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MEASUREMENT OF CYLINDER/PLATE COUPLING LOSS FACTORS AND ASSOCIATED PROBLEMS

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1. INTRODUCTION

Recent work has highlighted the importance of Statistical Energy Analysis modelling in helicopter structure borne studies. Such prediction models are very dependent, however, on a good knowledge of the coupling loss factors between elements, derived either from measurement or theory. To date only beam junctions have received a rigorous analysis of their coupling characteristics, whilst plate junction studies have been less well researched. Helicopter cabin and gearbox structures incorporate many beam and plate structures together with more complicated elements such as cylinders, cones, etc and it is important to extend the coupling loss factor knowledge to all these different junction types.

As part of an SEA study of a simulated gearbox casing structure, coupling loss factor measurements have been performed on cylinder plate combinations. The top cover of the gearbox casing consisted of a cone joined to plates either end by steel rings. Attempts to measure the coupling between the cone and its circular top plate proved fruitless, due to the low modal density of the plate and the difficulties of locating several measurement positions and drive points on a small structural element. It was decided, therefore, to represent (to a first order), the cone/plate interfaces at either end of the cone by cylinder/plate combinations and two such structures were built for testing purposes. Since it was not known whether the measured cylinder/plate coupling loss factors were realistic or not, a plate/plate structure was also tested to calibrate the measurement procedure.

The paper presents the results of the test programme based on the matrix inversion measurement procedure developed at Southampton University, whereby an assembled structure is excited on each element in turn to give averaged energy levels and input power levels. Problems encountered in the measurement procedure, the interpretation of the data and the validity of introducing the SEA reciprocity relationship $n_i \eta_{ij} = n_j \eta_{ji}$ will be discussed.

2. MEASUREMENT TECHNIQUE

The coupling loss factor measurement method developed by Clarkson and Ranky (1) and Lalor (2) involves exciting the assembled structure at several places in turn. Consider, for example, a 2 element structure for which the basic SEA equations are:-

1) $(\eta_1 + \eta_{12})E_1 - \eta_{12}E_2 = P_1/\omega$

2) $-\eta_{12}E_1 + (\eta_2 + \eta_{21})E_2 = P_2/\omega$

3) $n_1\eta_{12} = n_2\eta_{21}$

where equation 3) is the well known reciprocity relationship.

With element 1 excited, energy E' and power P' are obtained and with element 2 excited, E'' and P'' are obtained. Based on equations 1) and 2), this leads to the matrix equation:

4) $$\begin{vmatrix} E_1' & E_1' & -E_2' & 0 \\ 0 & -E_1' & E_2' & E_2' \\ E_1'' & E_1'' & -E_2'' & 0 \\ 0 & -E_1'' & E_2'' & E_2'' \end{vmatrix} \begin{vmatrix} \eta_1 \\ \eta_{12} \\ \eta_{21} \\ \eta_2 \end{vmatrix} = \begin{vmatrix} P_1'/\omega \\ 0 \\ 0 \\ P_2''/\omega \end{vmatrix}$$

where E is given by mv^2 or ma^2/ω^2 for acceleration level 'a'. η_{12} and η_{21} are then found by inverting the matrix, which also gives the in-situ internal loss factors η_1 & η_2. At this stage the reciprocity relationship has not been invoked and 4 unknowns are required from 4 equations.

The accuracy of the resulting η_{12} and η_{21} values depends upon the accuracy of measuring E and P values and small errors in the energy measurements will produce large errors in the coupling loss factors. Some investigators overcome this by using an iterative procedure to derive a symmetric matrix which gives the best fit (least squares method) to the measured matrix. This approach has not been adopted here.

3. TEST STRUCTURES

The first cylinder/plate structure consisted of a 3mm thick rectangular steel plate of dimensions 1.1 x 0.75m and a 1.5mm steel cylinder of height 0.3m and diameter 0.55m (denoted short cylinder). The plate was welded to the end of the cylinder and continued across the cylinder end face without a hole cut-out.

The dimensions were chosen to be compatible with the gearbox casing sizes so that the results of the cylinder/plate configuration could be read across to the cone/plate structure (in this case the larger end of the cone).

Since there was some doubt as to the suitability of a short squat cylinder as an SEA component, it was decided to repeat the tests with a long thin cylinder attached to a plate. The reasoning behind this approach was to choose a structure which more closely met the SEA ground rules of high modal density and non equipartition of energy distribution. A short fat cylinder (or cone), whilst representative of the gearbox cone dimensions, is not large enough to be considered infinite from an SEA point of view. Also difficulties were experienced in measuring average structural acceleration levels at positions sufficiently far away from the drive point, the cylinder ends and the cylinder/plate junction. The long thin cylinder (of smaller diameter 0.3mm, but of length 3 times the diameter) overcame some of these problems and had the added advantage of having a diameter matching the diameter of the top end of the cone, so that both ends of the cone were now represented. This time a hole was cut out of the plate to accept the end of the cylinder.

In addition a plate/plate structure was also tested to calibrate the measurement procedure, since plate/plate theory already exists in the literature. Once the method had been checked on the plate/plate structure, some degree of confidence could be placed on the cylinder/plate results. All three structures are illustrated in figure 1.

4. TEST PROCEDURES

The structures were tested with added damping material to produce a non-equipartition of energy distribution situation. Two drive points per element were used with 6-8 accelerometer positions on each. All positions and drive points were located well away from structural edges and junctions. Only bending wave transmission was considered, so that drive inputs were aligned perpendicular to the respective plate or cylinder surfaces. Input power levels were measured by a force gauge and accelerometer at the drive point.

Each acceleration level (in each 1/3 octave frequency band) was normalised to 1 watt of input power and the resulting levels were averaged over all measurement positions. This information together with measured masses of each element plus damping material, was then inserted into a computer programme, employing a matrix inversion routine to give η_1, η_2, η_{12} and η_{21} for each drive point.

Since modal density is an important parameter in assesing the cylinder results, both measured and predicted modal densities were obtained for the cylinder structures. The former was obtained from point mobility measurements in the usual manner with corrections for the added mass of the force gauge etc.

5. RESULTS

The modal density results for both cylinders show good agreement between measurement and theory, with the modal density (n) rising to a peak at the cylinder ring frequency and then decreasing to the equivalent plate value at high frequency.

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The measured coupling loss factors for the plate/plate structure also show good agreement with theory. The theory is given by $\eta_{ij} = \frac{C_{gi} L_{ij}}{\omega\pi A_i} \tau_{ij}$,

where C_g = bending wave group velocity, L = junction line length, A = area of driven element and τ = transmission coefficient. Unlike beams which assume normal incidence waves, $\tau_{12} \neq \tau_{21}$ for plates of different thicknesses because τ must be averaged over all angles of incidence. The wave number parameter K is introduced such that $K_1 \tau_1 = K_2 \tau_2$ and when this is included in the above formula, the reciprocity relationship $n_1\eta_{12} = n_2\eta_{21}$ is obeyed. Although the individual coupling loss factors show good agreement, the acid test is how well the ratio η_{21}/η_{12} agrees with n_1/n_2. This comparison is shown in figure 2 with predicted modal densities n_1 and n_2, and although the digression is rather large at a few frequencies, the η_{21}/η_{12} values fall either side of the n_1/n_2 line.

The two cylinder/plate structures produced similar shaped coupling curves to the plate/plate structure, but not entirely in the expected manner. For the short cylinder (figure 3) η_{12} decreases with increasing frequency, whilst η_{21} tends to be independent of frequency, but with rather wider fluctations about a mean line. Comparisons of the measured η_{21}/η_{12} ratio with the predicted n_1/n_2 ratio (figure 4) are surprisingly good throughout the frequency range. The long thin cylinder results show similar shaped characteristics, but at a lower level due to the reduced junction length (figure 5). When the η_{21}/η_{12} ratio is examined, however, (figure 6) it lies well below the n_1/n_2 ratio. It is not clear why the two cylinder sizes should show different results. Possible reasons for the differences are the effect of the hole cut-out, the presence of other wave types or changes in the direct and reverberant field distributions with cylinder length. If the two cylinders, however, were responding in different ways due, for example, to longitudinal wave effects, one would intuitively expect the long thin cylinder to respond correctly owing to its larger modal density and closer approximation to a semi infinite SEA element.

Since both cylinder modal densities show good agreement between measurement and theory, and plate modal densities are well predicted, there is no basis for suspecting the accuracy of the n_1/n_2 ratio. If, however, the ratio η_{ij}/n_j which is effectively a form of transmission coefficient, is plotted, then the η_{21}/n_1 values are consistently lower than the η_{12}/n_2 values for the long cylinder, but not the short cylinder. A question mark, therefore, hangs over the accuracy of the η_{21} measurement.

A reappraisal of the data, however, shows no obvious measurement errors. For example, the discrepancy between η_{21}/η_{12} and n_1/n_2 of figure 6 suggests a systematic error of a factor of about 1.5 (which is equivalent to about 2dB error in measured acceleration level), but any widescale adjustment to either E' or E'' values would result in changes to both η_{12} and η_{21}, which are inwardly linked and hence no improvement in the ratio agreement. A change in mass (m_1 or m_2) of the cylinder or plate element would suffice to produce better agreement, but a change of 1.5 in the measured mass is too significant an amount to be a measurement error.

The validity of using the matrix inversion procedure for measuring structural coupling loss factors is currently receiving widespread debate. The questions revolve around the minimum number of elements necessary for correct use of SEA, the use of the reciprocity relationship, redundancy of information in the equations, and the concept of equivalent mass. For the two element systems used here, where the reciprocity relationship was not invoked, there are 4 matrix equations and 4 unknowns. For complicated structures the modal densities n_1 and n_2 are also unknown, suggesting 6 unknowns to be derived from 4 equations. Thus it could be argued that the method can not be used on a 2 element system. If, however, n_1 and n_2 are known and the reciprocity equation is used, then the number of unknowns reduces to 3 (η_1, η_2 and η_{12}) and there is a redundancy of information from 4 equations.

Since the reciprocity relationship is an essential part of the SEA ground rules, it can be argued that the relationship should be used to force better accuracy in the matrix inversion procedure. In our case n_1 and n_2 are known and the insertion of these into the matrix equations should produce a more accurate set of loss factors.

The concept of equivalent mass, introduced in reference 3, has been adopted by Lalor to allow for non uniform structures and the transmission of non-bending wave types, which can result in an incorrect measurement of the true structural energy. Whether this approach is valid or not, it is of interest to assume that m_1 and m_2 are also unknowns and use the SEA equations to predict the m_1/m_2 ratio, for comparison with measurement.

Thus invoking the reciprocity relationship and inserting m_1 and m_2 into equation 4) leads to the matrix equation:

$$\begin{vmatrix} a_1'^2 & a_1'^2 & -(n_1/n_2)a_2'^2 & 0 \\ 0 & -a_1'^2 & (n_1/n_2)a_2'^2 & a_2'^2 \\ a_1''^2 & a_1''^2 & -(n_1/n_2)a_2''^2 & 0 \\ 0 & -a_1''^2 & (n_1/n_2)a_2''^2 & a_2''^2 \end{vmatrix} \begin{vmatrix} m_1\eta_1 \\ m_1\eta_{12} \\ m_2\eta_{12} \\ m_2\eta_2 \end{vmatrix} = \begin{vmatrix} \omega \\ 0 \\ 0 \\ \omega \end{vmatrix}$$

This equation can then be solved in two ways; either with m_1 and m_2 known to give revised values of η_1, η_{12} and η_2 or with m_1 and m_2 unknown to give the ratio m_1/m_2.

The revised values for η_{12} and η_{21} are shown in figures 7 and 8. For the short cylinder, forcing reciprocity into the system results in a slight smoothing out of the peaks and troughs in the curve, but no real change to η_{12} or η_{21}. For the long thin cylinder, η_{12} only changes marginally again but η_{21} increases throughout the frequency range.

Figures 9 and 10 compare the measured and predicted mass ratios and, in the case of the long thin cylinder, there is clearly a large discrepancy at all frequencies.

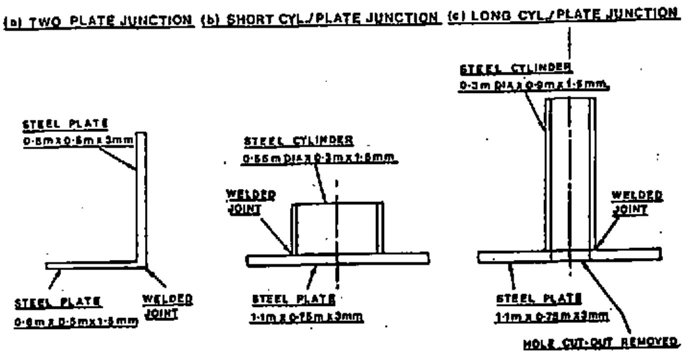
6. CONCLUDING REMARKS

The study has highlighted some of the problems of measuring coupling loss factors on relatively simple structures. The matrix inversion measurement procedure was successfully calibrated on plate/plate structures, but cylinder/plate structures gave conflicting results. Contrary to expectations, the long thin cylinder proved more troublesome than the short squat cylinder and it is hoped to investigate this further when theoretical coupling loss factors for cylinder/plate junctions become available. The energy matrix inversion method does work, but in effect it is only an SEA model in reverse and the more accurately the structure is subdivided into elements and the more representative the elements are of satisfying SEA rules of large size, high modal density and sufficient damping for non equipartition of energy distribution, the less room there is for experimental errors.

7. REFERENCES

(1) B L CLARKSON and M F RANKY, 'On the Measurement of the Coupling Loss Factor of a Structural Connection', J of S & V, 1984, 94(2) 249-261.

(2) N LALOR, 'The Measurement of SEA Loss Factors on a Fully Assembled Structure', ISVR Tech Memo, 150, August 1987



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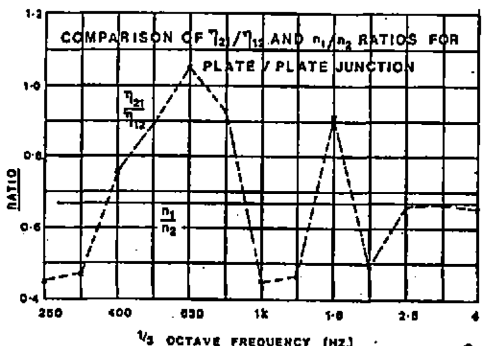

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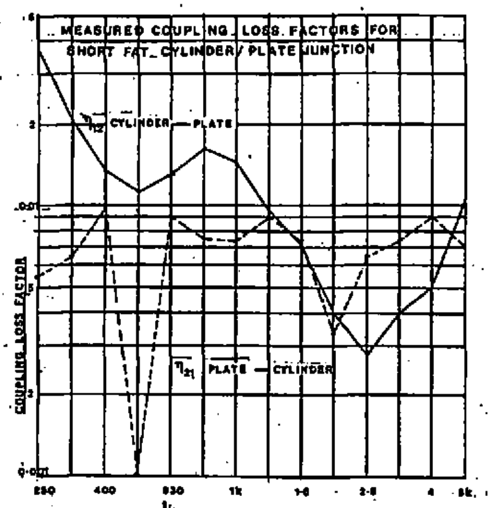

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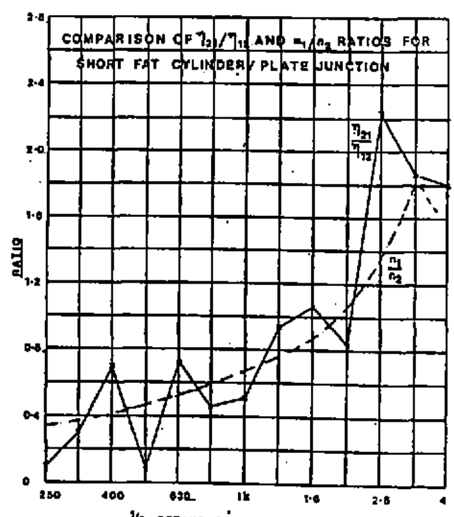

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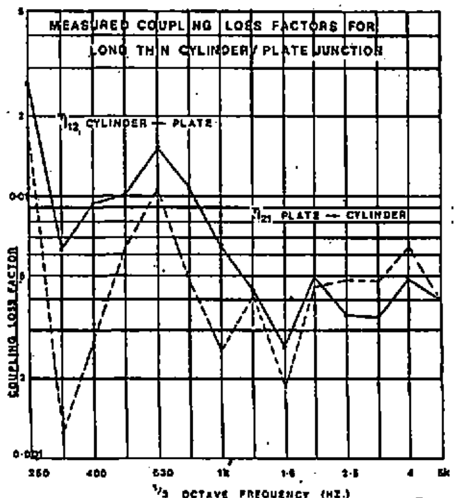

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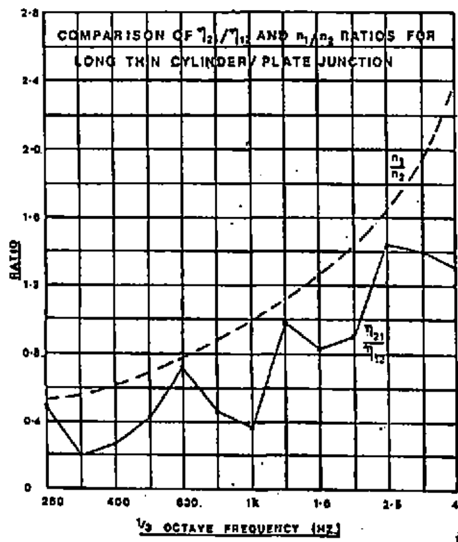

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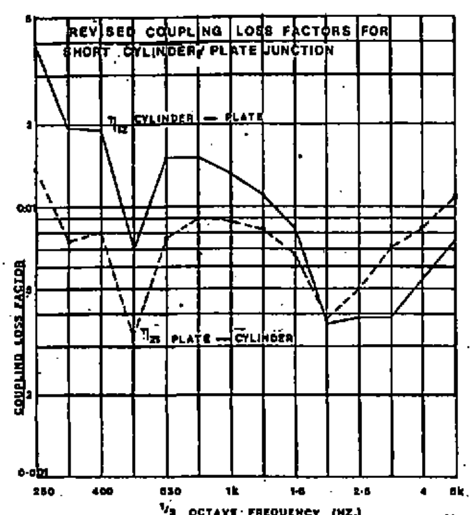

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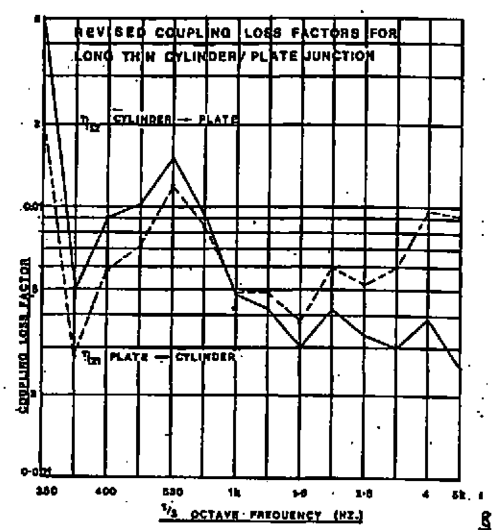

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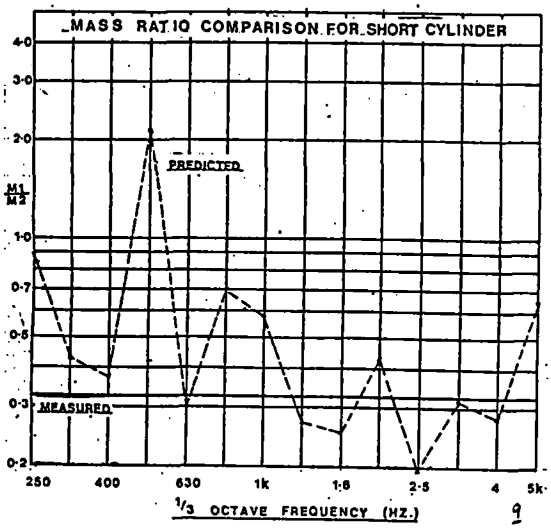
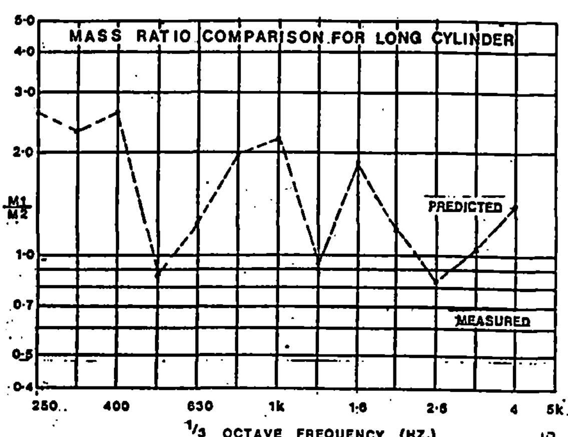

MASS RATIO COMPARISON FOR SHORT CYLINDER
4·0
3·0
2·0
1·0
0·7
0·5
0·3
0·2
M1/M2
PREDICTED
MEASURED
250
400
630
1k
1·6
2·5
4
5k
1/3 OCTAVE FREQUENCY (HZ.)

9


MASS RATIO COMPARISON FOR LONG CYLINDER
5·0
4·0
3·0
2·0
1·0
0·7
0·5
0·4
M1/M2
PREDICTED
MEASURED
250
400
630
1k
1·6
2·5
4
5k
1/3 OCTAVE FREQUENCY (HZ.)

10