Proceedings of the Institute of Acoustics

A METHOD FOR ASSESSING NOISE REDUCTION PROVIDED BY CYLINDERS

J.N. Pinder (1), F.J. Fahy (2)

(1) ISVR Consultancy Services, University of Southampton.
(2) ISVR (Institute of Sound & Vibration Research), University of Southampton.

1. INTRODUCTION

The theoretical estimation of the noise level inside a cylinder due to external acoustic excitation of the cylindrical surface can be performed by application of Finite Element and Boundary Element methods [1, 2], but this requires very intensive use of powerful computing resources to give results over a broad frequency bandwidth for multiple angles of incidence, which is disadvantageous for initial design studies. Statistical energy analysis (SEA) is much simpler to apply, but is only appropriate for a diffuse excitation field; cannot deal easily with non-resonant behaviour; and suffers from large confidence intervals at low frequencies where acoustic modal densities are low. The approach described here was developed to assist the design of a spacecraft launcher fairing by allowing parametric sensitivity studies to be performed for non-diffuse excitation fields at low cost and effort; it is tailored specifically for the intermediate frequency range between the practical upper limits of the computational approach and the lower limit of usefulness of SEA. This frequency range corresponds to that of maximum sound transmission and maximum concern for the integrity of the enclosed payload equipment. Modal Interaction Analysis (MIA) was selected as best suited to meet these requirements. The original implementation is described in [3].

2. DESCRIPTION OF THE MIA APPROACH

2.1 Introduction

The analysis is based upon the assumption of cylindrical geometry and has two basic steps: first the structural response of a cylindrical mode is calculated due to the external acoustic forcing field; then the coupling between that mode and the acoustic modes of the gas in the enclosed cavity is calculated (in general there is no need for fully coupled dynamic analysis). This is repeated for all the structural modes having their natural frequency in the frequency band of concern. The resulting acoustic modal energies are summed and then converted into the corresponding spatially averaged sound pressure level. The numbers of resonant mode pair couplings in a frequency band could range from none at low frequencies, up to several thousand at high frequencies. Even if there are no mode pairs with natural frequencies in a particular frequency band, the significant off-resonance contributions from selected mode pairs are included in the sound transmission estimate. It is unusual for the contribution of a single mode pair to dominate the resulting frequency band value.

A METHOD FOR ASSESSING NOISE REDUCTION PROVIDED BY CYLINDERS

2.2 External Coupling

2.2.1 Plane Wave Scattering. Each cylindrical structural mode has an axial and a circumferential order (m,n) (see Figure 1). The analysis developed in [3] assumes that the sound field consists of one or more incident progressive plane waves and that the ends of the cylinder are simply-supported. The blocked pressure field at the surface of the cylinder is calculated taking into account the diffraction of the incident waves by the 'rigid' cylinder. Any one plane wave can be decomposed into a set of cylindrical waves, the sum of which satisfies the boundary condition on the rigid cylinder. Each cylindrical component has a single circumferential order (n). Similarly, each structural mode of uniform cylindrical structure has a unique circumferential order; only acoustic components and structural modes of the same order can couple. The fraction of the plane incident wave which is matched to the circumferential mode of order n is termed the 'scattering coefficient' for mode n, giving a family of circumferential mode order curves of scattering coefficient versus frequency. These reach a maximum value which decreases with increasing mode order but occurs at a frequency which increases with mode order.

2.2.2 Generalised Force. When calculating the overall force driving a particular cylindrical mode (termed the 'generalised' force), the mode shapes of the flexible cylinder are taken into account, in the circumferential direction by virtue of the modal scattering coefficient, and in the axial direction by the Joint Acceptance Function (JAF) [4].

The resulting expression for the spectral density of the generalised force, $S_{F,(mn)}(\omega)$, for random excitation is given by:

$$S_{F,(mn)}(\omega) = S_{p,(n)(\omega)} . K . j_m^2 (k_z)$$

where $S_{p,(n)(\omega)}$ = spectral density of blocked pressure
m = axial mode order,
n = circumferential mode order,
K = function of (R/L),
R = radius of cylinder,
L = length of cylinder,
$j_m^2(k_z)$ = Joint Acceptance Function.

If the cylinder is assumed to be simply-supported at its ends:

$$j_m^2(k_z) = \frac{1 - (-1)^m \cos k_z L}{\left[\left(k_z^2 - (m\pi/L)^2\right)\right]^2}$$

where k_z = axial component of incident field wavevector (see Figure 1).

2.2.3 Structural Response. With the coupling, or 'joint acceptance', taken into account in the generalised force expression, the response is calculated as if for a single-degree-offreedom system. Structural response W_{mn} in the mn mode at frequency ω is expressed by the following relationship:

Generalised displacement response = [Gen.Force/Gen.Mass] [Dynamic Magnification factor]

as $$W_{mn} = (F_{mn}/M_{mn})\left[(\omega_{mn}^2 - \omega^2) + i\,(\eta_{mn}\,\omega\,\omega_{mn})\right]^{-1}$$

In spectral density terms

$$S_{w,(mn)}(\omega) = \left(S_{F,(mn)}(\omega)/M_{mn}^2\right)\left[(\omega_{mn}^2 - \omega^2)^2 + (\eta_{mn}\omega\,\omega_{mn})^2\right]^{-1}$$

where η_{mn} is the sum of the dissipation and external radiation loss factor and ω_{mn} is the natural frequency of mode m,n.

The resonance proximity term expresses the frequency matching whilst the degree of spatial coupling for mode m,n can be appreciated from the behaviour of the Joint Acceptance Function. This will be influenced by the structural mode shape, which will be potentially best coupled to the sinusoidal pressure distribution when the structural mode shape is also sinusoidal. This only occurs for simply-supported end conditions. The influence of boundary conditions is greatest for low order modes and is thus greatest at low frequencies. Radiation damping of the cylinder is also influenced by the mode shape at the ends of the cylinder. Remote from the ends, the mode shapes differ little from being sinusoidal for a structure of uniform bending modulus.

Applications of the model up to the present assume simply-supported ends; therefore the accuracy of predictions of performance of real installations is reduced for low mode orders and non-ideal boundary conditions. It has been found that although end plates fitted to a cylinder influence the natural frequencies by only up to 10%, the mode shapes are significantly changed [5]. However, detailed evaluation of the true mode shapes is often impractical and uneconomic for parametric design studies.

For the simply-supported case, the characteristics of the Joint Acceptance Function are illustrated by Figure 2. Under maximum spatial matching conditions, the acoustic trace wavelength equals the structural wavelength and the value of the JAF is about 0.25 for m > 2. Equi-spaced peaks occur at diminishing level both at lower and higher wavelength ratios. Thus, for example, as the angle of incidence increases from grazing towards normal, the coupling value represented by the JAF passes through half-cycles of increasing amplitude until the max-max or absolute peak is reached at the equal wavelength conditions, after which the value passes through half-cycles of decreasing amplitude. The 'coincidence' condition which occurs when the response and excitation wavelengths are equal at the excitation frequency is here termed 'external coincidence'.

2.2.4 External Coincidence Conditions. The cylinder's response is controlled by coincidence or near-coincidence conditions, and so it is the peak value of joint acceptance which is most important. As already shown, this is almost constant for m > 2. Other values of joint acceptance are required to estimate the coupling over the bandwidth of concern, but the detailed behaviour of the JAF is only of secondary significance because of its rapid reduction in value as coincidence conditions become remote.

The general effect on external coincidence of the parameters of the cylinder and of the incident acoustic field may be appreciated from Figure 3. External coincidence occurs when the linear curve of the excitation field's axial wavenumber intersects the structural dispersion curves. The structural dispersion 'curves' for resonant axial modes only exist at the axial modal frequencies, so that resonant-coincidence occurs only when the external wavenumber curve passes through a mode resonance point. However, the principle is clarified by reference to a complete dispersion curve.

In Figure 3, the effect of the grouping of curves in the vicinity of the ring frequency, ω_{ring}, can be seen, leading to high densities of coincidence in the frequency domain. The ring frequency is the natural frequency of the diametric breathing mode of zero circumferential order of the equivalent infinitely long uniform cylinder. For a given cylinder, external coincidence varies only with the angle of incidence, since this determines the gradient of the external wavenumber line. The gradient decreases as incidence angle changes from grazing to normal. Changes to the cylinder's dispersion curve will be produced by anything which changes the natural frequencies of the modes. Below the ring frequency, reducing the natural frequencies raises the dispersion curve, and would cause coincidence to occur nearer to grazing incidence, and at lower frequencies. Figure 4 shows this effect for a single circumferential mode. Each circumferential mode can be considered separately since there can be no coupling between modes of different circumferential order. Near to ring frequency, the S-bend nature of the dispersion curve causes less predictable changes; from Figure 3 it can be seen that the density of coincidences can suddenly increase for small changes in angle of incidence.

Criteria are applied to the selection of mode pairs to avoid the inclusion of weakly coupled pairs and hence to economise on CPU time.

2.3 Internal Coupling

To estimate noise levels inside the cylinder, the response of the acoustic modes to structural mode vibration is calculated, frequency by frequency, using a classical response analysis for coupled modes. The spectral density $S_p(\omega)$ of the pressure amplitude of the acoustic mode r can be expressed as follows [see Ch. 6.4 of ref 5]:

$$S_p(\omega) = \left\{ \frac{\rho_o^2 \omega^4 c^4 S^2}{\Lambda_r^2 \left[(\omega_r^2 - \omega^2)^2 + (\eta_r \omega \omega_r)^2 \right]} \right\} S_{w,(mn)}(\omega) . C_{mnr}^2$$

where C_{mnr} = coupling coefficient for acoustic mode r and structural mode mn,

Λ_r = $\int_v \Psi_r^2(r) dV$,

Ψ_r = acoustic pressure mode shape,

V = cavity volume,

A METHOD FOR ASSESSING NOISE REDUCTION PROVIDED BY CYLINDERS

S = cavity surface area,
ρ_o, C = density, speed of sound of enclosed fluid,
$(\rho_o C^2$ = fluid adiabatic bulk modulus)

The denominator of the term in curly brackets is related to a dynamic magnification factor, as was a similar form of term in the expression for $S_{w(mn)}(\omega)$. The whole of this term is an expression of response of an acoustic mode, and C_{mnr} is a form of joint acceptance between the acoustic mode and the structural mode. In addition to coupling of cavity modes of order (mm) to structural mode orders (m), (m-mm = odd-numbered integers), the cylinder modes couple to the cavity modes which have radial orders (p) as well as the same circumferential orders (n) (Figure 1). The term Λ_r can be expressed simply in terms of the cylindrical cavity dimensions and the roots of the acoustic wave equation in the cavity (e.g. as given in [6]). The coupling once again peaks when coincidence occurs, in this case as shown by the intersection of internal dispersion curves and structural dispersion curves, again for a common circumferential mode order as indicated by Figure 5. Both sets of dispersion curves now only exist in reality at resonance frequencies. Their intersections at these points are 'internal' coincidences. It will be understood that circumstances of maximum transfer of energy to the internal sound field occurs when an external coincidence and an internal coincidence occur at or near to the same frequency.

All cavity modes are coupled to each structural mode to some extent, but the analysis is restricted to cavity modes which exhibit good coupling. Since the analysis proceeds in two separate uncoupled stages, there is no mechanism for feedback of acoustic energy to poorly excited structural modes which acts as a form of damping for the acoustic modes. Hence the acoustic energy can build up to a level which contravenes the limiting condition of equi-partition of modal energy under broadband excitation. Equi-partition is imposed as an upper limit on the acoustic modal energy, using all available structural and acoustic modes in a band. The modelling has been coded so that the coupling is evaluated at a set of discrete frequencies and then summed over one-third octave bands. The contributions from the selected mode pairs are then summed to give the overall noise reduction in each band.

2.4 Comparison of Internal Coupling with Flat Plate Structure

The frequency regions of structural 'stiffness' and 'mass' control of a cylinder are different from those of a flat plate because of the effects of curvature. The reason may be clarified by reference to Figure 6. Structural and acoustic modes which are well matched have the same circumferential wavenumber n/a and similar axial wavenumbers. These mode pairs are illustrated in Figure 6 by filled circles (•). At frequencies well below the ring frequency, ω_{ring}, cylinder curvature causes the natural frequency of the cylinder mode, $\omega_{mn(cylinder)}$, to be greater than the natural frequency of the internal acoustic mode, ω_r, for good matching. Hence the acoustic resonance response is controlled by structural mode <u>stiffness</u>; on the other hand the equivalent flat plate mode frequency, $\omega_{mn(plate)}$, is less than ω_r and hence transmission at low frequency is <u>mass</u> controlled (mass law). Above ω_{ring} the behaviour of cylinders and flat plates does not differ, except in terms of the external field blocked pressure (scattering) coefficient. This explains why the NR curves show stiffness dependence at frequencies well below ω_{ring}, and mass dependence above ω_{ring}.

2.5 Multi-Component Incident Fields

The external sound field at launch is usually of primary concern and this can be modelled as several progressive plane-waves of the required relative level. The noise reduction is assessed for individual component plane-waves and then the overall noise reduction computed. A reverberant sound field can also be modelled, by an option within the analysis, by suitable weighting of the contributions from individual incidence angles spaced at equi-angular intervals. The internal coupling is not influenced by the external field although the available energy is governed by the structural response.

3. COMPARISONS OF PREDICTIONS WITH MEASURED NR VALUES

To test the model, a series of experiments was performed using sandwich wall construction cylinders scaled to represent structures similar to the full-size European Ariane 4 launcher fairing [1]. The wall construction of the models used orthotropic carbon fibre reinforced plastic (CFRP) faceplates of lay-up and properties close to the full-scale construction, but the aluminium honeycomb core thickness and overall dimensions were at one quarter scale. The test frequency range, scaled on the overall dimensions, covered the structural behavioural regimes of the full-size fairing when reduced to the equivalent cylinder. Properties of the model cylinders and the assumed fairing properties are shown in Table 1.

Predictions have been made using a development of the early analysis described in [1]. In the later analysis, the internal acoustic modes are excited by a spectrum shaped by the structural response rather than by a spectrum assumed to be flat. The structural eigenvalues are estimated separately from the MIA model for in-vacuo modes by an analysis based on Baker and Hermann's theory [7]. The internal acoustic field is assumed to be not coupled to the structural field. The modelling is performed using the computer code 'PROXMODE' developed by ISVR Consultancy Services. The predicted and measured cylinder performance is shown as noise reduction values (NR, dB) where

NR = (Incident sound pressure level at the structure's location in the absence of the structure) – (spatially averaged internal sound pressure level).

Some predicted spectra of noise reduction values are shown in Figures 7 and 8 with measured results superimposed. Also superimposed are corresponding measured and calculated changes, since it is the sensitivities rather than absolute values which the method is designed to predict. The predicted effects of changing the wall construction of the cylinder are shown in Figure 9.

4. APPLICATION TO LARGE SHELLS

Little data is available for full-size sandwich construction cylinders, but a comparison is shown in Figure 10 between some predicted and some preliminary measured values for an Ariane 4 fairing. The fairing incorporates a nose cone and is made of two halves, on a diametrical split-line. The prediction of very low values of noise reduction is due to the high modal density over the frequency range of interest, even for the relatively stiff real wall construction. For example, there are four flexural mode eigenvalues in the 32 Hz centre frequency octave band for the 4m diameter cylinder, but only 3 values in the 250 Hz band for the model cylinder. This difference triggers the invocation of the equi-partition limit for the model but not for the full-scale structure. Even with this limitation, negative values of noise reduction are possible. There is a difficulty in applying this criterion to a limited bandwidth, since a judgement has to be made as to which structural modes contribute significantly enough to the acoustic field response to warrant being included in the structural modal energy estimate. When the energy limiting criterion is invoked, the value of the internal absorption has no influence on the internal noise level. The physical reason is that the modal-average coupling loss factor exceeds the acoustic dissipation loss factor.

5. FURTHER DEVELOPMENT

It is expected that the structural modelling is the source of most inaccuracy at present at frequencies of the very low order modes, due to mode shape discrepancies arising from non-ideal boundary conditions and minor structural inhomogeneities. Also, in-vacuo eigenvalues are possibly significantly in error. Further investigations into these effects and their resolution by the application of modal decomposition of analytical (e.g. hyperbolic-function) mode shapes and numerically defined mode shapes is necessary to improve accuracy in this frequency range.

However, for large cylinders, the frequency range of most interest is usually higher, extending up to about 2 ω_{ring}, where the prediction is controlled by the criterion of equi-partition of modal energy levels as already mentioned. Optimisation of the structural modal energy estimate and establishment of confidence levels in this region remains to be undertaken.

Efficiency of the computation of response in a diffuse field could be improved by using the analytical expression for diffuse field cross-spectral density in the JAF, and by use of SEA modelling at well above the ring frequency where the structural behaviour approaches that of an equivalent flat plate.

ACKNOWLEDGEMENT

The authors thank the European Space Agency, the Centre Nationale d'Etudes Spatiale, and Dornier Luftfahrt GmbH, for their support of the work described, and also Dr Neil Ferguson of ISVR who undertook the structural analysis associated with it.

A METHOD FOR ASSESSING NOISE REDUCTION PROVIDED BY CYLINDERS

REFERENCES

[1] M.A. Hamdi: 'Méthodes de discrétization par éléments fines et éléments finis de frontière', in *Rayonnement acoustique des structures*, ed. C. Lesueur, Editions Eyrolles, 1988.

[2] M. Faust, G. Schweickert, F. Strobel: 'Structure-Acoustic Finite Element Analyses for Noise Reduction Investigations of Launcher Payload Compartment Structures Made of CFRP Sandwich Material', *Proc. Int'l. Conf. 'Spacecraft Structures and Mechanical Testing'*, Noordwijk, 24-26 April 1991 (ESA SP-321).

[3] J.N. Pinder, M.E. House, F.J. Fahy: 'A Preliminary Study of the Noise Reduction in the Ariane 5 Payload Bay Given by the Fairing and SPELTRA', *Final Report, ESA Contract No. 6675/86/F/Fl*, 1987.

[4] A. Powell: 'On the Fatigue Failure of Structures due to Vibration Excited by Random Pressure Fields', *J. Acoust. Soc. Am.*, 30, p. 1130, 1958.

[5] M. Ohlrich & K. Ronge: 'Modal Analysis of Circular Cylindrical Shells', *Report No. 49*, The Acoustics Laboratory, Technical University of Denmark, 1991. ISBN 0105-3027.

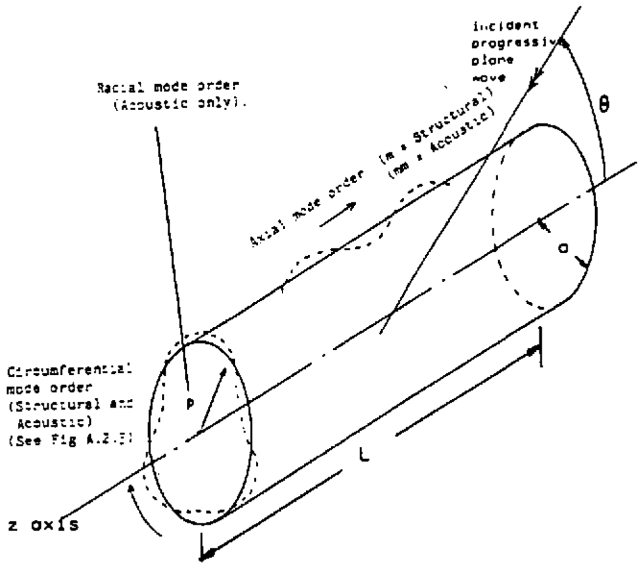
[6] F.J. Fahy: 'Sound and Structural Vibration', *Academic Press*, 1987. ISBN 0-12-247671-9.

[7] E.H. Baker & G. Hermann: 'Vibration of Orthotropic Cylindrical Sandwich Shells Under Initial Stress', *AIAA Journal*, 4, No. 6, pp. 1063-1070, June 1966.

Table 1. Properties and Dimensions of Empty Unlined Cylinders

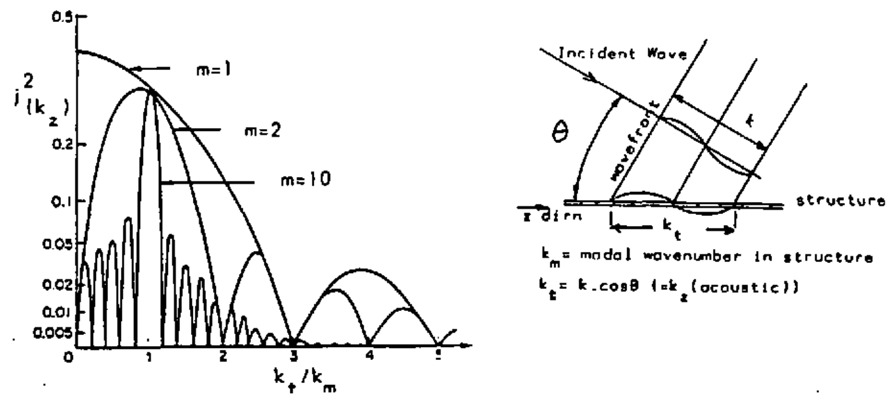
Parameter	Scaled	A4 (Assumed)
Face Plates (CFRP, common)		
Thickness mm	1.0	0.9
Elastic mod. N/m^2 E9		
Tensile circumf.	26.2	20.4
Tensile axial	69.5	66.3
Shear in plane	19.3	12.4
Density kg/m^3 E3	1.6	1.7
Core (aluminium honeycomb)		
Thickness mm	5	24
Shear mod. N/m^2 E6		
Circumf.	150	80
Axial	240	140
Density kg/m^3	48	48.7
Wall		
Length m	0.94	8.7
Diameter m	0.9	4
Mass/area kg/m^3	4.0	4.1
Structural damping (loss factor)	0.01	0.01
f_{ring} (Hz)	1349	234
f_{crit} axial/circumf. (Hz)	1038/1692	186/483

A METHOD FOR ASSESSING NOISE REDUCTION PROVIDED BY CYLINDERS



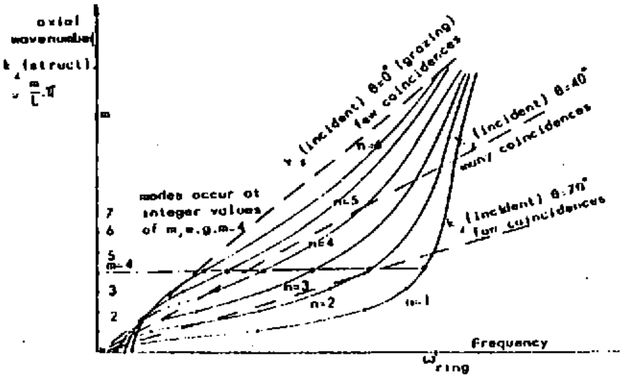
Cylinder and Mode Order Notation

Figure 1

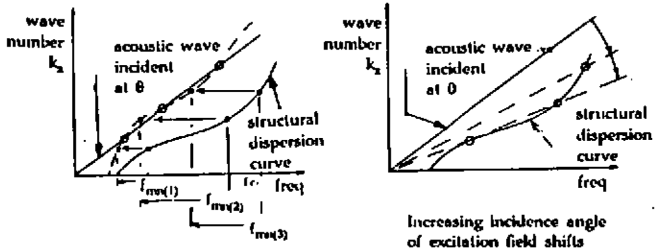


Variation of joint acceptance, $j^2(k_z)$, with Wavenumber Ratio k_t/k_m.

Figure 2



Structural Mode Dispersion Curves for Cylinder **Figure 3**

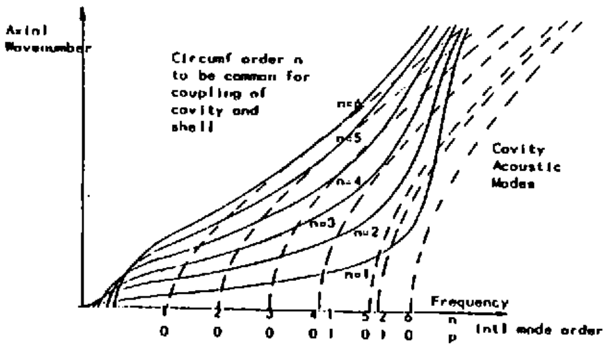


Reducing structural natural frequencies shifts dispersion curves towards a given incidence wavenumber line

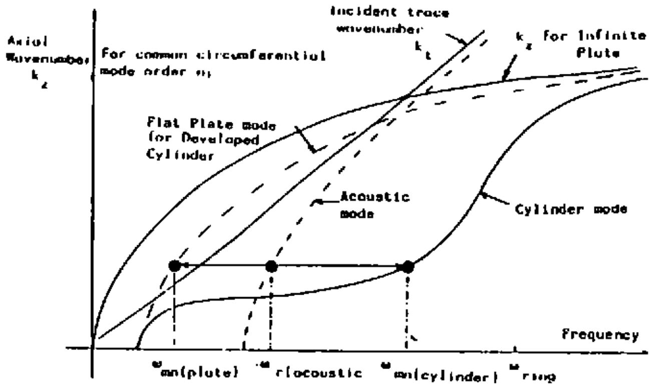
Increasing incidence angle of excitation field shifts external wavenumber curve towards a given structural curve, leading to single, then multiple coincidences

Effect of Structural and Incident Field Changes on Occurrence of External Coincidence

Figure 4



Structural and Internal Acoustic Mode Dispersion Curves
Figure 5



Non-resonant Modal Coupling for Cylinder and Plate **Figure 6**

A METHOD FOR ASSESSING NOISE REDUCTION PROVIDED BY CYLINDERS

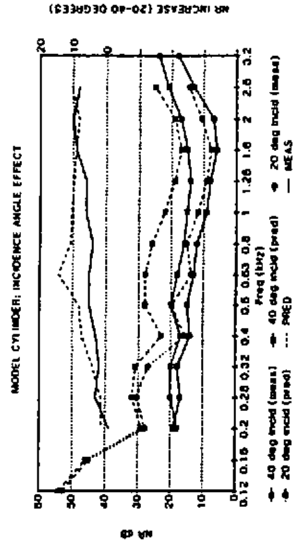


Figure 8

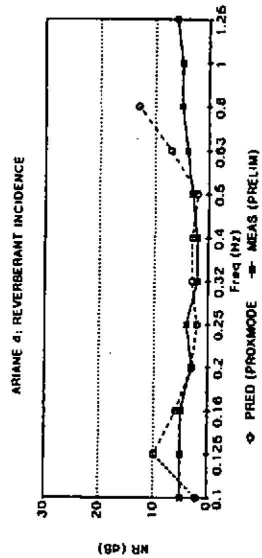


Figure 10

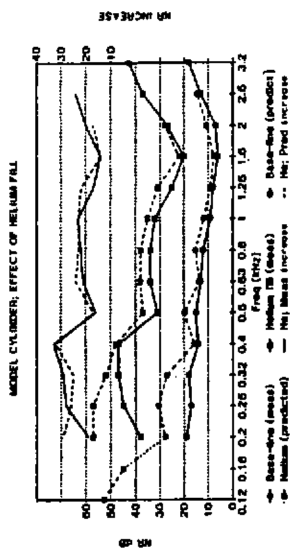


Figure 7

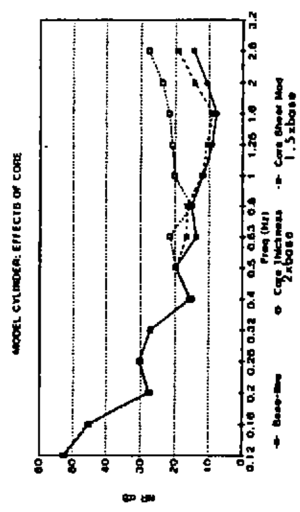


Figure 9