Proceedings of the Institute of Acoustics

IMPROVED ANALYSIS OF INPUT IMPEDANCE MEASUREMENTS

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1. INTRODUCTION

The design of acoustic impedance sensors has been the object of numerous works, especially for studying wind music instruments and characterizing acoustic materials [1 to 8]. Calibration of these sensors is a major problem, as it limits the actual validity of the measurements. Many calibration methods have therefore been used, but they all require one or more reference loads whose impedances must be known with a high accuracy. These loads are often cylindrical closed tubes, whose walls are assumed perfectly rigid; it is however very difficult to evaluate the importance of this last assumption.

The purpose of this paper is to describe a method allowing the analysis of impedance measurements without the "perfect walls" assumption, and even more the determination of the propagation constant in the tubes. This method is based on the fact that most calibration errors of linear impedance sensors lead to deviations between the estimated propagation constant and the actual one, which are related to the impedance of the load and show therefore alternating maxima and minima with a frequency spacing related to the length of the tube. This phenomenon is characteristic of the calibration errors, as the propagation constant has a much smoother frequency variation; it can then be used as a simple criterion to adjust slightly the calibration parameters in a minimization process.

2. PRINCIPLE OF THE METHOD

For sensors using two electric signals linearly related to the pressure and the volume velocity at the input of the load, the transfer function Q between these signals is related to the acoustic impedance Z_e of the load by the following relationship :

$$Q = K\, e^{\gamma} (Z_e + \beta) / (1 + \delta Z_e) \qquad (1)$$

β, δ, γ and K are four frequency-dependent calibration parameters. K is chosen so that $\gamma \ll 1$ and is obtained by a first step of calibration or by a theoretical evaluation.
The calibration of the sensor is done using a closed cylindrical tube whose input impedance can be expressed as :

$$Z_e = Z_c \tanh(\Gamma l + \operatorname{atanh}(Z_t)) \qquad (2)$$

l is the geometrical length of the tube, and Z_t it's termination impedance.

Z_c and Γ are respectively the characteristic impedance and the propagation constant in the tube. These quantities are influenced by the boundary conditions including viscous and thermal phenomena and the mechanical admittance of the walls (assumed negligible for "perfect walls") [2,9 and 10].

A first estimation of Γ is given by the quantity Γ' so that $\Gamma' l = \operatorname{atanh}(Q/KZ_c - j\pi/2)$. Denoting respectively α and k the real and imaginary parts of the actual propagation constant Γ and similarly α' and k' for Γ', Γ and Γ' can be related using the calibration parameters. Denoting $\delta' = (\delta Z - \beta/Z)/(1 - \beta\delta)$, $\gamma' = \gamma\ (1 + O(\beta\delta))$ and $\beta' = \delta Z_c + \beta/Z_c$ and assuming that the termination impedance is high, a first-order expansion respective to δ', β', γ' and αl leads to the following expression

$$k' l = kl + \frac{1}{2}\left[\left(\mathrm{Im}(\delta') - 2\alpha l\ \mathrm{Im}(\gamma')\right) \cos(2kl) - \left(\mathrm{Re}(\gamma') - 2\alpha l\ \mathrm{Re}(\delta')\right) \sin(2kl) + \mathrm{Im}(\beta')\right] \qquad (3a)$$

$$\alpha' l = \alpha l + \frac{1}{2}\left[\left(\mathrm{Re}(\delta') - 2\alpha l\ \mathrm{Re}(\gamma')\right) \cos(2kl) + \left(\mathrm{Im}(\gamma') - 2\alpha l\ \mathrm{Im}(\delta')\right) \sin(2kl) + \mathrm{Re}(\beta')\right] \qquad (3b)$$

The difference between Γ and Γ' leads then to periodic fluctuations of α' and k' with a frequency spacing inversely proportionnal to l, and very different from the variations of Γ. The complex amplitudes of these oscillations can therefore be extracted using a demodulation by the two components $\cos(2k''l)$ and $\sin(2k''l)$, where k'' is an arbitrary, but reasonable, estimation of k (for example $k'' = \omega/c$). Identification of these amplitudes with the factors of equations (3a and 3b) leads to four relationships defining a first evaluation of the real and imaginary parts of δ' and γ'.

As relation (3) are first-order approximations, they are not sufficient to determine accurate values of δ' and γ'. It would be possible to use a more complete expression, leading to non-linear relations between the amplitudes and the factors to be determined, but this would necessitate accurate values for k" and other physical constants. It is therefore easier and more efficient to simply iterate this process.

The further steps involve then small corrections to the first evaluation of δ' and γ' instead of their values, determined from the oscillations remaining in the difference between two consecutive estimations of Γ. These estimations are obtained from the initial one Γ', corrected using the last evaluation of δ' and γ', i.e. their first calculated values plus all the previous corrections added.

As δ' and γ' involve the three calibration parameters β, δ and γ, it is still necessary to obtain one more relation to achieve the calibration procedure. In a first step, as the main effect of β and δ can be interpreted as a correction to the length of the tube, it would seem possible to evaluate their mean value by the comparison of the geometric length of the tube and the one deduced from the measured impedance extrema. This would however necessitate some assumptions concerning Γ; a preferred solution is then to measure a second tube of different length and same shape, using only the previous estimations of Γl.

Denoting l_i the length and $(\Gamma' l)_i$ the estimated Γl_i for tube i, where $(\Gamma' l)_i$ is obtained from two measurements but only one estimation of γ' and δ', β is given by the following relation :

$$\beta/Z_c = \tanh\left(\frac{(\Gamma' l)_1 l_2 - (\Gamma' l)_2 l_1}{l_2 - l_1} - \operatorname{atanh}(Z_t) \right) \qquad (4)$$

It is then straightforward to calculate δ and γ from δ', γ' and β.

3. DISCUSSION

As exposed above, relations (3) are first-order approximations assuming that γ' and δ' are small enough. This assumption is necessary for the ease of deducing the parameters from the oscillations amplitudes; the iteration process permits however to deal with actual values of γ' and δ' near unity with only an increase of the number of iterations. Higher values may avoid the convergence of the iteration process; this point would require further investigations if necessary.

Another practical limitation of the method is the use of the estimate k" for the demodulation of the oscillations amplitudes. It's influence is to mix slightly the two components of the oscillation, therefore reducing the speed of convergence of the minimization process. It induce however no error on the resulting values of the parameters. Convergence is theoretically possible as long as $|k''-k|l < \pi/4$, and is almost identical for any small value of the difference. The estimate $k'' = \omega/c$ is then usually sufficient in most cases.

Similarly, the initial value of α required in the right-hand terms of relations (3) may be any reasonnable estimate such as the theoretical one, or a mean value, without compromizing the accuracy of the method.

For a better determination of δ' and γ', one needs to have enough impedance extrema for the first calibration step. This leads to the use of a long tube at this time, which permits also calibration over a lower frequency range. On the other side, the determination of β is better with two tubes of very different lengths; this can be achieved using a second tube as short as possible, keeping in mind that the minimum length should garantee that only plane waves are significant in the tube.

All the above remarks concern practical aspects of the method, but do not reduce it's validity. A more questionnable point is the use of Z_c and Z_t in the determination of the calibration parameters. This supposes that their values are well known, although the best mean available to determine them is to use the theoretical expressions including viscous and thermal losses. In fact, any perturbation modifying either the serial impedance or the parallel admittance per unit length (characteristic of the propagation in the tube) has an effect on both the propagation constant and the specific impedance, and these effects are of the same order of magnitude. It should therefore be possible to evaluate such a perturbation by comparing the measured value for Γ and the theoretical one, and to deduce the corresponding order of magnitude of the resulting errors on Z_c and Z_t.

4. RESULTS

We tested this method using an impedance sensor developped prior to this work [3 and 4], using an electrostatic transducer as volume velocity source, and an electret microphone as pressure receiver (fig. 1) :

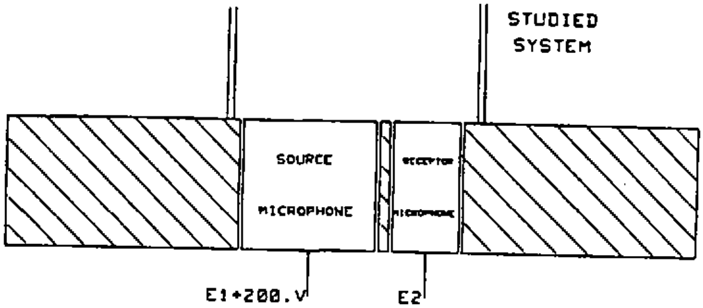

Figure 1 : principle of the sensor.

A first estimate for the term K of eq. (1) has been obtained from previous measurements, using classical methods. An example of initial impedance determination is given in fig. 2 :

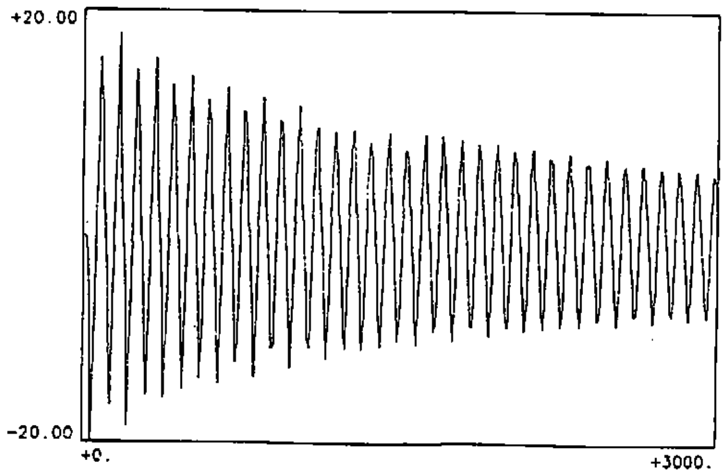

Figure 2 : first estimation of impedance modulus, tube of 16mm diameter and 2.01m length.
Frequency in Herz (300 points), modulus in dB.

Although this curve seems quite similar to the expected one, there are still some discrepancies. These can be emphasized when plotting the initial estimate $\alpha' l$ of αl (cf eq. 3) as in fig. 3 :

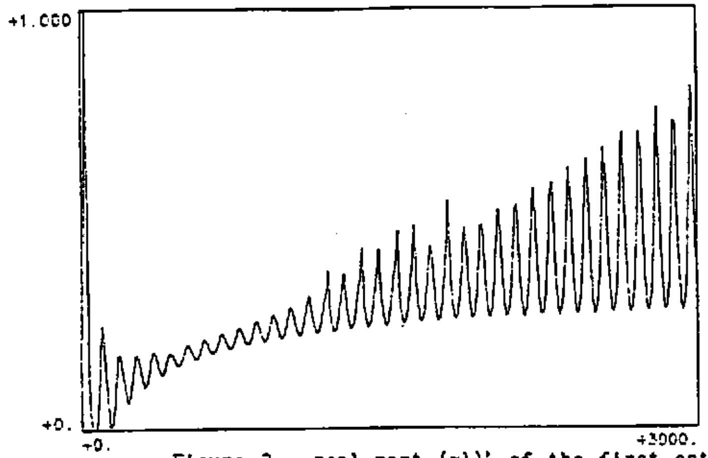

Figure 3 : real part (αl)' of the first estimation of the absorption.
Frequency in Herz, (αl)' adimensional.

This first estimation shows very sharp deviations from an average curve which could be the expected one. Clearly, these deviations are correlated with the impedance extrema. A new calibration has been done from the measurement of a 1m and a 0.1m tubes, and using the method described above; the resulting estimation of αl for the 2m tube is given by fig. 4 :

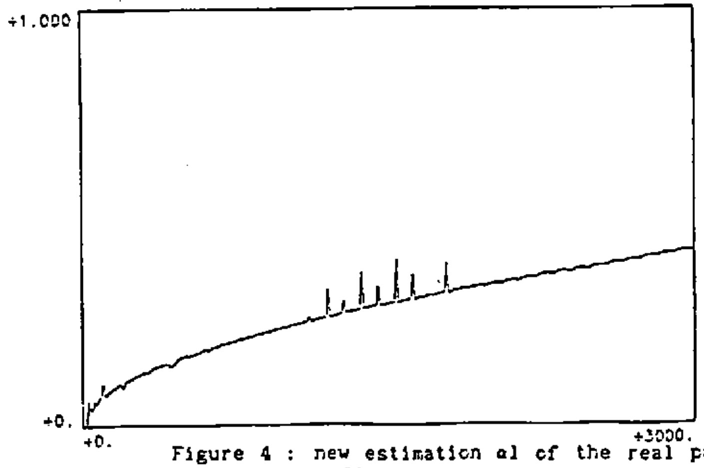

Figure 4 : new estimation αl of the real part of the absorption coefficient.
Frequency in Herz, αl adimensional.

This last result is much closer to the expected variations. It shows however several spikes between one and two kiloherz, which seem to result from residual noise in the pressure signal when measuring impedance minima. This phenomenon is likely to induce errors in the demodulation process, and is therefore avoided during calibration by interpolating the spurious points after a few iterations.

As can be seen by comparing figures 3 and 4, the resulting estimation of αl is different of the middle line of the initial one; this may be related to the constant terms in right-hand sides of eqs (3), although the difference is more important due to the numerous iterations involved.

The estimated values for α can be compared to the theoretical ones, taking into account the viscous and thermal phenomena for plane wave propagation, and assuming perfectly rigid walls. The ratio of these two quantities is given by fig. 5; it shows a good agreement, with a systematic deviation from unity above 1 kiloherz. This may be related to some other calibration errors, which could be responsible for the fluctuations around the average value. It seems nevertheless that many different measurements lead to about 5 percents more absorption than calculated using the previous assumptions.

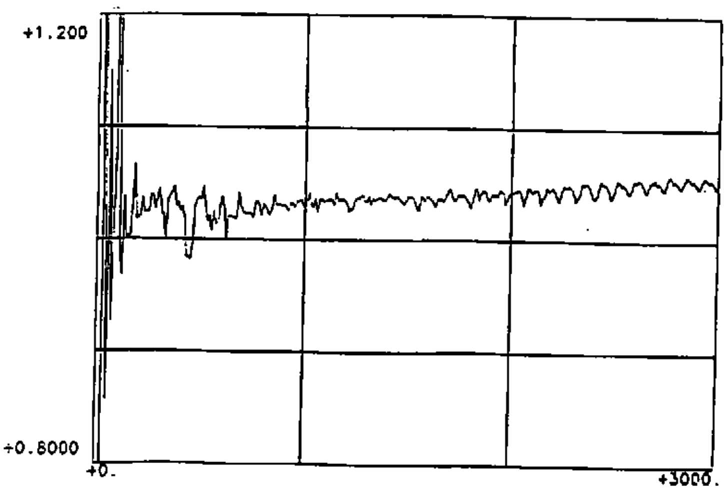

Figure 5 : ratio of measured and calculated absorption coefficient.
Frequency in Herz.

5. REFERENCES

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