Proceedings of The Institute of Acoustics

PROBLEMS IN THE REALISATION OF TRANSDUCERS WITH OCTAVE BANDWIDTHS

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GENERAL CONSIDERATIONS

This paper describes the progress that has been made into the theoretical development of the transducer elements for a multi-element array which has the form of a spherical cap; by a particular choice of excitation function this array has a constant beamwidth over a wide range of frequencies. A major requirement is for the minimum variation in the sensitivities on transmission and reception over the frequency range 27 to 54 kHz; the array itself is designed to meet this specification provided that the individual elements have a reasonably uniform response. The Marine Laboratory at Aberdeen carried out the basic design of the array according to the theory developed by Rogers and Van Buren [1], assuming ideal elements, and the present investigation was concerned solely with the elements under the constraint of their working into the radiation load on this array.

The design of the element is based on the traditional "Tonpilz" element, comprising essentially a head mass radiating into water, a compliant stack of piezo-electric rings and a tail mass acting as a counter-mass with modifications for achieving an extended bandwidth. By the use of electro-mechanical analogies this structure can be regarded as being similar to an electrical filter, and the design of the element with an octave bandwidth proceeds by adding sections to the basic filter structure, a mechanical section between the basic head mass and the radiation load and an electrical section at the electrical terminals of the ceramic stack. This kind of arrangement was investigated many years ago [2]; although the work was not fully published to the present authors' knowledge, brief reference was made to it [3]. An alternative, but essentially similar, solution has been described recently [4], in which one or two quarter-wave coupling sections are used between the basic head and the load. This was a purely theoretical analysis which ignored practical considerations such as bending modes of vibration of the head sections and, more, seriously, the variations of radiation load with frequency and with the arrangement of elements in an array; however the results indicated that bandwidths exceeding one octave can be achieved.

THE RADIATION LOAD

Because each element has a radiating face with a diameter which is small in wavelengths, this being a condition for the avoidance of bending resonances in the head of the element and for obtaining a smooth directional response from the array, the radiation load on each element varies significantly with frequency and with the geometry of the array.

The two limiting conditions which can be readily analysed theoretically are for an isolated single element in an infinite rigid baffle and for a completely filled infinite planar surface; fortunately the Rogers and Van Buren array has similar loading characteristics to the latter. However in a practical array the individual elements must be separated in some degree for purely mechanical reasons, and therefore the radiation load on each is variable and complex, being basically that on an isolated element modified due to inter-element coupling. This load can be estimated from some data given by Stansfield for planar arrays [5] as that for the single element corrected by terms dependent on the proportion of the array surface covered by active transducer faces, i.e. on the packing factor. The array design produced by the Marine Laboratory specified elements of 20 mm diameter, equivalent to 0.36 and 0.72 wavelengths at the ends of the frequency band, and with an inter-element spacing of 25 mm centre to centre, the elements being arranged in concentric circles. This gives a packing factor of 0.484, resulting in a rather poor radiation load on each element. If the elements can be packed on a hexagonal pattern with a centre to centre spacing of 22 mm, the packing factor is improved to 0.845; the radiation load on each element is then rather greater with a much reduced reactive component.

The radiation loads for the three conditions are shown in fig.3, as plots of resistance against reactance. The array loads were calculated from Stansfield's data at four frequencies and interpolated as cubic functions of frequency, and that on a single element from standard formulae [8]. The design of the element is based on the more closely packed arrangement, but it is recognised that some modifications will be needed to the original mechanical design of the array and to its excitation function.

EQUIVALENT CIRCUITS FOR "TONPILZ" ELEMENTS

The conventional element

The basic element as mentioned earlier comprises essentially two masses connected by a compliant member, one mass being loaded by radiation into water. The compliance is coupled to the electrical circuit through piezo-electric effects, and this coupling is represented in the equivalent circuit by a transformer. By using the electro-mechanical impedance analogies of inductance being equivalent to mass and capacitance equivalent to compliance, a "lumped" version of the "Mason" equivalent circuit can be derived [6] for the case where the lengths of the different sections are small in wavelengths, which is generally the case for wide-band "Tonpilz" elements. This is shown in fig.1, where Mh is the sum of the head mass, half the mass of the bolt and half the mass of the ceramic stack. Similarly Mt is the sum of the tail mass and half the bolt and stack masses. Cc, Cb and Ct are respectively the compliances of the stack, the bolt and the bonds (the compliances associated with the head and tail masses are neglected in the circuit shown). Zr is the complex radiation load, C1' is the total clamped capacitance of the stack and N is its electro-mechanical transformation ratio. If the bandwidth is relatively small ($Q > 5$) and the tail mass is appreciably heavier than the head , the circuit can be simplified by combining the head and tail masses in parallel and modifying the load resistance [7]. The form of the resulting equivalent circuit, including a parallel tuning inductance can be shown to be that of a single half-section band-pass filter having a low-pass equivalent. The element can now be designed using conventional filter theory, as long as it can be reasonably assumed that the radiation load is constant over the working band. Two simplifications have been made in this argument; firstly the bonds and connections within the stack have been included by implication using values for the compliance and clamped capacitance for the stack which are effective values, not those for the ceramic alone, and secondly by assuming that the usual pre-stressing bolt has been decoupled from both head and tail (this is necessary for mechanical reasons as discussed in a later section).

The element with coupling sections

In electrical filter design, the characteristics of simple filters can be improved by the addition of more sections, although the design process from first principles becomes more complicated due to interaction between the sections. The same ideas can be applied to "Tonpilz" elements; purely electrical sections can be added at the electrical terminals with very few limitations on their form, which can be of lowpass, band pass or high pass type, but there are severe restrictions on the form of sections added on the mechanical side. If the mechanical elements are short in wavelengths, so that they can be regarded as "lumped" compliances or masses rather than as short lengths of transmission line, then added masses can only appear in the series arms in the equivalent circuits, and added compliances, while appearing to be mechanically in series, can form only shunt arms. The only exceptions to this rule are the shunt mass representing the unbacked tail mass and the series compliance arising in the piezo-electric part of the circuit. Therefore in general the added mechanical sections can only be of low pass form, and this suggests that for symmetry the added electrical sections could also be of low pass form. Thus the general form of the complete equivalent circuit does not conform to any of the basic types of electrical filter, and so standard design procedures cannot be applied directly.

If a mechanical coupling section comprising an extra head mass and a compliant tube are connected to the head of the basic sandwich structure as illustrated in fig.2, then additional components are introduced into the circuit. These are depicted in fig.4(a), where Mf is the mass of the extra head, Cf is the compliance of the tube connecting the two heads and Zr is the radiation load impedance. Electrical sections can be added at the electrical terminals, also shown in fig.4(a). These may comprise one or more of a shunt tuning inductance L1, a series tuning inductance Ls and a parallel tuned circuit Cp and Lp. Although direct analysis of the circuit of fig.4(a) by computer enables admittance loci to be readily evaluated and hence bandwidths deduced, it is initially necessary to make some intelligent guesses regarding the likely values of the circuit components. Optimisation is then achieved by an iterative process as discussed in the next section. Another approach is to reduce the circuit by further transformations into a recognisable filter section so that optimisation by synthesis can be attempted, and this is discussed in a later section.

THE APPROACH BY ANALYSIS - OPTIMISATION

General principles

The basis of the optimisation was to aim for the smallest variation in conductance over the bandwidth of 27 to 54 kHz, so that on transmission from a constant voltage source the radiated acoustic power would be most nearly constant. Since the width of the beam and its shape are constant within fine limits, this should lead to the least variation in source level on axis. It is highly desirable that the susceptance is kept low, and this can be corrected to some extent by the addition of a carefully chosen parallel tuned circuit at the input, thereby reducing the reactive power demanded from the power amplifier. The variation in response can also be reduced by matching the source resistance to the load presented by the array, and by reciprocity the best response as a receiver is obtained by feeding the array into a matched electrical load.

The starting point in the design was a conventional "Tonpilz" element, designed for a modest bandwidth and including a constraint that the effective coupling coefficient of the piezo-electric stack should be significantly less than that of the ceramic alone as given in the manufacturer's data. The reasons for this were firstly that the stack includes electrical contacts and a number of bonds which are relatively compliant and secondly that head and tail masses are not rigid in practice so that their compliances have to be allowed for. Therefore the ratio of the clamped to the motional compliances C1/Cm was taken as 4.0 compared with the theoretical value for PZT-4 in the longitudinal mode of approximately 1.0; the effective coupling coefficient is thus 0.45, which is a practical value found for conventional elements resonant around 40 to 50 kHz, compared with 0.70 for PZT-4 alone. If the coupling coefficient in the actual element is found to be higher than this, then it can be easily reduced by the addition of parallel capacitance on the electrical side.

Internal resonant frequencies

It is obvious from the equivalent circuit that there are a large number of variables, and from some general rules for the setting up of electrical ladder filters and from trial runs specific rules were formulated for internal resonances in the equivalent circuit. The characteristic frequency used was the arithmetic mid frequency 40.5 kHz, since the response was found to be approximately symmetrical on a linear, not a logarithmic, scale of frequency, and various combinations of components were made to resonate at this frequency. The first combination was the outer (radiating) head mass Mf, including a contribution from the reactive part of the radiation load, with the coupling compliance Cf, this corresponding to the mechanical anti-resonance of the unloaded coupling section. The next resonance is for the parallel combination of the head and tail masses of the basic element, Mh and Mt, with the series combination of the motional and clamped compliances, Cm and C1, and this corresponds to the electrical anti-resonance without the coupling section. The final combination is the added series tuning mass Ms, corresponding to the inductance Ls transferred from the electrical side to the mechanical through the electro-mechanical transformer, with the clamped compliance C1, and this corresponds to the electrical tuning of the series inductance with the clamped capacitance. Parallel tuning of the clamped capacitance is not used in this version of the circuit. This series tuning was however subject to small adjustments, of the order of 5%, in order to equalise the two outer peak conductances; since in the final realisation Ms is represented by an inductance, this is an entirely practical adjustment with no mechanical implications. The parallel tuned circuit Mp and Cp (the mechanical equivalents of the electrical components Lp and Cp'), which is added for the compensation of the overall susceptance, would be designed on slightly different lines so as to minimise the variation in the phase angle of the admittance over the working bandwidth, but its resonant frequency is likely to be close to the mid frequency.

Design guidelines

The parameters which were varied in the search for optimum performance were the outer and inner head masses, the tail to head ratio Mt/Mh and the coupling coefficient represented by the ratio C1/Cm; the remaining components were determined by the internal resonances described in the previous section. The basic masses Mh and Mt were initially kept to a minimum with the aim of maximising the bandwidth of the basic "Tonpilz" structure, but in the end this minimum is dictated rather by practical limitations on the physical dimensions of the piezo-electric stack. The compliance of the stack is a function of the area/length ratio and its mass is a function of the length x area. Thus the size of the stack can be varied for constanct compliance, but its mass must be included by corrections to the actual masses of the head and tail. So that these corrections are kept to a minimum, the stack needs to be kept small, and the required shape in small sizes proved to be impractical at the frequencies under consideration. Hence the actual head and tail masses chosen for the final design were somewhat greater than desired, leading to greater variations in the conductance over the middle part of the frequency range. In the conventional "Tonpilz" design the tail/head mass ratio is generally much larger than unity, even as high as 10:1, but it was found that in this design a ratio 1.2 to 1.3 was optimum. The principal effect is that with a high ratio the response is essentially double peaked and with a low ratio it is triple peaked with an increased bandwidth. The correct choice of Mt/Mh should give three equal peaks (subject to some adjustment by the series tunign Ms); there was a negligible effect on the bandwidth when Mt and Mh were varied by 20% keeping the coupling section unchanged and adjusting C1 and Cm for correct tuning. The value chosen for the compliance ratio C1/Cm, namely 4.0, seems to be a suitable value, but large changes, from 3.3 to 5.0, have only small effects on the bandwidth. Increasing the ratio, thereby reducing the effective coupling coefficient of the stack, leads to marginally reduced bandwidth, but it also raises the conductance of the central peak. The design of the coupling section has the greatest effect on the overall performance, effecting both the bandwidth (to a small degree) and the "peakiness" of the response (to a large degree). In terms of its behaviour as an impedance transformer in increasing the effective radiation load presented to the basic element, as outlined previously, only modest step-up ratios were considered, of the order of 2.0, because for high ratios the variation in the impedance presented over the pass band would be too great.

THE APPROACH BY SYNTHESIS - FILTER THEORY

The strategy of the synthesis technique is to convert the mechanical components in the circuit of fig.4(a) into a half-section filter and then to match the load and source resistances to the characteristic resistance of the filter. With this in mind the T-network comprising Mh, Cs, Mt and Cm is transformed into a half-section with an ideal transformer, as shown in fig.4(b). Only the mechanical components of the equivalent circuit are shown for convenience in this figure. In this derivation the reasonable approximation is made that the parallel resonance of the tail mass, Mt, with the bond compliances, Cs, is very much higher than the operating frequency. p is the ratio of the ideal transformer introduced in the network transformation; it is expressed in terms of the parameter m, the ratio of tail to head mass. Finally the parallel components can be transformed into the parallel circuit shown in fig.4(c), which has the desired form of a half-section bandpass filter.

However, in this filter the load resistance, R0 and mass, M0, are unfortunately frequency dependent. Therefore, matching the load resistance to the characteristic resistance of the filter at the centre frequency, in order to optimize the bandwidth, will only be an approximation. Nevertheless, it does give some guidance as to the choice of component values and the results obtained by this method are in reasonable agreement with those obtained from the computer analysis.

For a matched design with specified resonant frequency, radiation load and ceramic stack compliance and fed from a matched source it may be shown that the ratio, m, between the tail and head masses of the basic sandwich, i.e. Mt/Mh, is given by:

$$m = \frac{(1 + WQ) \pm (3WQ - W^2 + W^3Q + Q/W)^{\frac{1}{2}}}{(1 + 1/W^2 - QW)} \qquad \text{-------------(1)}$$

where W is a filter parameter given by the square root of the ratio of the ratio of the series to parallel compliances in fig.4(c), and the parameter Q depends upon the radiation resistance, Rr, the centre frequency, fo, and the compliance of the stack:

$$Q = (R_r 2\pi f_o C_m)^{-1}$$

The value of W defines the filter bandwidth; for example for a one octave band W should equal 0.5. For a bandwidth greater than one octave W > 0.5. Therefore for a specified Q and bandwidth the ratio m may be determined. Setting the resonances of the series and parallel tuned circuits of fig.4(c) to fo, then enables the values of the components in the circuit to be evaluated. In this synthesis fo was chosen to be the geometric mean of the two half-power frequencies.

For example; with W = 0.5 and with the following specifications:

R_r = 312 rayls, f_o = 38.2 kHz and $C_3 = 7.6 \times 10^{-9}$ m/N

then Q = 1.76. Using these values in equation(1) gives m = 1.06. From this the component values shown in Table 1 may be evaluated.

If different values of stack compliances are specified then alternative designs will be obtained. The merits of any particular design would then be judged upon its ability to be realised practically.

RESULTS OF THE THEORETICAL DESIGN

The component values selected as the result of the two approaches are listed in table 1, and the calculated admittances, referred to the mechanical side of the electro-mechanical transformer, are shown in fig.5. In fig.6 there is shown the admittance for the analytical design with an extra parallel tuned circuit for correction of the susceptance; this has been optimised for the minimum phase angle over the working bandwidth.

It is important to note that the design by analysis has series tuning (without susceptance compensation for fig.5(a)), whereas that by synthesis has parallel tuning of the clamped compliance; in all other respects the two designs are very similar. These calculations are with a radiation load corresponding to a packing factor of 0.845 as shown in fig.3, and substantially different curves (not included here) are obtained for the load on an isolated element in a rigid baffle and for a constant resistive load without a reactive component. These are important practical considerations for the testing of individual elements, since in the absence of the complete array this can be done readily either by mounting a single element in a housing for radiation into water or by loading it by a long rod of a suitable material such as "Perspex", with a diameter chosen to present the right resistive load. The analytical design, which as noted earlier is not quite optimum due to restraints in the practical realisation, has almost exactly the required octave bandwidth between the points at which the conductance falls to half the outer peak values, and the variation in conductance within the passband is just within a range of 2:1 equivalent to a 3 dB change in sensitivity. By the fortuitous choice of the initial component values there were no difficulties in achieving this bandwidth, once it was realised that a near-unity value for the tail/head ratio is required. In fact it is not immediately obvious how the design can be modified for either a much wider bandwidth or a much narrower one; on the other hand the design by synthesis gives a direct indication of how to proceed in this way.

For the results by synthesis the bandwidth from the locus of fig.5(b) is apparently less than the desired one octave. This arises because the synthesis is based upon the transfer from a matched source to the load, and the manner in which fig.5(b) was computed takes no account of this. If the value of W is increased then for the same value of C1/Cm there is a considerable change in the shape of fig.5(b), although the bandwidth based on the variation of conductance is increased as expected. The synthesis shows that an increase in W requires an increase in m and a decrease in the ratio of C1 to Cm (if parallel tuning is used), i.e. a larger bandwidth requires a larger value for the ratio of tail to head mass and a higher coupling coefficient. It may be shown that the basic sandwich element, without the mechanical filter, obeys the same criteria.

PRACTICAL CONSIDERATIONS

At the time of writing no elements have been made, but some thoughts have been put into practical realisation. The basic element, without the coupling section, is not expected to present any problems, although the dimensions are quite small. The coupling section does introduce difficulties, because the level of the mechanical impedances is quite low. It is proposed to use a low acoustic impedance, low density material, such as "Perspex", for both the compliant section and the outer head mass. The mass of the compliant section can be easily allowed for by corrections to the masses of the inner and outer heads, but the compliance of the outer head is more of a problem. The effective resonance of the coupling section as a whole is being investigated by finite element techniques, which will also demonstrate whether the bending of the head is likely ot be significant. An added difficulty in the structure is that the usual pre-stressing bolt would add too much stiffness and mass if it is attached solidly to the head, and also its overall length is likely to be a quarter of a wavelength or more (it is used to retain the element in the housing and so it is appreciably longer than the element). Therefore it is intended that the bolt should be decoupled by the use of disc spring washers from both heads and from the tail, although this does lead to problems in the detailed design at the outer head.

Another practical point concerns the electrical loading. Ideally it would be advantageous to drive the array from a conjugately matched source, but a resistively matched one would be simpler and nearly as good if the susceptance correction referred to earlier is optimised. In this way the variation in source level could perhaps be reduced to 2 dB or less. On reception the array should be matched in the same way, and by reciprocity it should have the same small variation in sensitivity.

ACKNOWLEDGEMENTS

The development of this wide-band element has been sponsored by the Marine Laboratory of the Department of Agriculture and Fisheries for Scotland at Aberdeen.

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Table 1: Component values

Design Method Component	Analysis	Synthesis
Mf gm	1.6183	1.67
Cf 1E-9 m/N	8.0504	8.09
Mh gm	4.80	4.43
Mt gm	5.40	4.70
Cm 1E-9 m/N	7.5964	7.60
C1 " "	30.3854	30.40
M1 gm	-	0.5710
Ms "	0.4900	0
Mp "	0.2240	-
Cp 1E-9 m/N	83.43	-

OCTAVE BANDWIDTH TRANSDUCER

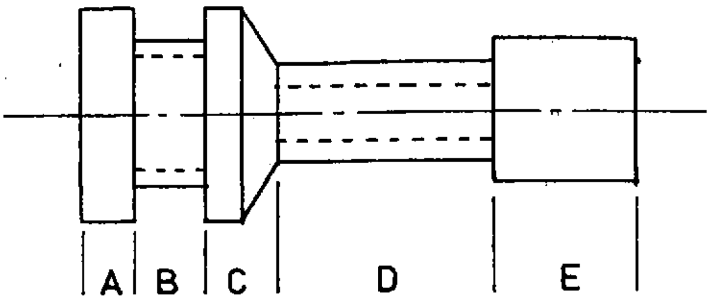


fig. 2 Mechanical structure of the element

A: Outer head mass } Coupling section
B: Compliant tube
C: Inner head mass } Basic element
D: Piezo-electric stack
E: Tail mass

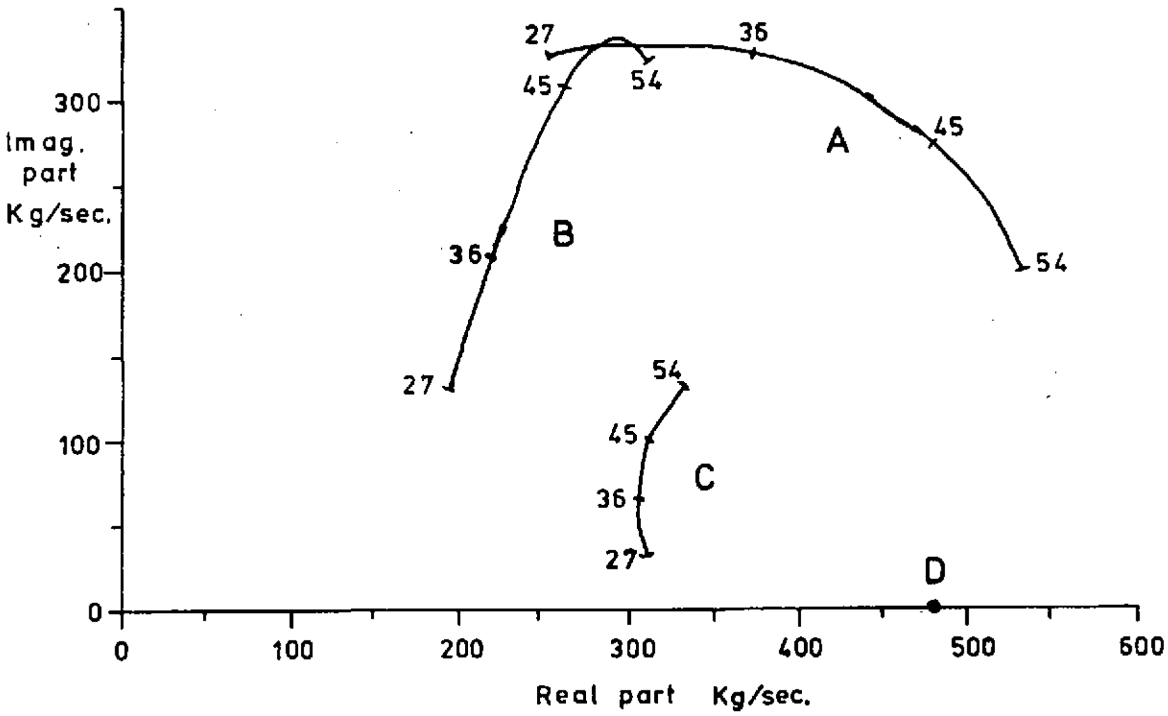


fig. 3 Average radiation load on the element

A: Isolated element in a rigid baffle
B: Each element in an array, packing factor 0.484
C: " " " " " " " 0.845
D: Each element in a completely filled array

(Frequencies in KHz)

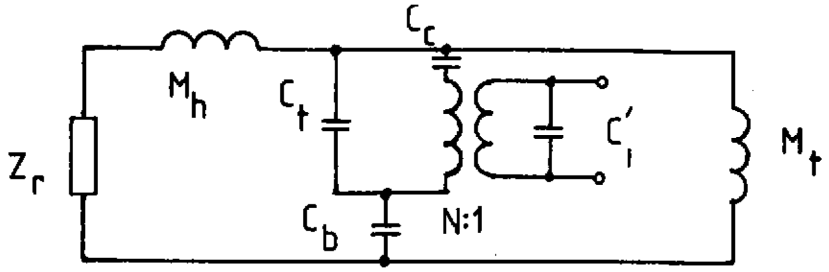


fig.1

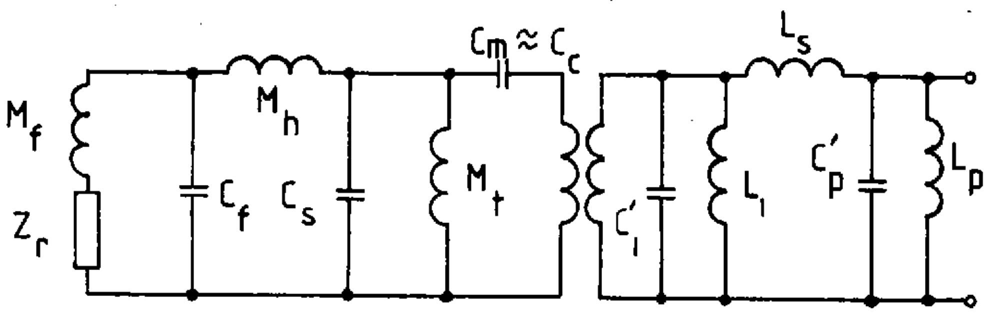


fig.4(a)

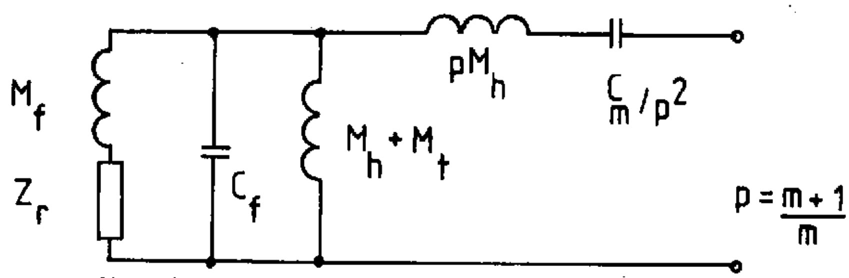


fig. 4(b)

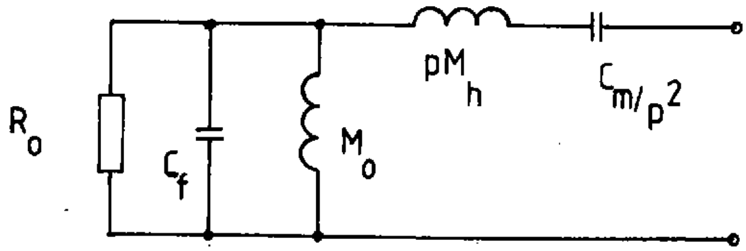
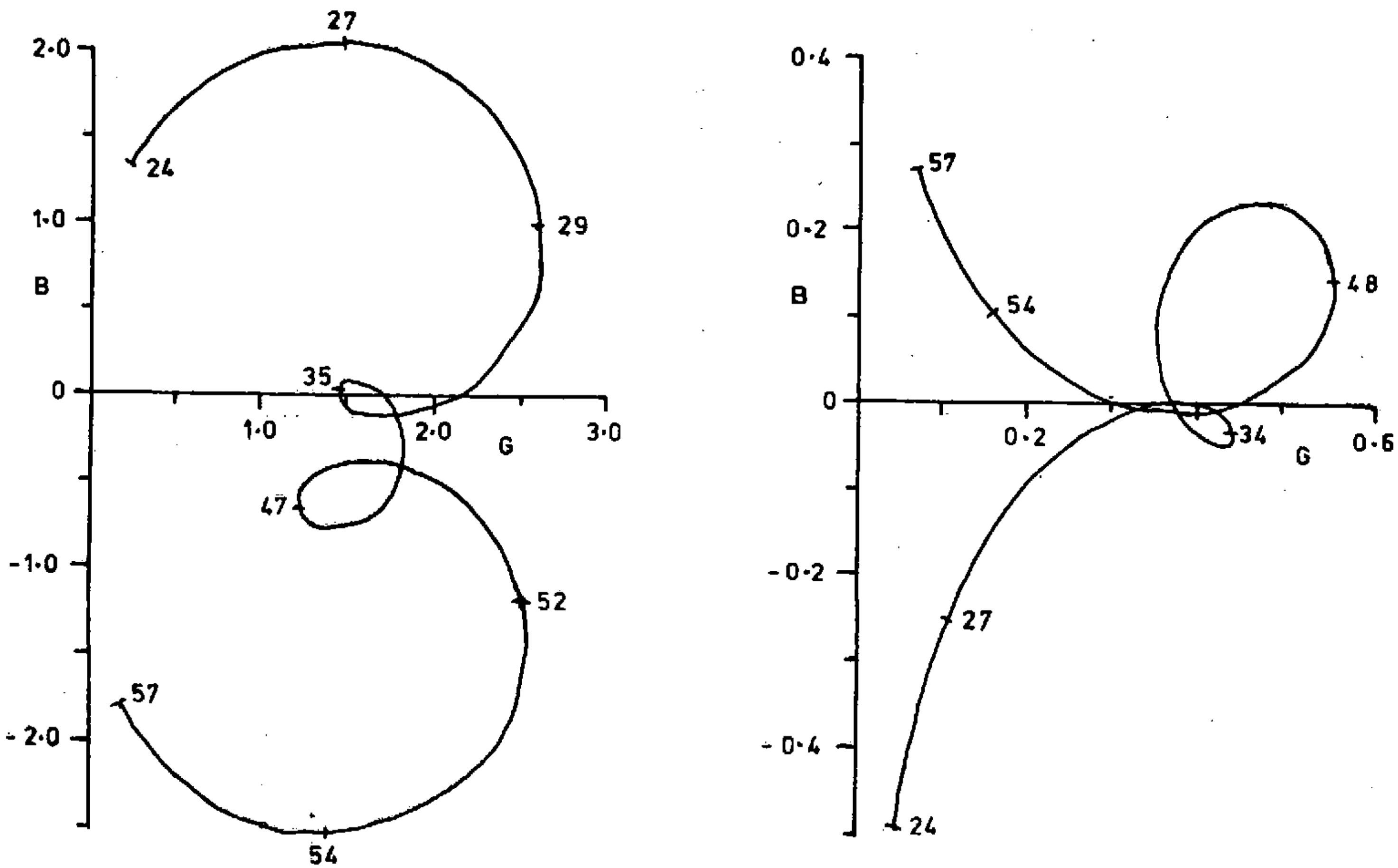


fig. 4(c)



(a) Design by analysis

(b) Design by synthesis

fig. 5 Mechanical admittances of the two designs
(Admittance in units of 10^{-2} sec/Kg., frequency in KHz)

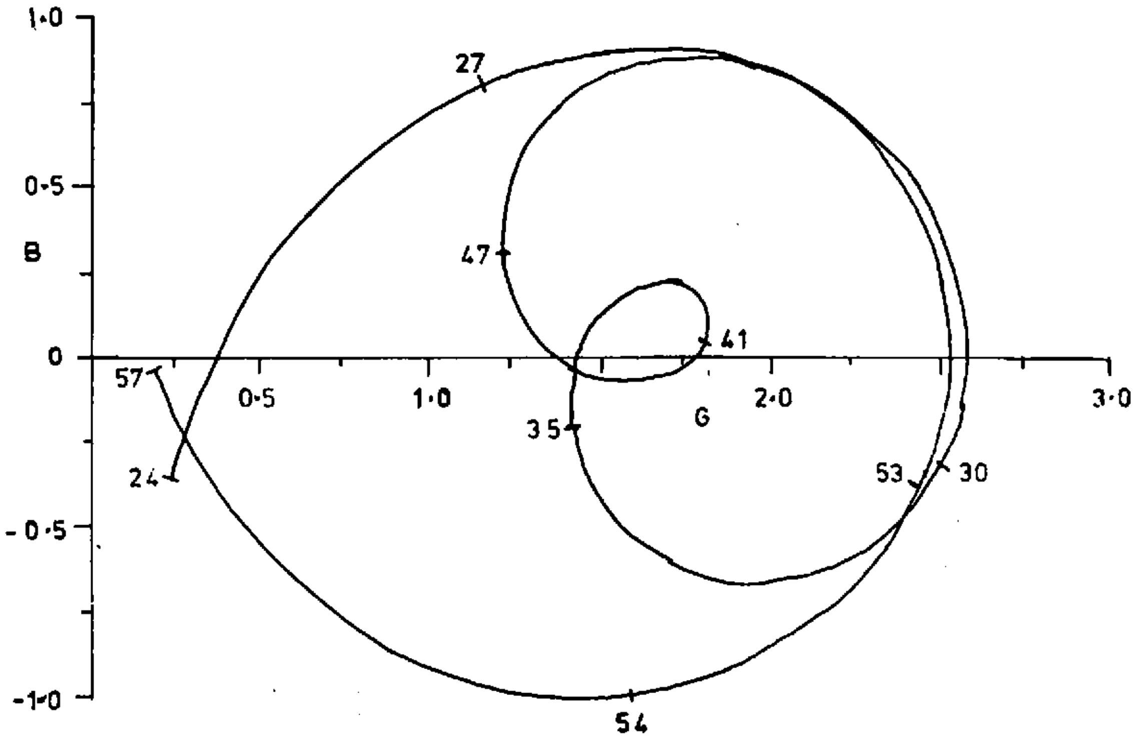


fig. 6 Modified admittance for the design by analysis (Units as in Fig. 5)