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AN INVESTIGATION OF POWER FLOW AND PROPAGATING PLANE WAVE ENERGIES IN ONE-DIMENSIONAL STRUCTURES

J T Lenaghan & F J Fahy

Institute of Sound and Vibration Research, University of Southampton, Southampton, UK

1. INTRODUCTION

The propagation of plane waves in coupled cylindrical ducts is being investigated as a part of a larger project which is, in turn, investigating the probability distribution functions of quantities employed in statistical energy analysis (SEA). The energy and power flow estimates within a structure, obtained from SEA, are means of the distribution functions of a population of similar structures. A population may result from the geometrical and manufacturing uncertainties of a structure's sub-systems, the variations in the fastenings and couplings between such sub-structures, as well as the uncertainty inherent in the methods of measurement of energy levels and coupling strengths. It is not evident that the means calculated are sufficient measure of the structure's behaviour. The 95% confidence limits would provide bounds against which the behaviour of a particular structure may be judged acceptable or, if at the design stage, that the probability of unacceptable levels of vibration when in service is sufficiently small. There are other questions to be investigated that are due to the inherent uncertainties in statistical energy analysis: these are due in part to the nature of the subsystems chosen and how they are chosen; the estimates furnished are either ensemble- or frequency-averaged or sometimes both. The initial theoretical support for SEA required the estimates to be considered valid only over ensembles of systems. This may be very difficult in practice, for example a rocket launch vehicle. This project in collaboration with A J Keane & C S Manohar of the Department of Engineering Science, Oxford University is seeking to analyse a large number of ensembles of rod, beam and plate structures so that the confidence limits (and other higher order statistical measures) may be deduced directly from the distributions.

One of the more elementary structures is two rods constrained to vibrate axially and coupled by a point spring. For this structure it is necessary to obtain a measure of the energies of each, and the power flows between them, for many examples of the two rods. The manufacturing difficulty of making sufficient number of these structures to form an adequate sample of a population would be prohibitive but, by analogy, a rod vibrating longitudinally may be replaced by a quiescent column of air restrained in a cylindrical duct and excited to vibrate below the duct's first cut-off frequency. By this substitution, the power flow within the duct may be deduced from the sound intensity and the energies of the forward and returning propagating waves from two measures of the sound pressure field. The total sound energy of the column may also be deduced with the propagation constant for the duct. Then the two-rod structure is replaced by two columns of air coupled by a compliant expansion. The problem of generating a sufficient number of structures with a known probability distribution for one of its parameters is transmuted into undertaking changes on the two columns of air. The geometric parameter of the acoustical length has been chosen and an automated method of generating this perturbation, produced. An experiment then consists of the measurement of the spectral estimates within each duct for one member of the ensemble; the length is changed, and a new set of spectral estimates measured until sufficient members have been determined. From the spectral estimates the power flows and energies are deduced. These are then used to generate the statistical distributions from which confidence intervals may be calculated.

2. MECHANICAL STRUCTURES IN ACOUSTIC ANALOGY

2.1 Acoustical-Mechanical Analogy

The mathematical model of longitudinal wave transmission is the same for a rod and a column of air. For a complex pressure field, p, in a medium with c speed of sound

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \mu\frac{\partial p}{\partial t} \tag{2.1}$$

where μ is a dissipation constant. Equation 2.4 may have solutions of the form:

$$\left.\begin{array}{l} p_+ = A_+ e^{\gamma x} e^{i\omega t} \\ p_- = A_- e^{\gamma x} e^{i\omega t} \end{array}\right\} \tag{2.2}$$

where p_+ and p_- represent the plane progressive harmonic waves of frequency in the positive and negative x directions respectively. A_+ and A_- are arbitrary complex wave amplitudes. γ is the complex propagation constant which in may be written as is $i\kappa + \alpha$ where κ is the phase constant and α is attenuation per unit length.

These progressive components interfere by superposition to produce the resultant field. If the complex reflection coefficient of a termination at $x = 0$ is represented by R, such that at that point $A_- = RA_+$, then the field becomes

$$p = p_+ + p_- = Q\cosh(\gamma x + \phi), \tag{2.3}$$

where $Q = A_+\sqrt{R}$ and $\tanh\phi = (R - 1)/(R + 1)$ for each frequency, ω. This result is used to determine the propagation constant from the transfer functions of an anechoically terminated tube.

There is the special case of a reflection coefficient $R = 0$ (anechoic termination), the field becomes one of a plane progressive attenuated wave in one direction. This result is important for the determination of the propagation constant which is used in the determination of the energy levels and coupling constants.

2.2 Power Flow

An alternative formulation for the pressures measured at the microphones is useful in determining the power flow, or intensity, from the spectral estimates:

$$p_1 = A_- e^{-ks/2} + A_+ e^{ks/2}, \tag{2.4}$$
$$p_2 = A_- e^{ks/2} + A_+ e^{-ks/2}. \tag{2.5}$$

The spectral estimates for the microphone signals are

$$\begin{array}{l} G_{11} = p_1 p_1^* = |A_-|^2 + |A_+|^2 + 2\Re e(A_- A_+^* e^{ks}), \\ G_{22} = p_2 p_2^* = |A_-|^2 + |A_+|^2 + 2\Re e(A_- A_+^* e^{ks}), \\ G_{12} = p_1 p_2^* = |A_-|^2 e^{-ks} + |A_+|^2 e^{ks} + 2\Re e(A_- A_+). \end{array} \tag{2.6}$$

These equations may be solved for the forward, backward amplitudes and the phase difference at the mid-point of the transducer.

The mean intensity, hence power flow may be deduced from the real part of G_{pu}, the pressure/particle velocity cross spectrum. For a p-p transducer this may be constructed as [2]

$$G_{pu}=(-j/2\rho\omega d)[G_{11} - G_{22} - j2\Im m\{G_{12}\}]. \quad (2.7)$$

$$I_m=-(1/\rho\omega d)\Im m\{G_{12}\}. \quad (2.8)$$

2.3 Energy Levels

The propagation constant for the tube shall be deduced in advance. This is then used to calculate the complex amplitudes of the two energy carrying waves in the tube from the measurements at the power flow measurement microphones. Using

$$p_1 = A_-e^{-\gamma d} + A_+e^{\gamma d}, \quad (2.9)$$
$$p_2 = A_-e^{-\gamma(d+s)} + A_+e^{\gamma(d+s)}, \quad (2.10)$$

where p_i is the pressure measured at microphone i, d is the distance from the far end to the first microphone and s is the inter-microphone separation. By integrating the power of the two waves with respect to distance over the length of the tube the total energy becomes:

$$\bar{E} = (1/4\alpha)\ [\ |A_-|^2(1 - e^{-2\alpha L}) + |A_+|^2(\ e^{2\alpha L} - 1)]. \quad (2.11)$$

From above the modal loss factor may be given as

$$\eta = P_{in}/\omega\bar{E}. \quad (2.12)$$

The power input when excited from the negative x direction is the intensity of the positive going wave minus the intensity of the negative going wave. For low loss systems this is

$$P_{in} = (1/2\rho_0 c)[|A_-|^2 - |A_+|^2]. \quad (2.13)$$

On combination and substituting $\theta^2 = |A_-|^2/|A_+|^2$ the modal loss factor is

$$= (2\alpha/\omega\rho_0 c)[(\theta^2)/((1 - e^{2L\alpha}) + (e^{-2L\alpha} - 1))]. \quad (2.14)$$

Hence the ratio of the square of the amplitudes may be used to estimate the loss factor.

For the single mode with a suitable number of estimating spectral lines the frequency–averaged loss factor is the ratio of the integral of the input power to the integral of the wave energy over the frequencies of the mode. For multimode estimates the integrals are taken over the band of frequencies of interest and the ratio is divided by the width of the band.

3. MEASUREMENT AND ANALYSIS

3.1 Apparatus

3.1.1 Pipes. PVC pipes of a nominal bore of one inch are used as the waveguides. The dissipation of vibrational energy is measured in terms of the attenuation of the plane wave amplitude. The attenuation will be a measure of the modal loss factors and the modal overlap. It is also essential for the calculation of the stored energy from the plane wave model. The data are collected with the tube terminated with an anechoic termination. The transfer function between two microphone positions within the tube is determined by

$$H_{12} = p_2/p_1 = e^{\gamma x_2}/e^{\gamma x_1}. \quad (3.1)$$

Hence,

$$\gamma = (1/s)ln(H_{12}), \qquad (3.2)$$

where $s = x_2 - x_1$, the microphone position separation. The attenuation for one such pipe is illustrated in figure 1.

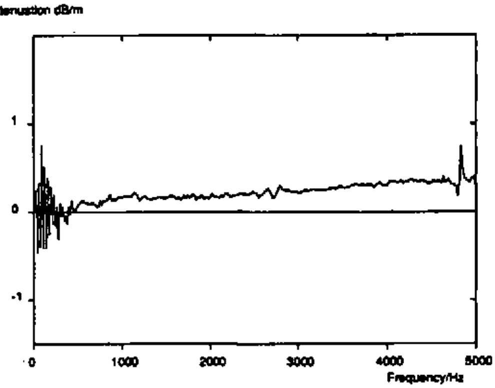

Figure 1: Attenuation(dB) per unit length(m) with respect to frequency.

3.1.2 Couplings. These form the point spring connections of the rod systems. They are simulated by acoustically compliant expansion chambers. The expansions have axial lengths and diameters chosen according to the transmission coefficient (equation 3.3). The minimum transmission should occur when the length of the expansion is equal to one quarter of a wavelength. In air with a free field sound velocity of 344ms^{-1} that minimum should occur at 4530Hz. The couplings required for the systems have been chosen to have minimum transmission coefficients (equation 3.4) that step down in a logarithmic series from 1 to, the least transmission coefficient of 0.01. This lower transmission coefficient is to reflect the coupling factors of the weakly coupled assumption of SEA.

$$T = \frac{4}{4\cos^2 kl + \left(\frac{S_2}{S_1} + \frac{S_1}{S_2}\right)^2 \sin^2 kl}, \qquad (3.3)$$

where k is the wavenumber, l is the axial length of the volume, S_1 and S_2 are the cross-sectional areas of the tube and volume respectively. The minimum transmission coefficient occurs at $kl=\pi/2$ and has a value of [1]

$$T_{min} = [2S_1S_2/(S_1^2 + S_2^2)]^2. \qquad (3.4)$$

3.2 Ensemble Generation

The waveguide properties may be altered in a number of different ways. The criterion to be satisfied by the perturbation is that there is a change in the frequency of the resonant modes in the perturbed subsystem with respect to the rest of the system. There are a number of methods that have been considered. These include variations in the properties of the waveguide wall, the air column and the wave path. But it is necessary to use a method that may be automated and straightforwardly characterised because of the required number. The perturbation in length may be comparatively easily implemented and the nature of this alteration may be characterised by one measurement, i.e. the length. The length of a subsystem cannot be perturbed at an intermediate position without introducing a change in the bore of the subsystem. However, tubes that are terminated by a high impedance (highly reflective) could be altered in length by changing the position of that termination with respect to the tube. For example, a piston may be slid within the tube. The air/water interface presents such a 'termination'. If the tube is dipped into a bucket of water and the water level is forced to change with respect to the end of the tube there is a resultant change in the acoustic length of that tube. This solution has been implemented. There is a problem in effecting a similar change mid subsystem. This would arise when the alteration is required in a subsystem that did not have a free end to terminated for is would be coupled to two others. What is proposed is that such a subsystem be changed be a branch tube. The impedance effects on the main tube by the side branch would be altered by the length of that branch.

A small peristaltic pump has been chosen to alter the level of water within the bucket into which the altered subsystem is placed. The pump is driven by a small DC motor which is, in turn, controlled by a variable frequency square wave from the analyser. The level is determined using an acoustic tube. A short length of tube is dipped end on into the water. The other end is rigidly capped. The free volume is excited by a small driver and the natural modes of the system are monitored by the analyser with a microphone. One such mode is chosen and its frequency noted to determine the length of the free tube. The ambient speed of sound is independently determined from the temperature.

The perturbation has a uniformly spaced uniform distribution. Other distributions shall be imposed on the data collected for this case to analyse the effect on the power flow and energy distributions.

3.3 Data Acquisition and Control

3.3.1 Microphones and preamplifiers. Small electret microphones have been used to sample the acoustic pressure field within the pipes. A special mounting block has been designed to hold the microphones and allow verification of the phase difference with respect to a B&K reference.

Like the electret transducers, the preamplifiers are not identically phase and gain matched and shall be compensated for with their own transfer function. This shall be tested for change periodically. It is expected that, barring failure, each electret shall be connected throughout the period of experimentation and testing to the same preamplifier.

3.3.2 Muliplexing signals. Only one pair of microphone signals need to be analysed at any particular time. However the apparatus requires many such pairs. A switching mechanism is used to select the pair to be tested. This selection is automatically controlled by the analyser during the experiment between the evaluation of spectral estimates.

3.3.3 FFT Signal Analyser. The calculation of spectral estimates is a fundamental function of the spectral analyser. The estimates may be generated on command from a controlling computer and the results passed back for post-processing. This post-processing may be carried out independently of the continued signal capture and initial estimation. The choice for data capture and processing is the Hewlett-Packard 35665A FFT signal analyser. The analyser incorporates a computer of its own which has been programmed to act as IEEE bus controller and process requests from the computer which may include direct commands to the analyser and switches but more often to retrieve files of collected spectral estimates. The general control and flow of the experimental apparatus and data collected is indicated in figure 2.

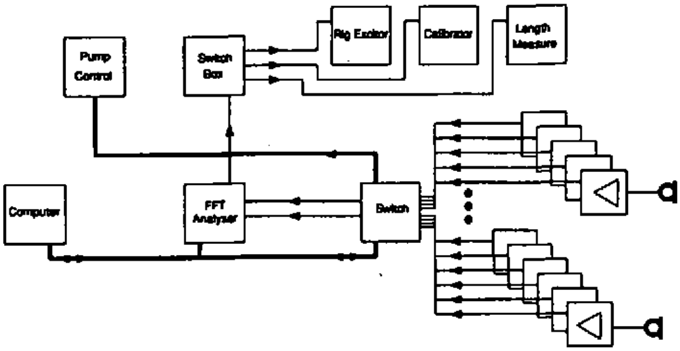

Figure 2: Signal communication within the instrumentation.

3.3.4 Signal Generator. The analyser's signal generator is used to provide a random noise driving signal to the system. It will also provide a drive to the calibration/performance testing rig and the load speaker for the perturbation length monitor. These are buffered and present a high electrical impedance to the generator output so may be connected in parallel.

3.4 Data Processing and Storage

The environment within which the apparatus control and data post-processing is carried out is Matlab. The main processing is the generation of statistical estimates from the data recovered from the analyser. The control of the rig consists of the commands to the analyser to send the captured data files as they become available. In the course of a measurement many more files are generated in by the analyser than is room to store them in the memory of either analyser or computer. The programs within Matlab manage the storage of the files prior to conversion to data formats for later processing. A tape drive is used to provide the required mass storage.

Once the measurement is completed the files are converted, compensation is made for the phase and magnitude inaccuracies of the microphones and the various power flows, energies *et cetera* are calculated for each member of the ensemble. Only at this stage are the statistical measures deduced from the ensemble of results.

4. INITIAL RESULTS

The results presented here are taken from the two pipe system described above with a coupling with a transmission coefficient of 0.1. There are 75 members to the ensemble over an interval of 150mm, altering the length of the second pipe from 8.1m to 8.25m. The pipe that is directly driven is 4m in length. Auto-spectra and cross-spectra over the frequency interval between 500 and 1300Hz are collected from each pair of microphones for each member of the ensemble. These are gathered together in the computer and used to calculate the power flows by equation 2.7. From the ensemble of power flows the mean, standard deviation and distributions illustrated below have been calculated for each spectral line. Figures (a) are for the unperturbed driving subsystem and those marked (b) are for the driven perturbed system. Although these results are not completely compensated for the effect of the microphone phase effects and not normalised to the driving power, which changes from frequency to frequency for each member of the ensemble it is evident in the perturbed case that the mean and standard deviation settle as more modes are perturbed through the higher frequencies. The figures 3.3 illustrate the cumulative probability distributions with respect to the power flow at the frequencies of 598, 898, 1198Hz.

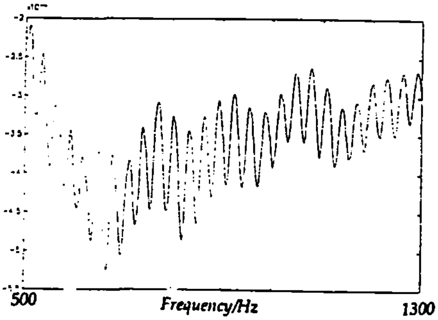

Figure 3.1(a): Mean Power In

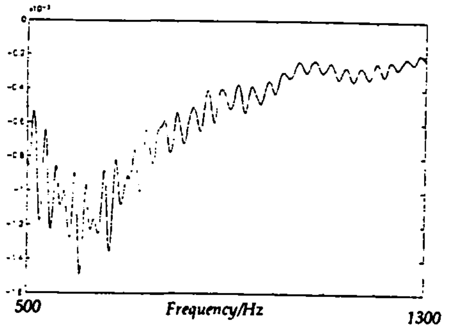

Figure 3.1(b): Mean Power In

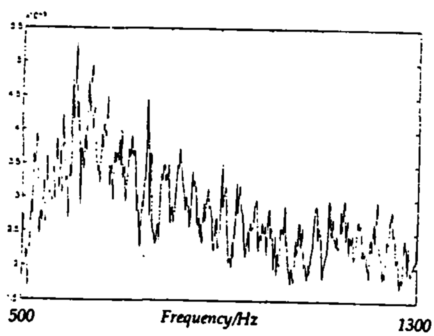

Figure 3.2(a): Standard Deviation of 3.1(a)

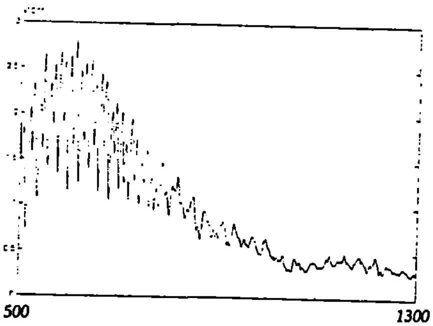

Figure 3.2(b): Standard Deviation of 3.1(b)

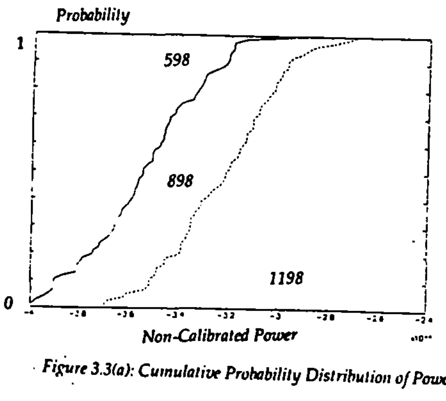

Figure 3.3(a): Cumulative Probability Distribution of Power

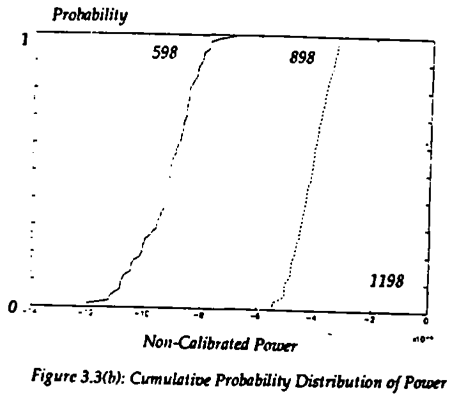

Figure 3.3(b): Cumulative Probability Distribution of Power

5. CONTINUING AND FUTURE WORK

5.1 More Complex One-Dimensional Structures

There is an intention to investigate the power flows in systems where the perturbed system is intermediate in an in-line series of sub-systems and recirculating systems such as a 'delta' arrangement of pipes. These are required to examine the effect of complexity of systems on the statistical measures. For this three short tubes have been machined into an aluminium block at an angular separation of 120°. The acoustic coupling between subsystems shall be altered by the two subsystem couplings attached to the exits of the three way coupling. This will set up a system of short subsystems between each tube. The energy level within the three way coupling shall be monitored by a single electret mounted at the junction.

5.2 Two Dimensional Structures

The investigation shall be extended to the measurement and analysis of power flows and energy levels of connected two dimensional structures. These structures will be the acoustical analogy to membranes, that is, a set of 'rooms' of 25mm height and side of about one metre. Right angled triangular and rectangular rooms shall be connected in various configurations by regularly spaced pipes along cojoint edges. These pipes shall have power flow monitored as for one dimensional systems. The energy levels within each room may be deduced from an array of randomly positioned microphones in the floor. An ensemble shall be generated by 'dipping' one edge of a room into a tank of water the level of which should be altered as described above.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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[2] Fahy, F J (1989) **Sound Intensity** Elsevier Science Publishers Ltd., Barking, UK