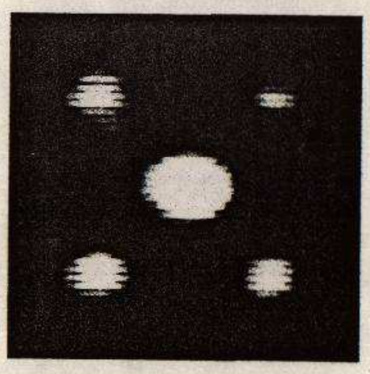

Figure 3(c). Self-focused image

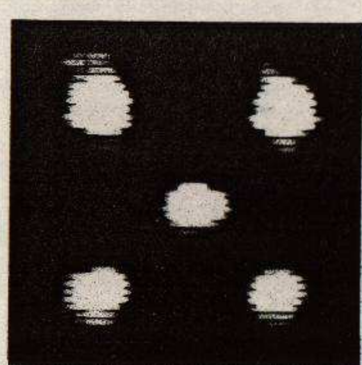

Figure 3(d). Backward propagation image

In order to simplify the argument, without decreasing the validity, p(x,y,o) is assumed not to vary in the y direction, thus we can write (13) as:

$$P_o(f_x) = \int_{-\alpha}^{\alpha} p_o(x,o,o) \exp\,(-j2\pi f_x x)\, dx \qquad (15)$$

A particular positive component in an object plane where there is no variation in the y direction can be written as:

$P_{xn+} \exp (j2\pi f_{xn} x)$

This is in fact a coefficient of exp(j2πft) where f is the temporal frequency of the ultrasonic variation.

This is the normal complex representation of a sinusoidal waveform, and P_{xn+} is in general complex and may be written in the form:

$|P_{xn+}| \angle \phi_{n+}$

In real terms the actual pressure variation due to this single spatial frequency is given by:

$$p = |P_{xn+}| \cos (2\pi ft + 2\pi f_{xn} x + \phi_{xn+}) \qquad (16)$$

Similarly the corresponding negative exponential term:

$P_{xn-} \exp (-j2\pi f_{xn} x)$

represents a real pressure variation of

$$p = |P_{xn-}| \cos (2\pi ft - 2\pi f_{xn} x + \phi_{xn-}) \qquad (17)$$

The positive (or negative) component on its own gives a variation in temporal phase across the object plane but constant r.m.s. amplitude. Suitable combinations of positive and negative terms represent variations in both amplitude and phase.

In the special case under consideration, where there is a constant temporal phase front given by cos (2πft) in the object plane, it is necessary that:

$|P_{xn+}| = |P_{xn-}| = P_{xn}$ say

and

$\phi_{xn+} = -\phi_{xn-} = \phi_{xn}$ say

Thus in real terms the pressure variation at the real spatial frequency f_{xn} is given by:

$$p = P_{xn} \cos (2\pi ft + 2\pi f_{xn} x + \phi_{xn}) + P_{xn} \cos (2\pi ft - 2\pi f_{xn} x - \phi_{xn})$$

$$= 2 P_{xn} \cos (2\pi f_{xn} x + \phi_{xn}) \cos 2\pi ft \qquad (18)$$

The relationship between the sampling plane and object plane in the spatial frequency domain can be derived on the basis of plane ultrasonic waves and is given by:

$$P_s(f_x,f_y) = P_o(f_x,f_y) . \exp (-j\gamma z)$$

where

$$\gamma = 2\pi\sqrt{\frac{1}{\lambda^2} - f_x{}^2 - f_y{}^2}$$

When a positive (or negative) complex spatial frequency is considered exp $(-j\gamma z)$ represents a phase shift which in real terms gives:

$$p_s = \left|P_{xn+}\right| \cos\,(2\pi ft + 2\pi f_{xn}x + \phi_{xn+} - \gamma_n z) + (P_{xn-})\cos\,(2\pi ft - 2\pi f_{xn}x + \phi_{xn-} - \gamma_n z) \qquad (19)$$

It can be seen from (19) that it is not possible to distinguish between temporal and spatial phase difference. If however the positive and negative spatial frequencies are considered together, and a constant phase front in the object plane is again assumed we have in real terms:

$$p_s = P_{xn}\cos\,(2\pi ft + 2\pi f_x x + \phi_{xn} - \gamma_n z) + P_{xn}\cos\,(2\pi ft - 2\pi f_x x - \phi_{xn} - \gamma z)$$
$$= 2\,P_{xn}\cos\,(2\pi f_x x + \phi_{xn})\cos\,(2\pi ft - \gamma z) \qquad (20)$$

This is the same distribution as for the object plane, as given in equation (18), except for a temporal phase lag of γz. Thus it can be seen that for any pair of spatial frequencies (positive and negative) the amplitude and spatial phase is the same in the sampling and object planes, i.e. there is no dependence on z.

There will however be both amplitude and temporal phase variation across the sampling plane in the space domain, since in general more than one pair of spatial frequencies will be present, and γ is a function of f_x (and f_y).

The problem now is to find a procedure, based on the above theoretical background to give the pressure variation in the object plane from measurements in the sampling plane in the space domain, without a knowledge of z.

IMAGE RECONSTRUCTION TECHNIQUES

The process described in the section on self-focusing image reconstruction can be related to the above analysis in the spatial frequency domain by relating the functions $p_1(f_x,f_y)$ and $p_2(f_x,f_y)$, of stage (c) of the self-focusing algorithm to equation (18). The in-phase components represented in $p_1(f_x,f_y)$ give pairs of complex spatial frequencies of the form $p_{xn+}\cos\,(\gamma z)\underline{/\phi_{xn+}}$ and $p_{xn-}\cos\,(\gamma z)\underline{/\phi_{xn-}}$ whilst the quadrature components give $p_{xn+}\sin\,(\gamma z)/\phi_{xn+}$ and $p_{xn-}\sin\,(\gamma z)/\phi_{xn-}$.

Thus $P_1(f_x,f_y)^2 + P_2(f_x,f_y)^2$ contains components of the form $\left|P_{xn}\right|^2\underline{/2\phi_{xn}}$, whence the square root contains terms such as $\left|P_{xn}\right|\underline{/\phi_{xn}}$, with a phase ambiguity of π radians as stated in stage (e).

Equation (18) indicates an alternative algorithm, since the Fourier transform of the measured values in the sampling plane will give pairs of complex components of the form $|P_{xn+}|\angle\Theta_{xn+}$ and $|P_{xn-}|\angle\Theta_{xn-}$ where

$$\Theta_{xn+} = \phi_{xn} - \gamma z$$

and

$$\Theta_{xn-} = \phi_{xn} + \gamma z$$

whence

$$\phi_{xn} = (\Theta_{xn+} - \Theta_{xn-})/2$$

Thus the amplitude, P_{xn}, of a particular spatial frequency of equation (18) can be determined from the square root of $P_{1xn}{}^2 + P_{2xn}{}^2$ and the phase ϕ_{xn} from $(\Theta_{xn+} - \Theta_{xn-})/2$. The division by 2 implies a 180° phase ambiguity.

CONCLUSIONS

The analysis given in the above sections has verified the basic self-focusing algorithm but has further highlighted the problem of the π radian phase ambiguity in the spatial frequency components of the reconstructed image. Alternative algorithms have been tried which in certain circumstances can give satisfactory results. However when these are checked against computed, or analytically derived, Fourier transforms of object geometries their success appears to depend on the particular properties of the object. The best technique may be a compromise between self-focusing and maximum rate of change of image. On this basis the π radian ambiguity of the self-focusing technique could be resolved by assuming the spatial frequencies to be in phase in areas where there are multiple zero crossings.

REFERENCES

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[3] S. Sepehr, 'Ultrasonic imaging by means of frequency domain processing', PhD Thesis, C.N.A.A., (1980).