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'OBTAINING GOOD EXPERIMENTAL DATA FOR THE MATHEMATICAL MODELLING OF ENGINE STRUCTURES'

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SYNOPSIS

External noise legislation and the demand for lower interior noise levels have put manufacturers under increasing pressure to reduce the emitted noise from their engines. This paper describes a combined analytical and experimental approach that allows the designer to optimise the effects of possible modifications, such as increase of stiffness, damping or mass, on structure-borne noise, without recourse to a series of expensive hardware changes. In particular, the addition of beam stiffeners to various areas of the engine block and of lumped damping material can be readily evaluated. To demonstrate the application of this approach, examples are given of engine system models obtained from mathematically combining either the measured or calculated dynamic characteristics of the engine component parts. Vibration reductions are related to noise reductions by the use of space averaged velocities as part of a statistical energy method to calculate radiated noise. A second method of relating noise reductions to the change in the surface velocity distribution is described. This technique uses a volume integration method to predict acoustic pressure and is presently being developed to accept analytical or experimental information.

INTRODUCTION

The development of the mini-computer and microprocessor has made the measurement and analysis of the frequency response characteristics of complex structures a useful and practical tool for the dynamics engineer. From measured estimates of these structural characteristics it is possible to calculate the natural frequencies, damping and modal masses of a set of independent single-degree-of-freedom systems whose combined response will approximate to that measured by the test engineer. Associated with this abstract representation of the structure are characteristic deformation patterns (mode shapes) at each of the natural frequencies. Interpretation of these modeshapes by the engineer can provide great insight into the controlling elements of particular vibration phenomena. However, it is absolutely essential that the physical understanding of the structure is not lost by resorting to modal representations of real world structures and that the common interface plane which this representation brings to the experimental and analytical dynamics engineer is fully exploited.

The reduction of a complex system into an assembly of well-defined components through the concept of SDRC's* Building Block Approach (BBA) enables the test engineer, analytical engineer and designer to study the response of individual or combinations of dynamic systems. The technique relies upon the sub-systems being represented by a linear mathematical model (or a set of linear models) and uses modal synthesis in conjunction with constraint equations to couple the components together to predict the response of the total system. Since

*SDRC is a service mark of Structural Dynamics Research Corporation modal representations are obtained from both the test and analytical approaches, the modal synthesis method allows the combination of test and analytical data to model structural modifications. SDRC's program to do this is known as SABBA (System Analysis via the Building Block Approach). Using this tool the engineer can evaluate the effect of beam stiffeners or tuned absorbers, for example, to be added to the structure that has been tested. Such changes can be assessed before committing to hardware.

THE EXPERIMENTAL MODELLING APPROACH

In order to apply these techniques, the experimental modal data base obtained from the artificial excitation tests must be of a very high standard. To establish the quality of test data, the ability to reconstruct the original data from the mathematical model is essential. Great care has also to be exercised in the calibration of the test data. These considerations are discussed in detail in Reference 1.

This paper describes how the application of a combined experimental and analytical mathematical model could be used to produce a stiffened diesel engine block design which would radiate less noise than the existing casting. The work would fall into the following phases.

Phase 1: Acquisition of good frequency response data for the block.
Phase 2: Modal analysis of this data to establish an experimental modal representation of the structure.
Phase 3: Check on validity of data base using synthesis.
Phase 4:
- 4.1 Setting up SABBA system files to allow the structural modifications which the interpretation of the experimental mode shape data suggested would improve the design.
- 4.2 Validating the system files and setting up the load files for the forced response work.
- 4.3 Establishing modal representation of the beam stiffeners and adding sub-systems to the experimental system files at the SABBA level.
- 4.4 Calculating the forced response of the modified block.

Phase 5: Post-processing the output data with and without the modifications to obtain the change in the space-averaged velocity on the block and hence the change in the radiated sound power of the modified region.

These phases are described in greater detail in the following sections.

MEASUREMENT OF FREQUENCY RESPONSE DATA

Classically, the Inertance Frequency Response Function of a linear system due to a single point excitation is defined as

$$H_{ik}(j\omega) = \frac{\text{Fourier transform of the acceleration at point } i}{\text{Fourier transform of the force applied at point } k}$$

This definition is not suitable for experimental measurements, especially when a randomly varying force is used to excite the structure. A practical measurement with a digital analyser is achieved by calculating

$$H_{ik}(j\omega) = \frac{\text{Cross Spectral Density between input and response}}{\text{Power Spectral Density of the input force}}$$

By implementing this definition, ensemble averaging can be used to improve the statistical quality of the computed frequency response functions. In particular, when this definition is applied to random excitation the result is the best least squares linear approximation for the frequency response function over the range of amplitudes and frequencies excited.

Provided that sufficient ensemble averages are taken, the cross spectral representation has a number of other important properties:

(a) contaminating noise in the response signal is eliminated;
(b) any additional uncorrelated inputs which also excite the test structure (e.g. from adjacent rigs in the test area) will not affect the measured frequency response functions.

The quality of each measurement may be assessed by examining the corresponding Coherence Function defined as

$$\gamma^2_{ik}(\omega) = \frac{\text{Cross Spectral Density}^2}{\text{Force}_k \text{ PSD} \times \text{Acceleration}_i \text{ PSD}}$$

For single point excitation of an ideal linear system, the computed coherence function should have a value of 1.0 at all frequencies. In practice however, signal processing errors, noise and structural non-linearities will tend to reduce the observed coherence.

For the majority of frequency response functions measured in the modal survey tests, the coherence should be greater than 0.95 in the region of each resonant frequency. Figure 1 expresses the confidence limits for frequency response functions in terms of the number of ensemble averages and the measured coherence value. The majority of the test data was calculated with 32 independent ensemble averages and Figure 1 confirms a high level of statistical accuracy when γ^2 is greater than 0.95.

Single point excitation using an electromagnetic shaker in conjunction with a force transducer and accelerometers can be employed to measure the frequency response characteristics at sufficient points on the block to enable the mode shapes to be recognised.

Once all the frequency response functions have been measured and stored, a detailed analysis can be carried out in the following manner for each exciter location:

1. A sufficient number of Bode and Nyquist plots are displayed and examined to enable the approximate frequencies of all resonant peaks in the range of interest to be identified.
2. A Multi Degree of Freedom (MDOF) curve fit is applied to each driving point frequency response function to extract accurate modal parameter information.
   For a single-input measurement the frequency response function monitored at location n may be written as

$$H_{mn}(i\omega) = \sum_{r=1}^{2N} \frac{\psi_n^r \, \psi_m^r}{a_r(i\omega + \zeta_r\omega_r \pm i\omega_r \sqrt{1-\zeta_r^2})} \qquad (1)$$

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$$= \sum_{r=1}^{2N} \frac{A_{mn}^r}{(S-S_r)} \qquad (2)$$

where A_{mn}^r is the complex residue at the pole S_r

ϕ_m^r, ϕ_n^r are the complex eigenvectors (mode shape amplitudes) for the rth mode

ω_r is the undamped natural frequency of the rth mode

ξ_r is the viscous damping ratio for the rth mode

The complex (Laplace domain) nature of this expression results from the fact that the distribution of damping throughout the structure may not be proportional to the distribution of the mass and/or stiffness. This is true in many real-world structures, for example when the crankshaft is added to a bare engine block, there is significant lumped damping at the main bearings which causes the modal data to become non-normal (ie complex) for the combined system although the modes of the bare block, with its light proportional damping are real and normal. This effect is shown in Figure 2.

The driving point parameters (frequency, damping and complex residue) form the basis of a modal parameter table which will be updated and improved later by curve fitting additional frequency response functions. In general, the mode shapes are not obtained using an MDOF analysis since every frequency response would have to be curve fitted which would be extremely time consuming.

3. A Single Degree of Freedom (SDOF) circle-fitting technique is applied to each modal circle (ie every mode at all response locations for each exciter position) to extract all the mode shape vectors. The frequency limits for fitting each mode must be carefully chosen to ensure extremely good correspondence between the analytical and measured circular arcs in the region of a resonance. The mode shapes can then be stored for future use. Figure 3 shows a typical circle fit for an identified mode and illustrates the high quality of measured data required.

4. The mode shape vectors can usually be extracted assuming them to be real and normal. The justification for such a simplification can be found by examining the modal circles. Typically in most cases, the phase angle of the complex vector (the angle between the centre of the circle and the resonant frequency point) is approximately $\pm 90^0$ even though some of the circles can be displaced due to contributions from adjacent modes.

5. For each mode, animated mode shapes obtained at different exciter locations can be compared to confirm that they are repeatable from each exciter location and therefore true eigenvectors for the system.

For each natural frequency, the best set of mode shape vectors are stored separately to become the final complete eigenvector matrix.

6. For each mode, the normalising point or a point of large response can be identified and an MDOF curve fit applied to its measured frequency response function. The accuracy of the fit (amplitude and phase) can be confirmed by examining the Bode and Nyquist plots in the frequency range containing the mode. The parameters for the mode can then be moved into the parameter table replacing the original values obtained from the driving point frequency response. This procedure must be repeated for each mode to give a final parameter table containing accurate modal properties. (Table 1). Figure 4 is a typical MDOF curve fit for one frequency response function in this table.

POTENTIAL PROBLEMS WITH MODAL ANALYSIS

1. Damping assumption

The circle fit theory is only valid for lightly damped systems ($\zeta<0.15$). As the damping level is increased, the circle representation no longer holds and the technique breaks down.

2. Frequency resolution requirements

When performing a modal test using digital Fast Fourier Transform techniques there is a conflict between the desirable constant bandwidth frequency resolution and the maximum frequency of interest. Typically, the computed frequency response function (FRF) will contain 1024 equi-spaced spectral lines. Consequently, once the upper frequency limit is chosen, say f_{max}, the frequency resolution is fixed at

$$\Delta f = \frac{f_{max}}{1024}$$

To extract good estimates of both mode shapes and damping from either MDOF or SDOF curve fitting routines, there should be at least five measurement points within the half-power bandwidth for each mode of vibration.

The minimum frequency resolution therefore needs to be

$$\Delta f_{min} = \frac{2}{5}\,\xi f_n$$

where ξ_n and f_n are the critical damping ratio and natural frequency of the rth mode respectively.

In order to satisfy this criteria with engine block frequency response function measurements, two frequency ranges are needed (0-1000 Hz and 0-4000 Hz) to ensure adequate frequency resolution for the lower frequency modes.

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3. Closely spaced modes

In the case of the engine structure, there are some modes of vibration very closely spaced in frequency and there are a number of problems associated with performing a circle-fit analysis with this type of data.

(a) It becomes difficult to use the maximum frequency spacing criteria to identify the resonant frequencies and to choose limits for the circle fits.
(b) It is not clear whether the modes are real or complex.
(c) The mass and stiffness contribution from the close adjacent mode is no longer constant and distorts the circle.
(d) Non-linearities can produce similar distortions of the Nyquist plot.

When this occurs, the MDOF procedure must be used to determine good estimates of the mode shapes as well as the modal parameters.

A major feature of the MDOF curve fitting procedure is its ability to separate and quantify very closely spaced modes. This results from the fact that the complex frequency response functions are represented analytically as Laplace rather than Fourier Solutions. This is only true if the basic model of the system is assumed to exhibit non-proportional viscous damping.

Measurement noise

Because the MDOF curve fitting is a least-squares solution, small amounts of statistical variability in the frequency response function have a negligible effect on the modal parameters extracted. In order to improve accuracy, the statistical variability in the measured frequency response spectrum must be reduced. For a practical measurement this is achieved either by smoothing the data, (which reduces the frequency resolution), or by increased ensemble averaging.

Real or complex modes?

The SDOF and MDOF curve fitting techniques are capable of extracting complex mode shapes from measured data. However the subsequent use of the modal data base is greatly simplified if the modes can be assumed to be real. Therefore the analyst must decide on the validity of a real mode assumption. The 'normality' of the modes can generally be assessed by examining the Nyquist plots. For example Figure 5 was measured on a large built up structure where the structural damping was uniformly distributed throughout the structure (ie proportional damping) and the modal circles are very normal since, in each case, the resonance frequency point lies directly above or below the centre of the circle. On the other hand, Figure 6 was measured on a motor vehicle where the damping elements are discrete (shock absorbers in the suspension assemblies) and the modes are definitely non-normal due to the non-proportional damping.

When the modes are closely spaced, it becomes more difficult to assess the normality since the modal circles become distorted due to the influence of adjacent modes. Figure 7 demonstrates the large phase error which can result if a complex mode is assumed to be real. Note that the magnitude of the Bode plot is the same in both cases.

VALIDATING THE MODAL DATA BASE

One of the most important aspects of the experimental modal analysis is the determination of the accuracy of the information estimated from the frequency response test data. The test and analysis phase of the modal survey must be followed by a 'correlation' process which shows whether the combined inaccuracy of testing methodology and parameter estimation was held small. The frequency response function is related to the complete set of modal parameters and mode shapes by the following expression:

For a system with proportional viscous damping the frequency response at point i due to an input at point k is given by

$$H_{jk} = \sum_{r=1}^{N} \frac{\psi_j^r \; \psi_k^r}{m_r \left(\omega_r^2 - \omega^2 + j2\xi_r \, \omega\omega_r\right)}$$

where $m_r = \dfrac{\psi_m^r \; \psi_n^r}{2A_{mn}}$

Or $$H_{jk} = \sum_{r=1}^{N} \frac{\psi_j^r \; \psi_k^r}{k_r \left(1 - (\omega/\omega_r)^2 + j2\xi_r\omega/\omega_r\right)}$$

Where:

ψ^r ----Eigenvector (ie, mode shape coefficients) for the rth Mode

m_r ----Effective Mass for the rth Mode

k_r ----Effective Stiffness of the rth Mode = $m_r\omega_r^2$

ξ_r ----Equivalent Viscous Damping Ratio for the rth Mode

ω_r ----rth Natural Frequency

ω ----Excitation Frequency

A_{mn} ----Residue calculated for rth Mode from H_{mn} ie value in Parameter Table

This reconstruction can be performed for any pair of structure test locations (using any test location for either the exciter location or the response location). For example, if all modal parameters were estimated from one exciter location, these parameters can be used to analytically predict the response of the system for a different position of excitation. The exciter can then be moved to the position that was analytically simulated so that the analytically predicted frequency response function can then be compared with measured data. Since all of the mode shape data for the given points, as well as the complete set of modal parameter estimates, are involved in this reconstruction, it provides a very thorough check of the entire analysis.

It is worth studying the controlling elements in the synthesis expression. Once the important influences are understood by the engineer then sources of discrepancies between measured and calculated frequency response functions can be quickly traced to errors of judgement during the analysis of the frequency response functions. In the general case there are seven major controlling parameters:

1. The mode shape coefficient at the original driving point
2. The mode shape coefficient at the response point on the parameter table
3. The mode shape coefficient at the new driving point
4. The mode shape coefficient at the new response point
5. The residue calculated from the frequency response function
6. The damping of the mode
7. The natural frequency of the mode

Of these seven quantities, the influence of poor estimates of damping and natural frequency can readily be observed during data analysis. However, phase errors in the frequency response function calculated from the synthesis expression can be due to errors in sign in <u>any</u> of the first five parameters.

The sign of 2 must correspond to the phase of the residue, so there are only four parameters to check when there is an obvious phase error between calculated and measured frequency response functions.

Finally, if the calculated frequency response function is always over or under-predicting then the error will lie in the estimate of the residue and/or damping. Local fluctuations are caused by errors in the mode shape coefficients or non-constant damping across the structure.

MODEL SYSTEM DEFINITION

The experimental modal data base established by the methods described in the previous section are used to produce the system files for the SABBA program.

The SDRC SABBA computer program can be used to couple the structural characteristics of various components or sub-systems in order to predict the response parameters (e.g. force displacement, velocity, acceleration) of a total system. SABBA is based on an analytical formulation known as the SDRC Building Block Approach. The basic philosophy behind this approach involves generation of a total system analytical model from separate analyses of components which make up the system.

For example, in this case the original engine would be one sub-system and the beam stiffeners other sub-systems. Each of these sub-systems can be analysed by existing experimental techniques, finite element routines or classical beam theory. The data generated for each sub-system is then input to SABBA to form a model of the total system. Once a force or displacement input excitation is defined to the SABBA model, the resulting response output can be calculated.

Using SABBA, the selection of sub-systems is arbitrary. Sub-systems are usually determined by the geometric or physical make-up of the system under study. The dynamic behaviour of each sub-system is determined from separate studies on each component. A discussion of the structural parameters required for each sub-system can be found in Reference 2. For sub-systems not easily described by available finite element routines, the methods of dynamic testing previously described have been formulated to obtain these parameters from laboratory test data. Software has been written to interface the laboratory test data with SABBA. Mathematical relationships are derived for each sub-system. The structural parameters for each sub-system are then provided as input to the SABBA computer program to analyse the total system. Once the total system mathematical model is formulated by the program the user then defines force or displacement excitations.

Unlike most finite element simulation programs where accuracy depends on the number of elements used, SABBA allows the user to choose only those points of interest in his system which he desires to study. If properly modelled, a multiple degree-of-freedom finite element component (e.g. 1000 D.O.F.) can be represented by only a few degrees-of-freedom with no loss of accuracy. Because of this flexibility, the use of SABBA is limited only by the user's imagination and his ability to model his structure as an assemblage of separate sub-system components.

For this type of analysis, the points taken across from the experimental model fall into three categories:-

(i) Force input points

(ii) Sub-system connect points

(iii) Response monitoring points.

A typical system file is given in Appendix 1.

The modal parameter table, mode shapes file and geometry file are used to obtain the SABBA system file information automatically. The additional sub-systems are edited on to the end of the file.

FORCED RESPONSE OF THE ENGINE BLOCK

Force inputs to the block can be initially concentrated at the main bearing location, which have been shown to be a major energy route. Using the measured cylinder pressure diagrams in conjunction with a bearing load program, the main bearing load time-histories can be predicted and then transformed into phased frequency spectra for input to the forced response calculation. In this way representative force spectra produce typical block surface velocity distributions which have to be post-processed to define the acoustic coupling between the structure and the surrounding air.

Two methods have been developed. The first uses third octave space-averaged velocity spectra in conjunction with an acoustic model of the engine (Ref.3) consisting of flat plates from which the radiation efficiency of the various surfaces can be estimated and the radiated sound power calculated. In summary the method consists of the following:

(1) Setting up an acoustic model of the crankcase consisting of flat plates in order to calculate their radiation efficiency for different third octave bands

(2) Calculating the space-averaged velocity for each panel in each third octave band

(3) Predicting the sound power radiated by each plate from the expression Sound Power Level (SWL) = $\rho c < u^2 > A\sigma$ rad.

(4) Summing the SWL of each constituent plate to obtain the total acoustic power

The output from the program is a ranking of the sound power radiated by each of the plates constituting the acoustic model of the structure and the total sound power radiated by it.

The second method uses a finite element approach to discretize the surface of the block and then employs an approximate Green's function solution to obtain the volume integral of the surface velocity distribution to calculate the acoustic pressure at some point in the far field.

This method requires the following information:

(a) Geometric definition of the finite elements representing the surface

(b) The element connectivity to describe the area and velocity distribution points

(c) The velocity distribution at each of the modal points of the element

(d) The far - field point co-ordinates for which the acoustic pressure is to be calculated.

The output from the program is the acoustic pressure at a specific point in the far field and the contribution of individual elements to the calculated sound pressure level.

In both cases, the surface velocity distribution can be obtained from experimental or analytical data. For the second method, both magnitude and phase of the velocity distribution are required as inputs to the program.

The second method is far more accurate than the statistical energy approach at low frequencies but as the modal density of the structure increases in the third octave band and the directivity effects of acoustic elements become less important at high frequencies there is justification for using the first method above 1000 Hz for engine structures and the second method below 1000 Hz.

APPLICATION OF THESE TECHNIQUES TO DIESEL ENGINE DESIGN

Figure 8 is a synthesis of a driving point frequency response function in the crankcase region of a diesel engine. The similarity between the measured and calculated frequency response functions providesthe necessary confidence in the quality of the test data.

An analytical representation of a ladder frame was added to the crankcase region of the engine. The modified structure showed a 3 dB reduction in vibration levels from this region of the block. Since the sump is a major radiator of noise such a reduction in the force input to the sump will reduce overall noise levels significantly. In practise a 1½-2 dBA overall engine noise reduction was achieved.

CONCLUDING REMARKS

Modal analysis used in conjunction with SABBA and noise from vibration plate theory is being used by the test engineer and engine designer to investigate noise and weight reduction in diesel engines. However, it should be stressed that the test engineer and designer must still apply sound engineering judgement to the data generated.

The technique does not have the flexibility of purely analytical methods in the modal data base and therefore modifications are restricted to the three translations at each point and limited to discrete stiffness, mass and damping or modal representation of a restricted type of structure,with the similar limitations regarding translation. Furthermore, changes to geometry cannot be made, and the complete removal of stiffness, mass or damping from the model is not possible.

To obtain this level of flexibility, SDRC again combine analytical and experimental techniques but use the measured modal properties to correlate a finite element model of the engine structure. Once confidence has been established in the analytical model, the insight available to the test engineer, analytical engineer and engine designer regarding the structural behaviour of the engine is considerably increased.

The test engineer must rely on the mode shapes, aided by animation, to determine which elements are controlling the particular modes of vibration, whereas with finite element methods this information can be obtained from a strain energy tabulation. Like the finite element method, the combined approach to systems analysis only permits changes to be made at the measurement points, therefore, close cooperation between the designer and test engineer is essential to ensure that points where possible modifications can be made are measured. Modal models obtained from measured test data are, however, not subject to the assumptions, errors and costs of largely analytical models and when used judiciously have been found to be a cost effective way of tackling engine noise and a variety of other mechanical engineering related problems.

The final proof of whether a modification is of any consequence, however, depends upon building the hardware and testing it under operating conditions. These techniques allow the design to be optimised without recourse to many such tests.

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2. KLOSTERMAN, A.L. SAE Paper 720092 'A combined experimental and analytical procedure for improving automotive system dynamics'

3. HAWKINS, M.G., O'KEEFFE, J.M. 1975 CIMAC Conference 'A method of determining the effects of design changes on diesel engine noise'

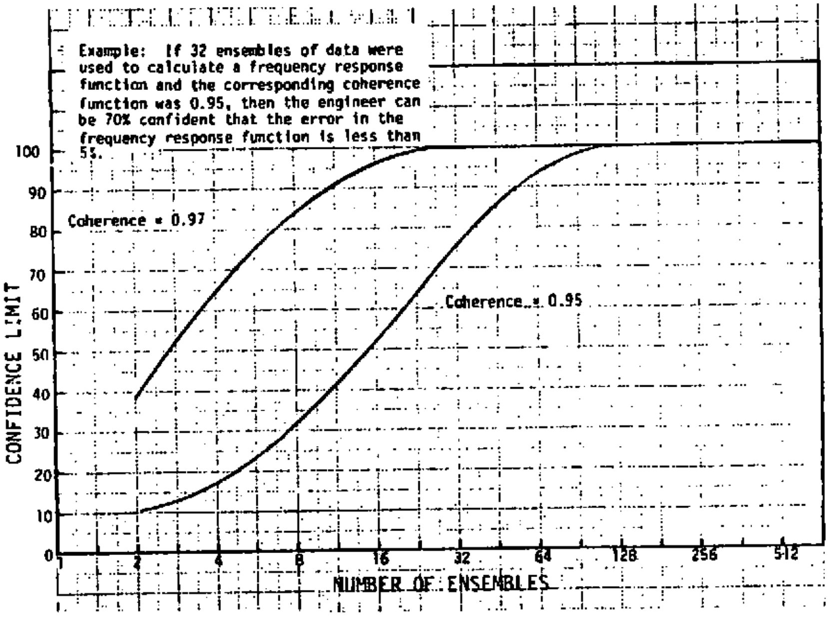

Figure 1. Confidence Limits for 5% Error Based on Number of Ensembles and Coherence Value.

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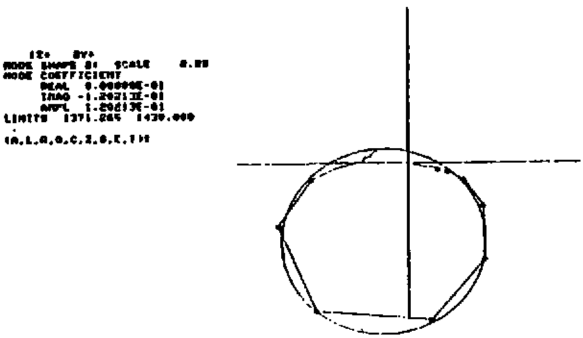

Figure 2a.

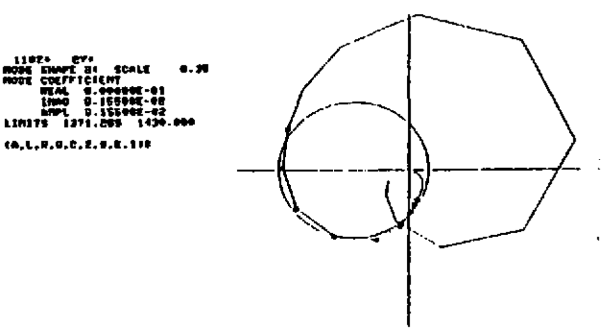

Figure 2b. Comparison of Nyquist Plots for block on its own (2a) and with crankshaft installed (2b).

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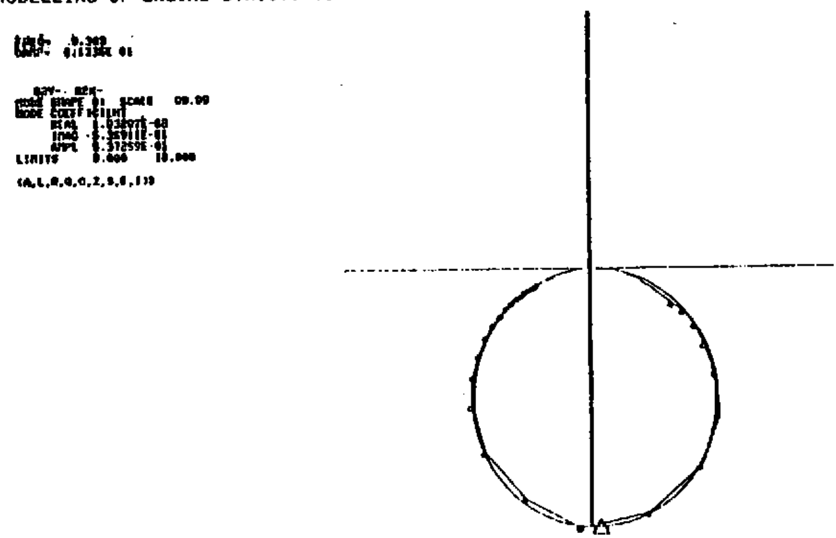
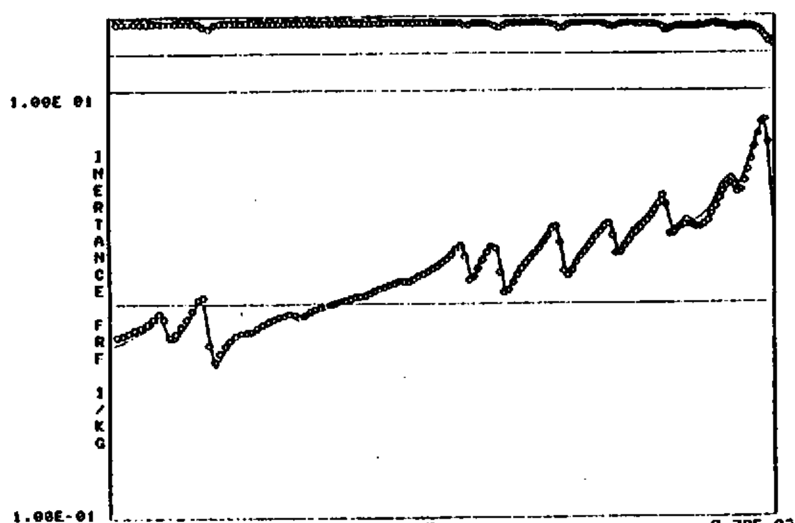


Figure 3. Typical Single Degree of Freedom Circle fit.



MDOF CURVE FIT AT DRIVING POINT WITH RESIDUALS - Figure 4

RESIDUALS

COMPLIANCE	MOBILITY	INERTANCE
1.0534E-09-5.6851E-10	0.0000E-01 0.0000E-01	-4.5734E-01-2.4263E-01

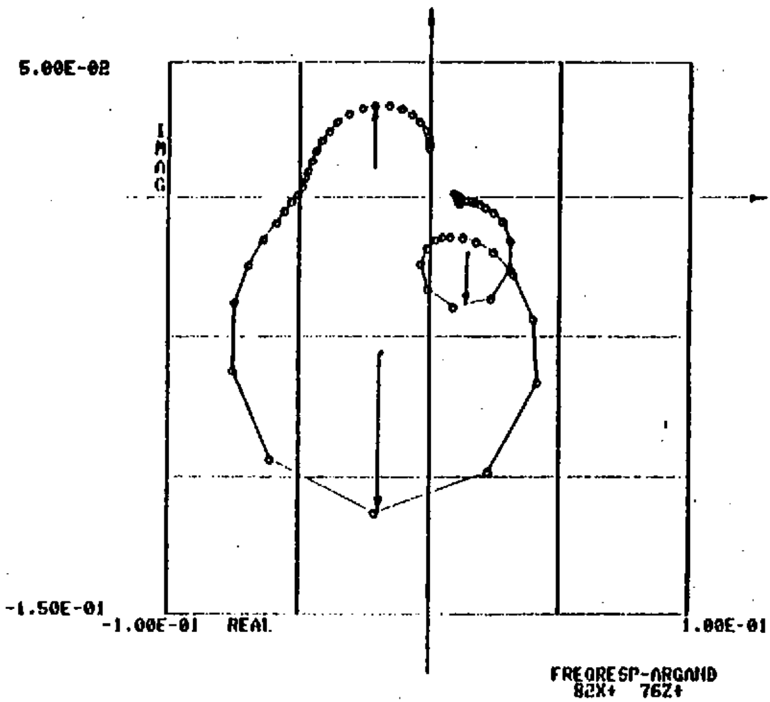


Figure 5. Nyquist Plot for Proportionally Damped Structure

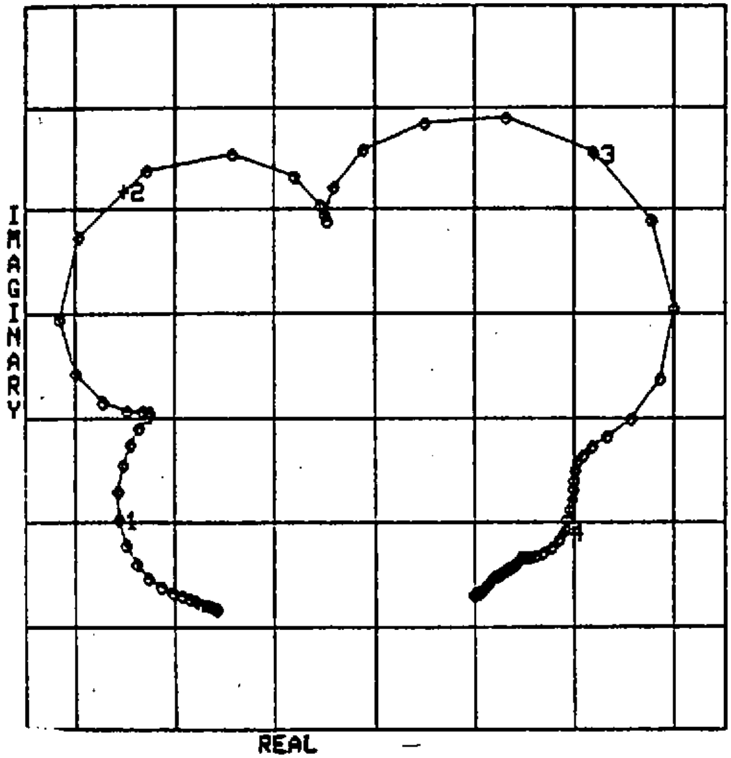


Figure 6. Nyquist Plot for Non-proportionally Damped Structure.

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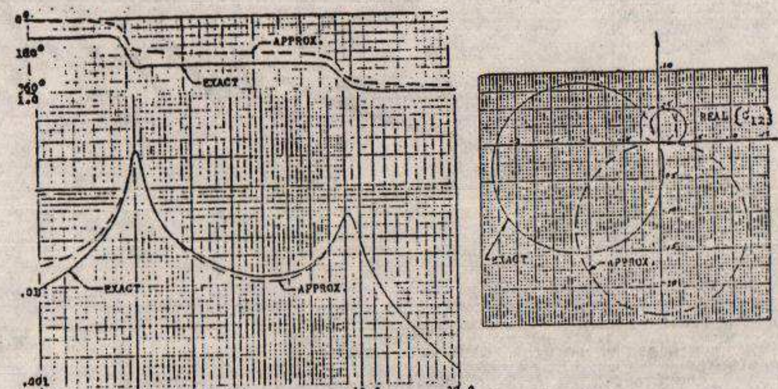

Figure 7. Phase Discrepancy Produced by Selecting Normal instead of Complex Modes

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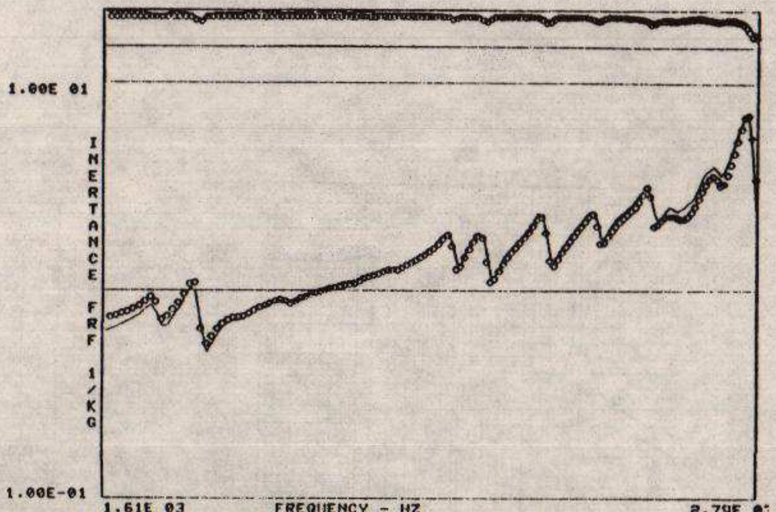

SYNTHESIS OF DRIVING POINT WITH RESIDUALS - Figure 8.

RESIDUALS

COMPLIANCE	MOBILITY	INERTANCE
9.0615E-10-4.1486E-10	0.0000E-01 0.0000E-01	-4.8924E-01-1.3781E-01

$LP

MODE PARAMETERS

LABEL	FREQ	DAMPING	AMPLITUDE	PHASE	REF	RES	MODE	FLAGS
1	504.918	0.034420	3.790	0.3969	1Z-	1Y+	1	0 0 0 0 0
2	580.475	0.015774	0.9212	-1.4608	1Z-	1Z-	2	0 0 0 0 0
3	673.336	0.016548	0.7344	-2.7813	1Z-	1Z-	3	0 0 0 0 0
4	712.600	0.008200	0.7057	-1.7644	1Z-	1Z-	4	0 0 0 0 0
5	805.094	0.006100	6.400	-1.8567	1Z-	1Z-	5	0 0 0 0 0
6	830.662	0.006607	11.34	-1.1002	1Z-	1Z-	6	0 0 0 0 0
8	867.973	0.004837	1.662	-1.5461	1Z-	1Z-	8	0 0 0 0 0
9	932.846	0.004293	10.01	-2.1235	1Z-	1Y-	9	0 0 0 0 0
11	1398.622	0.004359	1.788	1.6827	1Z-	1X+	11	0 0 0 0 0
12	1416.206	0.005240	1.845	1.2029	1Z-	1X+	12	0 0 0 0 0
13	1526.285	0.006564	19.24	-1.9056	1Z-	1Z-	13	0 0 0 0 0
14	1688.695	0.003558	14.42	-2.1535	1Z-	1Z-	14	0 0 0 0 0
15	1747.016	0.002785	28.60	-1.8226	1Z-	1Z-	15	0 0 0 0 0
16	2157.869	0.005888	50.75	-1.4887	1Z+	1Z-	16	0 0 0 0 0
17	2220.081	0.006489	82.38	-1.8861	1Z+	1Z-	17	0 0 0 0 0
18	2334.647	0.006563	112.8	-2.0440	1Z+	1Z-	18	0 0 0 0 0
19	2438.927	0.005021	63.92	-1.9667	1Z+	1Z-	19	0 0 0 0 0
20	2545.361	0.006255	95.81	-1.2152	1Z+	1Z-	20	0 0 0 0 0
21	2688.462	0.010638	353.1	-1.3158	1Z+	1Z-	21	0 0 0 0 0
22	2771.842	0.005473	574.0	-2.4291	1Z+	1Z-	22	0 0 0 0 0
23	2862.414	0.002870	38.90	-2.3423	1Z+	1Z-	23	0 0 0 0 0
24	2942.125	0.005207	563.3	-2.1275	1Z+	1Z-	24	0 0 0 0 0

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TABLE 1. Modal Parameters Extracted from Measured Frequency Response Functions.

'OBTAINING GOOD EXPERIMENTAL DATA FOR THE MATHEMATICAL MODELLING OF ENGINE STRUCTURES'

APPENDIX I SYSTEM FILE FOR CRANKCASE MODIFICATION

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TT:=RK0:CRMS06.DAT

100,MODIFIED CRANKCASE
110,23
120,70Z,81Z,92Z,72Z,83Z,94Z,74Z,85Z,96Z,71Z,82Z,93Z,73Z,84Z,
122,1,2,3,A1,A2,A3,B1,B2,B3
130,MODAL DATA
140,A,14,14,1
150,620.243,736.000,767.758,811.790,830.715,988.314,1032.841,
160,1123.873,1442.611,1667.021,1877.703,2032.784,2251.250,2320.389
170,7193.750,1232.688,0.1119E+05,4052.581,6271.514,2158.531,
180,9541.136,357.234,3423.616,406.077,575.604,3800.208,1225.761,
190,1235.319
200,0.012,0.017,0.019,0.038,0.019,0.024,0.010,0.024,0.025,0.022,
210,0.029,0.019,0.014,0.017
220,MDM
230,1,14.931,21.861,105.440,-2.283,30.352,168.779,3.843,31.840,
240,125.575,4.521,28.086,158.518,5.563,42.553
250,2,2.979,3.801,14.348,10.094,6.562,30.744,6.734,24.599,19.625,
260,9.797,7.517,21.717,14.032,9.045
270,3,-15.494,-21.364,-80.581,5.054,-21.353,-150.071,8.617,-20.583,
280,-160.949,-0.698,-17.843,-100.294,1.556,-38.450
290,4,33.468,28.126,123.028,-3.067,40.987,223.440,-9.809,61.343,
300,221.826,4.703,32.060,156.846,-17.259,77.582
310,5,33.939,28.456,130.596,-3.427,44.236,232.720,8.499,64.411.
320,234.107,4.749,32.834,165.519,17.909,81.377
330,6,12.493,42.139,134.528,-15.363,-6.790,145.784,-34.090,-47.585,
340,-27.630,-4.925,14.402,120.223,-22.375,-53.084
350,7,18.712,34.707,113.069,-3.906,21.908,146.441,31.217,36.289,
360,23.356,4.572,21.599,138.593,17.425,22.449
370,8,-6.806,-17.768,0.000,-8.742,-13.966,35.637,18.292,28.910,
380,-12.584,-17.686,6.785,34.195,1.512,-10.779
390,9,44.409,-23.452,36.172,-41.387,43.430,234.538,45.581,117.624,
400,288.914,-30.925,25.419,127.860,16.000,89.893
410,10,13.586,-40.373,-130.699,-25.998,21.644,77.810,11.319,35.681,

420,109.354,-25.188,6.448,-24.367,32.995,76.321
430,11,27.857,-28.229,-117.380,-15.554,20.834,68.415,-14.896,24.914,
440,89.053,-19.937,-10.535,41.506,31.980,52.630
450,12,-43.766,22.609,19.776,20.981,16.916,137.892,-5.648,25.295,
460,229.149,-33.169,-36.784,43.278,30.171,64.370
470,13,-17.835,15.089,27.431,0.000,-18.899,37.171,-18.643,-14.777,
480,-41.576,14.491,13.438,29.114,31.356,-44.879
490,14,-31.788,-15.531,-37.920,9.350, 8.818,-62.044,-40.720,-30.912,
500,71.637,-20.642,-39.087,-53.561,20.034,19.841
510,1,70Z,81Z,92Z,72Z,83Z,94Z,74Z,85Z,96Z,71Z,82Z,93Z,73Z,84Z
520,0
730,BEAM A PROPERTIES
740,A,3,3,1
780,0,0,10017
790,.56,.14,.16
800,0.0001,0.0001,0.0001
810,MDM
820,1,1,1,1
830,2,1,0,-1
840,3,-1,.6,-1
850,3,A1,A2,A3
860,0
730,BEAM B PROPERTIES
740,A,3,3,1
750,0,0,10017
760,.56,.14,.18
770,0.0001,0.0001,0.0001
780,MDM
790,1,1,1,1
800,2,1,0,-1
810,3,-1,.6,-1
820,4,B1,B2,B3
830,0
730,BEAM STIFFENERS
770,A,3,3,1

780,0,0,10017
790,.56,.14,.18
800,0.0001,0.0001,0.001
810,MDM
820,1,1,1,1
830,2,1,0,-1
840,3,-1,.6,-1
850,2,1,2,3
860,0
870,BEAM TO CRANKCASE ATTACHMENT
880,SPRING,1,70Z,10E9
890,SPRING,2,81Z,10E9
900,SPRING,3,92Z,10E9
910,SPRING,A1,72Z,10E9
920,SPRING,A2,83Z,10E9
930,SPRING,A3,94Z,10E9
940,SPRING,B1,74Z,10E9
950,SPRING,B2,85Z,10E9
960,SPRING,B3,96Z,10E9
970,0
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