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MEASURED TRANSDUCER FIELD DISTRIBUTIONS FOR IMPROVED MEASUREMENT AND IMAGING

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ABSTRACT

Real transducers often behave in a non-ideal way and radiated field measurements must be used in order to obtain more realistic theoretical models. Two approaches are described here, (1) the determination of effective geometrical parameters from a limited number of measurements, (2) source reconstruction from extensive amplitude and phase measurements over a suitable (usually plane) surface together with appropriate signal processing. The limitations of the measurement process are emphasised.

1 INTRODUCTION

This paper discusses the ways in which direct measurements of the field radiated by a transducer can be used to obtain better approximations to a realistic transducer model. The first approach is based on the definition of effective geometrical parameters for the analysis, using a very few measurements of the field, while the second is based on extensive measurements of the radiated field distribution. In discussing these two approaches, emphasis is placed on the limitations that arise from practical measurement constraints.

2 EFFECTIVE GEOMETRICAL PARAMETERS

The diffraction corrections used for an axially aligned pair of transducers of the same nominal radius, a, have as their parameters the ratio of the radius to the wavelength, λ, and the ratio of the transducer spacing, d, to the wavelength (or to the radius). These are often combined into the dimensionless parameter $a^2/\lambda d$ [1]. Given that, for velocity measurements, it is λ that is to be measured, it is clear that the use of any diffractive corrections must involve an iterative procedure until the magnitude of the corrections is considered to be insignificant. The measurement of d should present little problem, but the value of 'a' that it is appropriate to use will depend critically upon the closeness with which the actual behaviour of the transducer approaches the ideal (pistonlike) model assumed for it.

Over the last ten years advances in design and calibration of miniature hydrophone probes [eg 2,3,4] has opened up new possibilities in the quantification of ultrasonic fields. For a pistonlike disc source of radius a, the position of the last axial minimum is at a distance z_0 from the disc surface, where z_0 is given by $z_0 = a^2/2\lambda$. Location of this minimum, which is spatially well defined, in a fluid of known speed of sound, permits the calculation of an effective radius for the transducer at a given frequency of excitation. Early experiments [5] showed their effective radius to be, in general, frequency dependent, and to differ by as much as 30% from the nominal geometrical radius of the transducer element.

With the development of techniques for making measurements of the phase distribution over planes in the field perpendicular to the axis [6,7] it was possible to show, in a fairly comprehensive study [8] that the use of an effective radius gave greatly improved prediction of the actual fields radiated by four transducers. The effective radius was used in the theoretical pistonlike model to predict the phase and amplitude distributions in a given plane and these were compared with experimental measurements. The degree of agreement improved, the closer the effective radius was to its geometrical value.

The major advantages of this type of approach are several. It is relatively rapid and it requires no significant computational facilities. Most importantly it places minimal demands on the hydrophone. The hydrophone does not have to be much smaller than a wavelength (although if λ is comparable with 'a', problems will arise), and it does not need accurate calibration either in terms of directivity or frequency response.

The disadvantages of this approach are that it attempts to relate a real situation to a theoretical model which may be inappropriate and that it cannot provide information about the actual distribution. Current work indicates that the effective radius phenomenon has a physical interpretation such that in some circumstances a vibrating thin plate representation may be more realistic than a rigid piston model. In particular it appears that a smaller number of additional measurements could yield a qualitative estimate of the actual distribution, or at least be able to indicate which of a small range of possible distributions is the most appropriate [11].

This approach has been applied successfully to focused bowl transducers [9,10] although at least two measurements are needed to cover the two independent geometrical parameters that arise for such a transducer. The most appropriate appear to be the axial positions of the two minima closest to, but inside, the true focus of the transducer.

3 MEASUREMENT OF FIELD DISTRIBUTIONS

A sufficient number of measurements made over a single surface of regular geometry can be forward and back projected [12] to obtain the distribution over any other similar surface. The remainder of this contribution is concerned with the practical apsects of this procedure. The surfaces over which the measurements are made are parallel planes perpendicular to the transducer axis. Having measured the field distribution over a given plane there are a number of features which will limit the information available at any other plane (including the surface of the transducer). These are the necessarily finite aperture over which the measurements are taken, the finite size of the probe (of growing importance as the frequencies used in ultrasonic applications increase), the spatial sampling of the measurements, the presence of 'noise' (of positional or electronic origin), and the relation of the phase of interest to the plane ofmeasurement. Of these the finite area of the probe, and positional noise will be discussed in some detail. The discussion assumes linearity and a harmonic time dependence.

3.1 The effects of finite probe receiving area. In acoustic near field reconstruction measurements the measuring probe can be regarded as an ideal point detector. At ultrasonic frequencies even miniature probes have dimensions which are significant compared to the wavelength. The output signal of such a probe represents some average value of the field over the probe's sensitive area and will not, in general, be the same as that which an ideal probe would give. Limited calculations of the effect of a finite probe have been performed for the far field of a disk source [13] where it was shown that the effects of a probe of one wavelength in diameter are no greater than the errors usually associated with the measurements. A more detailed discussion is needed for the near field.

Consider a plane radiator located in the plane x=0. The field distribution over a parallel plane at a distance z from the source in the near field can be expressed as

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x,k_y)\ \exp\ [ik_x + ik_y]\,dk_x dk_y \qquad (1)$$

This is what would be measured by an ideal point probe, whereas a finite probe would yield an apparent distribution given by

$$f_m(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x,k_y)\ B(k_x,k_y)\ \exp\ [i\,k_x x\ +\ ik_y y]\,dk_x dk_y \qquad (2)$$

where $B(k_x,k_y)$ is the spatial frequency response of the measuring probe. When the usual transform [12,16] is applied a corresponding apparent source distribution $f_{sm}(x,y)$ will be reconstructed, given by

$$f_{sm}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x,k_y)B(k_x,k_y)\ \exp[ik_x x+ik_y y]\exp[-ik_z z]dk_x dk_y \qquad (3)$$

Thus if measuring probe response is known then the true source distribution can be recovered.

The Fourier transform of (2) yields

$$F(k_x,k_y)\,B(k_x k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_m(x,y)\ \exp\ (-ik_x x-ik_y y)dxdy \qquad (4)$$

The sampling theorem indicates that the complete information about the source is achieved by determining the field at a lattice of points with a spacing $\leq\lambda/2$ over the measurement plane. In the case of ultrasonic measurements the significant probe dimension may well exceed $\lambda/2$ [the probes mentioned earlier have $D/\lambda \sim 0.75$ at 1 MHz], in which case the successive areas covered by the probe will overlap. If the significant dimension of the probe is much greater than $\lambda/2$ the sampling will more closely resemble a continuous scan and the resolution will be effectively that of the probe significant dimension. The question arises, what amount of overlap [i.e. what is the maximum probe dimension], given $\lambda/2$ sample spacing, will ensure an effective recovery of the true source distribution? The answer will be dictated ultimately by the system noise. In general terms, if the probe spatial frequency response $B(k_x,k_y)$ falls to very low values over a certain spatial frequency range then a correspondingly high gain will be required in the source recovery processing, with the consequent enhancement of noise errors in that range. In particular, if the repsonse $B(k_x,k_y)$ should fall to zero, full compensation will be impossible.

Assuming a probe response of the form $J_1(\rho)/\rho$ then the first null occurs for p ~ 3.8. The null will appear in the non-evanescent spatial frequency response for probe diameters D greater than ~ 1.28λ. This suggests that reconstruction using compensation should be possible for probe diameters of at least 1λ. The sample spacing cannot be correspondingly reduced since aliassing errors would then be introduced.

3.2 PROBE POSITION ERRORS. Probe positional errors in the field sampling process, give measured values of amplitude and phase at positions $ns+\delta_n$ which differ from those at the nominal positions nz [where s is the sampling interval, n is an integer, δ_n the positional error at the nth sample, hence introducing a source of noise into the measurement and thus the reconstructions also. Consider 1-dimensional sampling of the field f(x) = with an ideal probe at a sampling interval $s = \lambda/2$. The spectrum of the strictly uniformly sampled field is

$$F_s(k_x) = \sum_{n=-\infty}^{\infty} f_n(ns) \quad \exp[-insk_x]$$

When there are positioning errors the resulting spectrum is

$$\tilde{F}_s(k_x) = \sum_{n=-\infty}^{\infty} f_n(ns+\delta_n) \exp\,[-insk_x]$$

where s_n is a random positional variable. Thus the spectrum itself is a random variable. By applying statistical estimation procedures it is straightforward to show [17] that the estimated spectrum $\hat{F}_s\,(k_x,x)$ is given by

$$\hat{F}_s(k_x,x) = F_s(k_x) \int_{-\infty}^{\infty} \exp\,[i\delta_n k_x]p(\delta)d\delta$$

where p(δ) is the probability distribution function (pdf)ξ of the positional errors. We note that the integral is a function of k_x, i.e.

$$\int \exp[i\delta_n k_x]p(\delta)d\delta = \Phi(k_x)$$

and this will modify the true spectrum in a manner which depends on the form of the pdf. For a uniform pdf $\Phi(k_x)$ is of the form

$$\frac{\sin(ak_x)}{ak_x} \quad \text{where } a = \frac{\beta k_x \lambda}{2} \text{ and } 0 < \beta < 1.$$

It is evident that the positional errors introduce an attenuation at high frequencies. Since the source reconstruction algorithm is linear and in particular only modifies the phases of the individual spatial frequencies it will yield the same signal/noise ratio at the reconstructed source plane. The signal/noise ratio is given by [17]

$$\frac{S}{N} = \frac{\int_{-\infty}^{\infty} |F_s(k_x)|^2 dk_x}{\int_{-\infty}^{\infty} \left(|\hat{F}(k)_x|^2 - |F_s(K_x)|^2 \right)} = \frac{1}{\int_{-2\pi/\lambda}^{2\pi/\lambda} [1 - \mathrm{sinc}^2 (ak_x)] dk_x}$$

Evidently the average noise spectrum increases with increasing frequency so that high frequency components will predominate. Thus an attempt to restore the true source distribution by simple inverse filtering will only be achieved at the expense of degrading further the S/N ratio of the reconstructed image.

However, efficient algorithms have been devised which are able to recover the true field distribution with good efficiency.

4 PRELIMINARY RESULTS

A series of model calculations has been commenced in order to evaluate the method prior to its experimental application. The field of a circular piston source, diameter 10λ, as determined by circular receiving probes of various diameters in planes at different distances from the source, have been calculated using the method described in reference [8]. Figure 1 shows clearly the effect of probe smoothing on the field distribution at a distance equal to the source diameter (10λ).

Figures 2(a)-(d) show a selection from a series of reconstructions of the source distribution. The data were taken over successively larger radii of the image plane. There is no significant difference between the 2(c) [30mm radius] and 2(d) [60mm radius] reconstructions.

In fact the reconstructions show negligible changes for radii of 20mm or greater. This represents the region where the image plane distribution has fallen to less than 10% of its peak value. The method of reconstructinoyields an acceptable approximation to the postulated 'top hat' source distribution. In particular the constancy of the phase over the source diameter is tobe noted.

5 CONCLUSION

Initial investigations of source reconstructin from data deduced as relatively short distances from the source have yielded promising results and work is continuing in improving the reconstruction algorithm.

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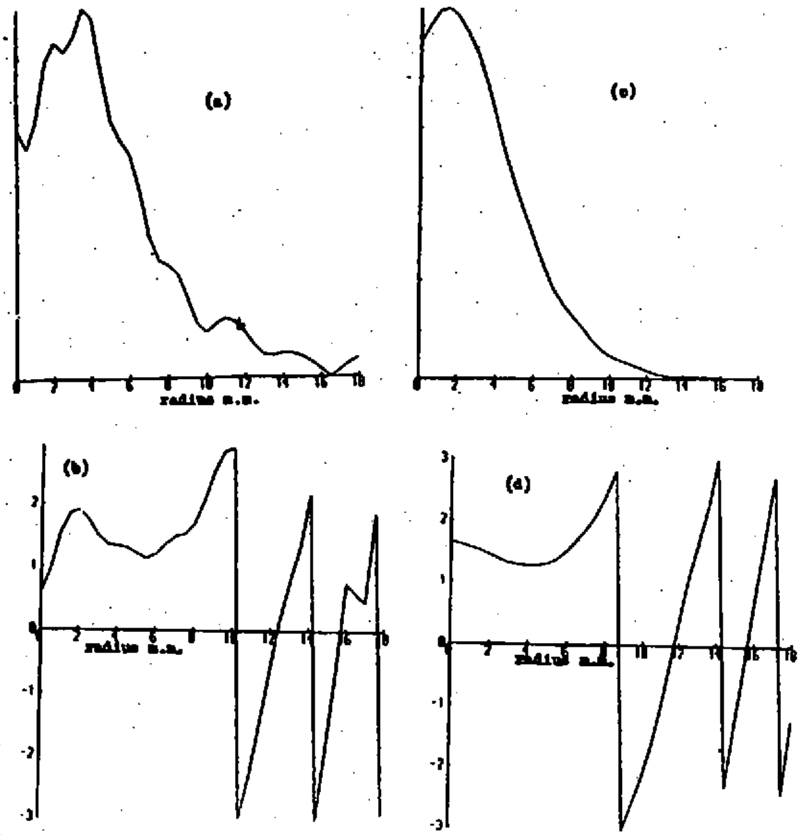

Fig 1. Smoothing effect of finite probe size on measured field distribution of a circular piston source.
(a) Amplitude (b) phase for 0.67λ dia. probe (c) amplitude (d) phase for 2λ dia. probe.

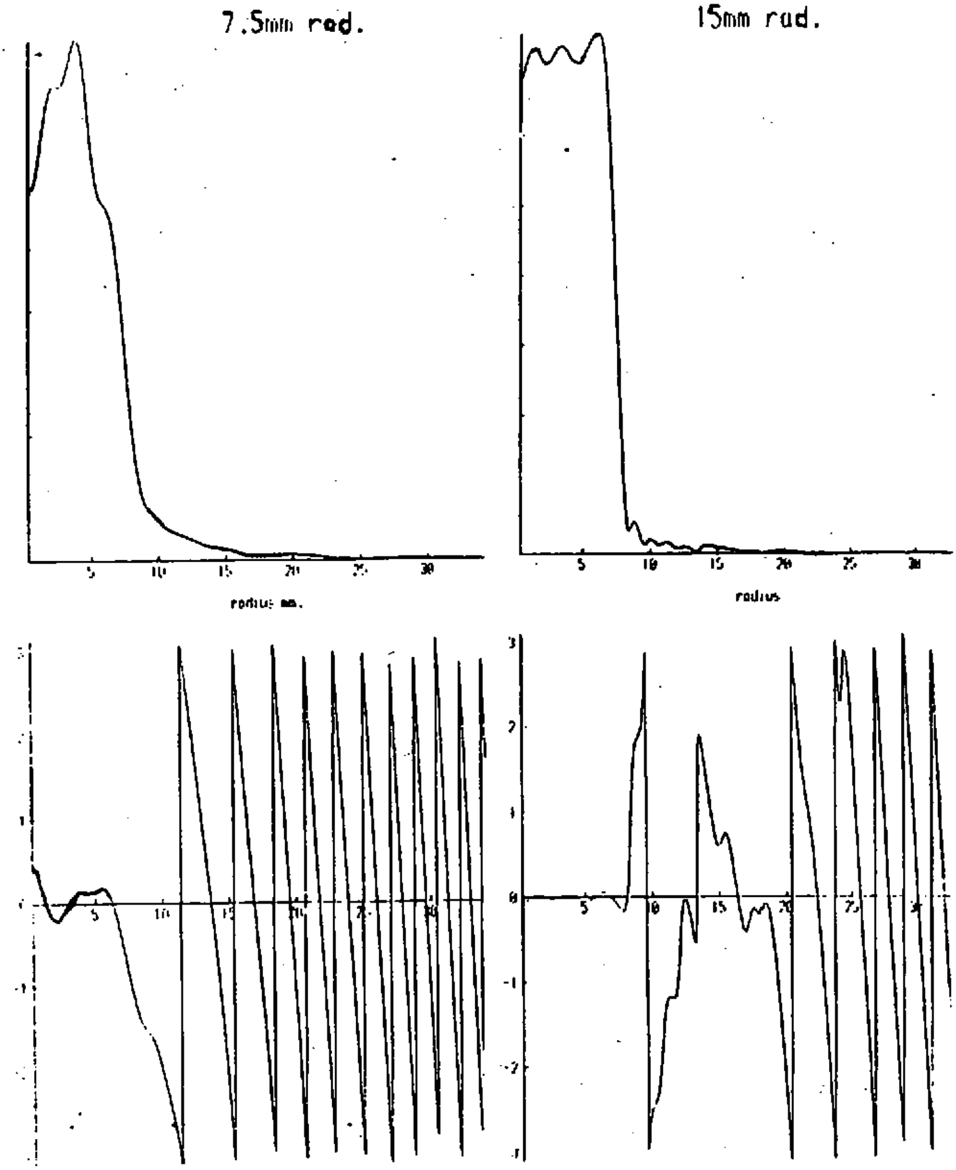

Fig 2 (a),(b) Reconstructed amplitude and phase distributions from image plane data out to radii 7.5 mm. and 15 mm.

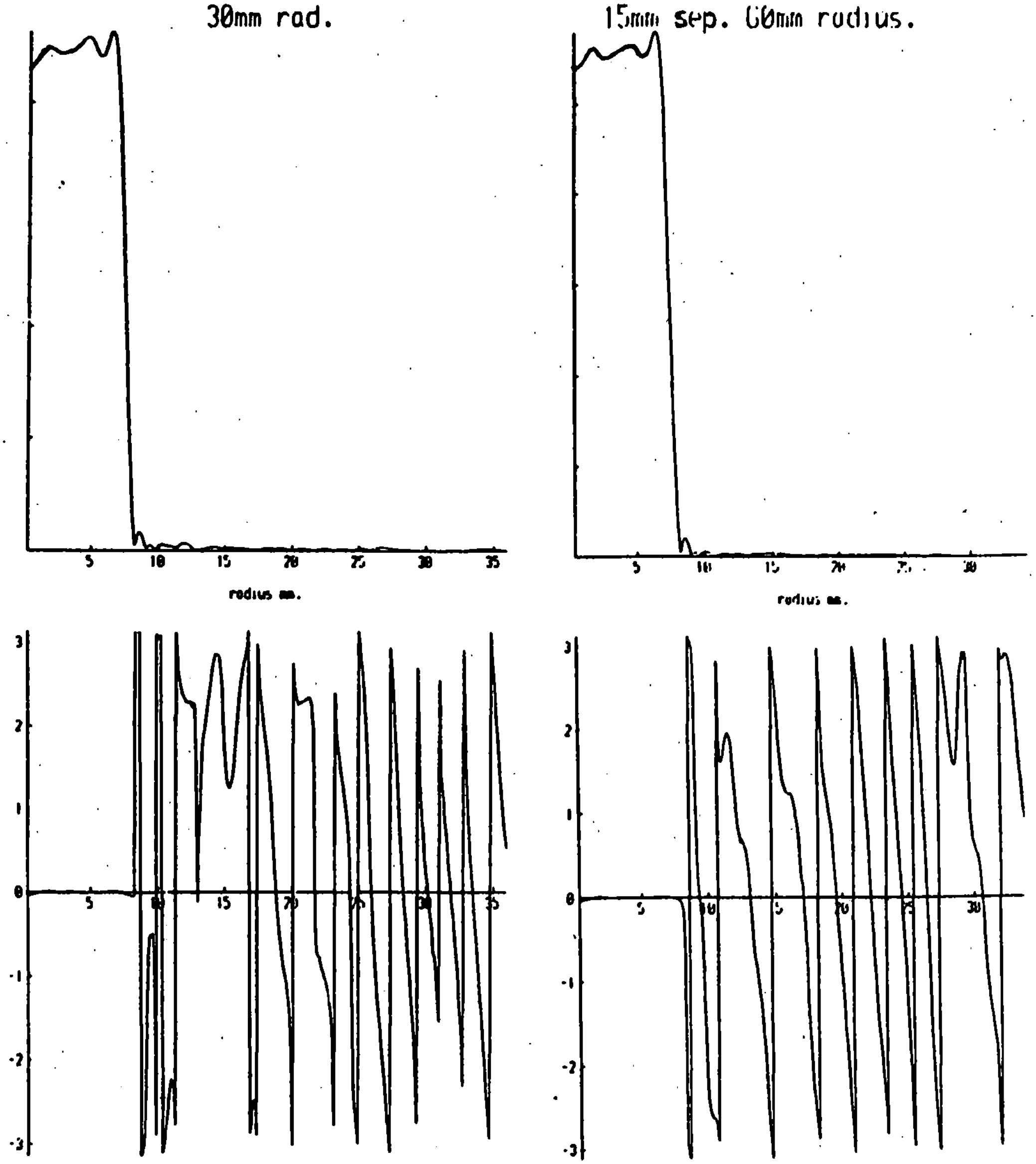

Fig 2 (c),(d) Reconstructed amplitude and phase distributions from image plane data out to radii 30 mm and 60 mm.