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THE EFFECTIVE RADIUS CONCEPT FOR PIEZOELECTRIC ULTRASONIC TRANSDUCERS AND ITS PHYSICAL INTERPRETATION

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1. INTRODUCTION

The effective radius concept has proved to be a useful indicator of the radiation properties of both ultrasonic plane disc and focused spherical cap transducers [1,2]. In particular the position of the last maximum or minimum of the axial pressure variation in the fresnel region of a plane disc radiator provides a simple method for determining the effective radius of the equivalent piston-like source. In the case of spherical cap radiators the effective geometric parameters can be deduced from a measurement of the positions of the two minima closest to the focal maximum, towards the transducer.

An understanding of the nature of the effective radius phenomenon would evidently be valuable in attempting to characterise the nature of the vibration of the transducer radiating surface.

The starting points of this investigation were the observations that (i) effective radii could differ from the physical radii by up to 30% in some situations, (ii) axial pressure variations exhibit minima rather than the zero values predicted for ideal piston and spherical cap sources.

The present contribution offers a physical interpretation of the effective radius and explores the possibility of gaining a qualitative estimation of the vibrational properties of the radiating surface.

2. THEORETICAL BASIS

The well known theory of radiation from a uniform piston disc in an infinite, rigid baffle predicts extrema at axial distances Z_n, given by

$$Z_n = \frac{4a^2 - n^2\lambda^2}{4n\lambda} \tag{1}$$

where a is the disc radius, λ is the wavelength and n is an integer.

The maxima and minima are given by odd and even values of n respectively and the values of the minima are identically zero.

The expression (1) shows a simple relationship between the positions of the extrema and the radius of the radiator. In practice the position of the last maximum (n=1) or, preferably since it is more precisely defined, the last minimum (n=2) are used to determine the effective radius. The positions of the extrema have a simple physical interpretation in terms of fresnel zones. If n is even there is an axial zero and at this axial distance the disc comprises exactly that (even) number of fresnel zones. Similarly if n is odd then there is an axial maximum and at the corresponding axial distance the disc comprises exactly that (odd) number of fresnel zones. Thus maxima result from a single uncompensated zone and consequently all maxima have the same amplitude. Consider the last minimum (n=2). In this case the disc comprises exactly two fresnel zones. Since the phase relationship between the zones is fixed by the geometry it is evident that, to a first order approximation, any amplitude shading access the disc cannot appreciably affect the position of the axial maximum but only its magnitude. This suggests that the non-zero values of the minima gives information about the amplitude distribution over the disc, while any significant change of the position of the minima away from those predicted for the uniform disc is an indication of non-piston-like behaviour due to a phase distribution over the source.

The question then arises; how might a phase distribution be produced? A more realistic representation of the transducer could be that of a plate or membrane clamped at its edge rather than that of a piston in an infinite baffle. This gives a justification for the assumption of some form of amplitude shading adopted by some investigators [3]. This idea can be carried further. It is well known that clamped plates and membranes exhibit complex higher order modes of vibration, in particular pure radial modes. These modes are characterised by a series of concentric nodal circles which divide the disc into a set of zones whereby adjacent zones vibrate in antiphase. A simple model based on this idea has been developed [4] and a series of model calculations has been carried out.

3 THE MODEL

(a) DISC RADIATORS

The initial model assumed that the radial modes can be usefully approximated by a central uniform piston surrounded by a set of concentric annular zones behaving in a piston-like way, but with adjacent zones vibrating in antiphase. The rectangular approximation for the "n=1 mode" is shown in figure 1. The pressure amplitude at a position R from the system is readily deduced by extension from the result for a single piston [5] and is found to be

$$p(R,O,t) = \frac{i\rho_o \; c \; U_o k}{} \, e^{i\omega t} \left[\int_0^{x_1} \frac{\exp[-ik\sqrt{r^2+\sigma^2}]}{\sqrt{r^2 + \sigma^2}} 2\pi\sigma \, d\sigma + \sum_{i=1}^{n} b_i \int_{x_i}^{x_{i+1}} \frac{\exp[ik\sqrt{r^2 + \sigma^2}]}{\sqrt{r^2 + \sigma^2}} 2\pi\sigma \, d\sigma \right] \qquad (2)$$

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where ρ_o, c are the density and the speed of sound in the medium and U_o is the excitation velocity amplitude. For the n=1 case the axial pressure amplitude, neglecting the harmonic time dependence, can be shown to be, for R $\gg$ a

$$p(z,O) = 2\rho_o\, c\, U_o \left[(1+b)\sin^2\left[\frac{kx^2}{2z}\right] + b(b+1)\sin^2\left[\frac{k(a^2-x^2)}{2z}\right] - \sin^2\left[\frac{ka^2}{2z}\right]\right] \tag{3}$$

A series of computations has been carried out for the "n-1 mode" in which the amplitude factor b_1 of the outer annulus was varied in the range $0 \leq b_1 \leq 1$ for $x_1 = 0.8a$ and $a = 5\lambda$. The results are summarised in Fig.(2).

It can be seen that positons of the extrema have been shifted towards the transducer thus indicating a smaller effective radius. Furthermore the minima are no longer zero as predicted for the uniform piston and the depths of the minima are directly related to the amplitude distributions.

A further set of calculations were carried out in which the value of b_1 was held constant at $b_2 = -0.3$ and the radial position of the node, x_1, was varied. These results are summarised in Fig.(3) in which the effective radius a_{eff} is plotted against x_1. A linear relation between a_{eff} and x_1 as calculated from the position of the last axial minimum is evident - indeed it is seen that $a_{eff} = x_1$.

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These results suggest that the effective radius can be identified with the radius of the nodal circle; the central zone provides the piston-like behaviour and the outer antiphase annulus serves to modify the magnitudes of the extrema, in particular contributing to the non-zero values of the minima. This interpretation appears to hold only for values of x_1 down to $x_1 = 0.29a$ when the positon of the last axial minimum suddenly changes to a much greater distance from the transducer and increases as x_1 is decreased further. The reason for this behaviour is as follows. At this critical axial distance ($z = 7.7\lambda$) x_1 can be considered as one of the three fresnel zones within the overall disc radius of 5λ. For values of $x_1 < 0.29a$ the position of the last axial minimum is now determined by the distance at which the outer annulus represents two fresnel zones and the calculated values of effective radius will now increase as x_1 decreases. It is interesting to note that the effective radius determined from the penultimate minimum using Eqn.(1) follows the original $a_{eff} = x_1$ relationship below the critical value of $x_1 = 0.29a$.

3b WEAKLY FOCUSED SPHERICAL CAP TRANSDUCERS

The model just discussed has also been applied to weakly focused spherical cap transducers. O'Neill [6] has developed an approximate theory of spherical shell radiators which is valid for aperture radii $a \gg \lambda$. He shows that the axial pressure

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$$p(z,0) = \frac{\rho_o\ c\ U_o}{(1-{}^{z}/A)} \int_{z}^{B} \exp[-ikz]\ ds \qquad (4)$$

can be expressed as where A is the radius of curvature of the transducer, z is the axial distance from the pole of the transducer to the point ofmeasurement, s is the idstance of a point on the spherical surface to the point of measurement and s =B for points on the transducer edge.

A series of investigations by one of us has extended the effective radius concept to the effective geometrical parameters (effective radius a_{eff}, effective cap depth h_{eff} and effective radius of curvature A_{eff}) for the spherical cap transducer. These quantities are readily deduced from measurements of the positions of the last two minima towards the transducer away from the focal maximum (the "two node" method) (Chivers et al [7]equations (14) to (18) inclusive).

In general it is found that the axial pressure distributions calculated from the effective parameters agree reasonably well with the measured axial pressure distributions. However, as with the disc radiators the amplitudes of the extrema differ from the predictions of the O'Neill theory, which assumes a uniform displacement amplitude over the transducer surface.

By analogy with the model for the disc radiator discussed earlier the O'Neill model has been modified to accommodate phase and amplitude variations by dividing the spherical surface into a series of concentric radial zones centred on the transducer's primary axis.

The axial pressure distribution for this model is given by

$$p(z,0) = \frac{\rho_o\ c\ U_o}{(1-{}^{z}/A)} \left[\int_{\omega}^{B_1} \exp[-ikz]ds + \sum_{i=1}^{N} b_i \int_{B_i}^{B_{i+1}} \exp[-ikz]ds\right] \qquad (5)$$

where b_i is the amplitude factor of the i-th zone ($0 \leq |b_i| \leq 1$), B_i is the distance from points on the i-th nodal circle to the point of measurement and B_{N+1} =B.

Preliminary calculations have been carried out for a simple two-zone model. The results are broadly similar to those obtained for the disc radiator. The position of the focus and the positions of the other extrema depend upon the radius of the nodal circle and the values of the pressure amplitude extrema depend upon the relative zone amplitudes.

A comparison with an experimentally determined axial pressure distribution is given in Fig.4 which shows the results for a transducer with A = 10 cm. a = 0.75 cm and f = 6 MHz. The model was chosen with these parameters and also x_1 = 0.65a and b_1 = -0.3. It can be seen that the predicted positions of the focus and the other extrema correspond well with the measured values and there is also good agreement with the values of the minima, but the calculated values of the maxima are too large.

4 CONCLUSIONS

The model presented here offers a physical explanation for the effective radius concept for the disc radiator and can be extended to focused spherical cap radiators. The simple model presented here can be readily extended to more realistic amplitude distributions. Conversely, the model appears to offer the possibility of suggesting the true amplitude distribution of a radiator. As such this would represent a useful adjunct to the effective radius concept. Its ultimate validity and usefulness must, however, be verified by comparison with reliable determinations of true source distributions using field reconstruction techniques from extensive measurements of the radiated acoustic field.

5 REFERENCES

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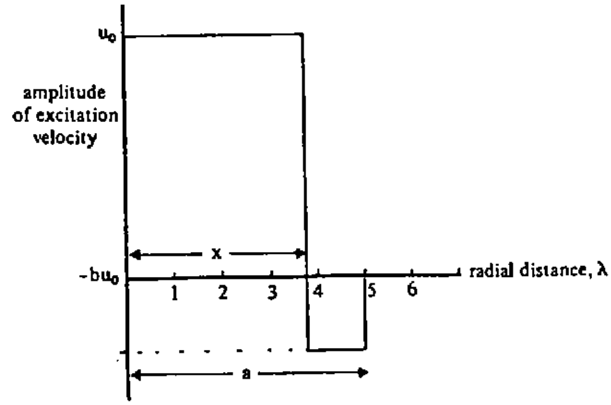

Figure 1 Amplitude distribution used for the "n=1 mode" model.

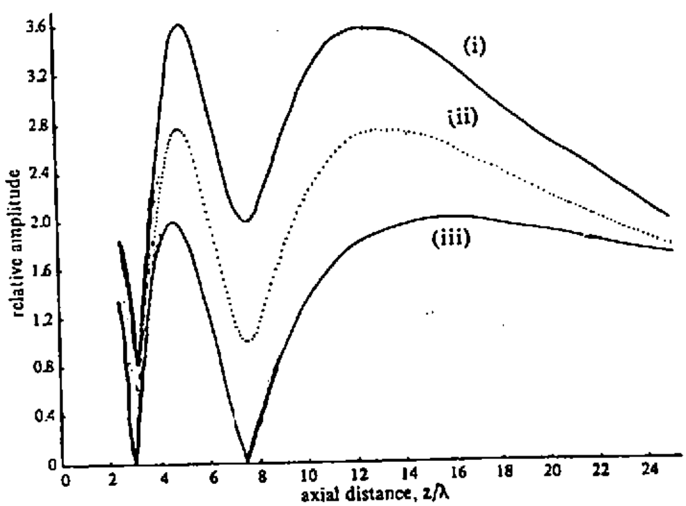

Figure 2 Calculated axial pressure distributions for the "n=1 mode" model for various values of the parameter b_1.
(i) b=0, (ii) b=-0.3, (iii) b=-1.0

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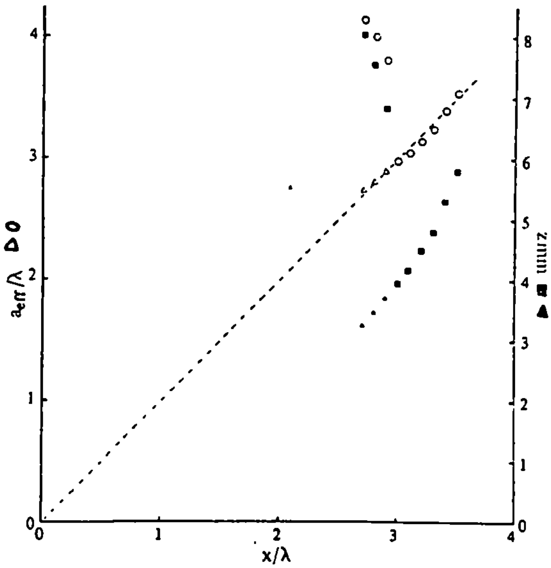

Figure 3 The dependence of a_{eff} and z_{min} on the position of the node x_1

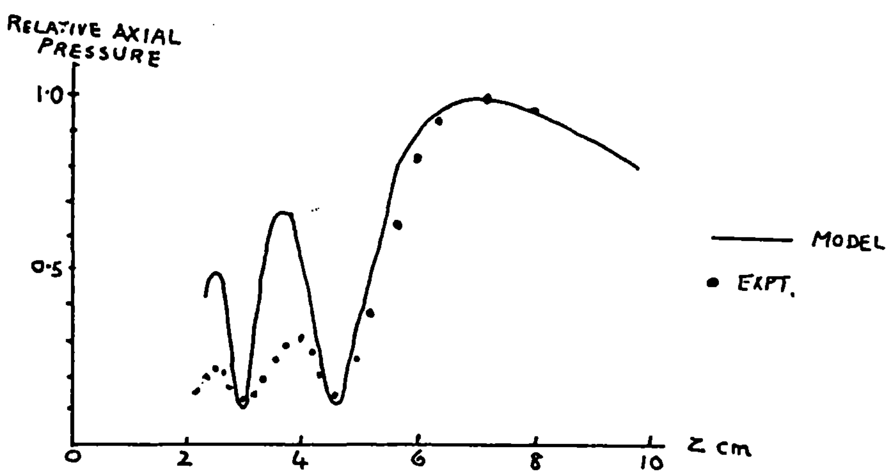

Figure 4 Calculated axial pressure distribution for a spherical cap transducer with A=10cm, a=0.75cm, x_1=0.65a at f=6 MHz, measured values