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NONLINEAR INTERACTIONS IN COUPLED BEAM SYSTEMS

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1. Introduction

A general and comprehensive description of the role of nonlinear analysis in structural dynamics has been given recently by Barr [1]. It is clear that the dynamicist must (reluctantly) include nonlinearities in his predictive models of structural behaviour to account for a wide range of complex response phenomena, which cannot be explained on the basis of linear system response. With the retention of quadratic nonlinear terms system equations may be seen to admit the possibilities of internal resonance when, if simple coincidental relationships exist between the undamped natural modal frequencies complex scattering of vibrational energy may result during forced or motional excitation and modes having natural frequencies remote from the excitation frequency may respond significantly.

There is no attempt in the present paper to extend the general analysis of such systems, but rather it is a description of work on a specific example of a configuration which gives an interesting range of responses. The system considered is a system of two beams coupled at right angles (Fig. 1). The primary beam AB is considered to be force excited in the vertical plane. The secondary beam CD is arranged to have low stiffness for bending out of the vertical plane. This type of arrangement is common in many structural forms. During an investigation of random vibration response of a model of this type [2] it was found that a narrow band excitation centred at a relatively high frequency (52 Hz typically) produced large responses in the first out-of-plane bending mode of CD at typically 4.5 Hz and in the first in-plane bending mode of the structure at 9 Hz. It was subsequently established that the effect also occurred with deterministic force excitation and was due to a four mode nonlinear interaction involving two plane bending modes, together with torsion and out-of-plane bending of the coupled beam CD. Fig. 2 shows some representative transducer traces of the effects. The analysis summarised below leads to an approximate prediction of the zone of interaction as a stability boundary for the onset of out-of-plane motion, and a qualitative explanation of the response.

2. System Equations of Motion

We represent the in-plane motion of the primary beam by an n degree of freedom lumped parameter model, with coordinates $q_1 .. q_n$, excited by an external force and the interaction loads at the coupling point. We model the coupled beam CD as a massless symmetric element with a single discrete mass, driven by the transverse displacement W and slope angle α at the coupling point C. (Fig. 3). Three cartesian displacement functions u, v, w specify the deformed elastic line and three Euler angles specify the section principal directions at any point. To quadratic order, the local curvatures and torsion rate are reduced to:

$$\kappa_1 = -v'' + u''\phi \quad ; \quad \kappa_2 = u'' + v''\phi \quad ; \quad \tau = \phi' + u''v' \qquad (1)$$

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The following constraint relations, based on inextension of the elastic line and negligible curvature in the local stiff plane of bending may be applied:

$$w(z) = -\tfrac{1}{2}\int_0^z u'^2 dz_o \; ; \; v(z) = \int_0^z (z-z_o)u''\phi dz_o \qquad (2)$$

Then using Galerkin's method with assumed deformation forms:

$$u(z) = f(z)u_o \; ; \; \phi(z) = h(z)\phi_o ; \; f(\ell) = h(\ell) = 1 \qquad (3)$$

leads to a two degree of freedom representation of the coupled beam in terms of its out-of-plane bending coordinate u_o, and torsion coordinate ϕ_o; retaining up to quadratic terms. Thus:

$$\ddot{u}_o + 2\delta_b\omega_b\dot{u}_o + \left[\omega_b^2 + \ddot{W}B_3 + \ell\dot{\alpha}^2 B_3\right] u_o + \ell\ddot{\alpha} B_4\phi_o = 0 \qquad (4)$$

$$\ddot{\phi}_o + 2\delta_t\omega_t\dot{\phi}_o + \omega_t^2\phi_o + \frac{m\ell}{I_o} B_4 \ddot{\alpha} u_o = 0 \qquad (5)$$

together with equations of plane motion:

$$\underline{M}\ddot{\underline{q}} + \underline{D}\dot{\underline{q}} + \underline{K}\underline{q} = \underline{P} \; ; \; q_1 \equiv W \; ; \; q_2 \equiv \alpha \qquad (6)$$

$$P_1 \equiv -m\dot{\alpha}^2\ell - mB_3(\dot{u}_o^2 + u_o\ddot{u}_o) \qquad (7)$$

$$P_2 \equiv -m\ell B_4(\ddot{u}_o\phi_o + 2\dot{u}_o\dot{\phi}_o + u_o\ddot{\phi}_o) \qquad (8)$$

$$P_k \equiv P\cos\Omega t \qquad (9)$$

It is clear from (4) and (5) that in-plane motion may excite out-of-plane bending and torsion through the quadratic coupling terms. Possible forms of interaction are discussed below.

3. Discussion

Two mode Interaction: For the case of Ω close to an in-plane natural frequency, a single mode representation of the in-plane motion may be considered by using $\underline{q} = \underline{r}_j \zeta_j$ in (6) where $\underline{r}_j$ is an eigenvector of the linearised plane problem with corresponding natural frequency ω_j. (6) then has the form

$$M_j(\ddot{\zeta}_j + 2\delta_j\omega_j\dot{\zeta}_j + \omega_j^2\zeta_j) = -mr_{1j} B_3 (\dot{u}_o^2 + u_o\ddot{u}_o) -$$
$$-mr_{2j}\,\ell B_4(\ddot{u}_o\phi_o + 2\dot{u}_o\dot{\phi}_o + u_o\ddot{\phi}_o) + r_{kj} P\cos\Omega t \qquad (10)$$

with

$$r_{1j}\zeta_j \equiv W \; ; \; r_{2j}\zeta_j \equiv \alpha$$

If there is some transverse displacement at the coupling joint C associated with this mode, then W will act as a parametric load in the u_o equation, while u_o motion will generate a reaction force from (7). This is autoparametric coupling as described in [3] and is known to be significant if

$$\Omega = \omega_j = 2\omega_1 . \qquad (11)$$

Three Mode Interaction: Once more considering a single mode representation of the in-plane motion excited by the external force as in (10) and assuming significant pitch angle at the coupling point, the arrangement of the quadratic coupling terms in α in (4) and (5) suggests a close parallel with parametric excitation of a two degree of freedom system and the possibility of combination type instability when Ω is in the neighbourhood of $\omega_t + \omega_b$, with the tangential acceleration $\ell\ddot{\alpha}$ as the excitation parameter [4]. We may expect in addition that α will be dynamically magnified by the external resonance if $\Omega \approx \omega_j$ so that the condition $\Omega = \omega_j = \omega_t + \omega_b$ may be expected to denote a region of strong three mode interaction. In the coupled motion u_o and ϕ_o will respond more or less at their individual resonant frequencies, so that the quadratic reaction moment term (8) on the in-plane system will contain significant frequency content at $\omega_t + \omega_\phi = \omega_j$ and will modify the in-plane motion. Note that purely in-plane response is a solution of the system of equations (4) - (6). The stability of this solution for small out of plane disturbances may be considered by neglecting products of u_o and ϕ_o in (6). The equations then represent a parametric excitation problem with excitation parameters V and α taken in the form of steady-state frequency response functions, derived from the primary excitation. The well known solution for the boundary curve of the combination instability zone [4] may then be applied in the form:

$$\frac{\Omega}{\omega_b+\omega_t} = 1 \pm \frac{1+d}{2\sqrt{d}}\,(\mu^2-\beta^2)^{\frac{1}{2}} \; ; \; d = \frac{\omega_t\delta_t}{\omega_b\delta_b} \; ; \; \beta = \frac{2\sqrt{\omega_b\omega_t\delta_b\delta_t}}{\omega_b+\omega_t} \qquad (12)$$

where the excitation parameter μ is given by:

$$\mu = \frac{E_4\sqrt{\dfrac{m\ell^2}{I_o}}\;\Omega^2\;\alpha_o\;(\Omega)}{2(\omega_b+\omega_t)\sqrt{\omega_b\,\omega_t}} \; ; \qquad (13)$$

(12) predicts a V shape instability zone in the (μ, Ω) plane centred on ($\omega_b + \omega_t$). α_o is the steady state pitch angle at the coupling point for forced vibration at frequency Ω, and will itself be subject to dynamic magnification at resonance of Ω with ω_j. This simple approach gives good correlation with regions of three mode interaction observed on the model.

Four Mode Interaction: As an extension of the previous case we may examine four mode interaction formally by taking a two mode representation of in-plane motion $\underline{q} = \underline{r}_i\,\zeta_i + \underline{r}_j\,\zeta_j$. This will lead to second equation of the form of (10) representing a mode excited at a frequency remote from its resonance by the external force and the quadratic reaction moment term. We will consider these effects to be small. The remaining quadratic term in u_o admits an interesting possibility. During out-of-plane motion u_o response will occur in the close neighbourhood of the out-of-plane bending frequency ω_b. The quadratic term will have strong content at a frequency $2\omega_b$ and this may be resonant with the second in-plane mode, leading to a complex pattern of four mode motion if the conditions $\Omega \approx \omega_j \approx \omega_b + \omega_t$; $\omega_i \approx 2\omega_b$ are realised. This is precisely the type of response observed on the model and the system resonant frequencies plotted in Fig. 4 show that the resonance conditions are satisfied.

4. Conclusions

Following derivation of system differential equations we have given a qualitative explanation of the complex interactive motions observed with this structural arrangement. Work is proceeding on response prediction by numerical integration.

References

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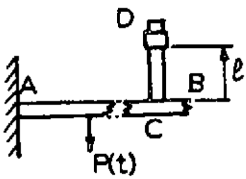

FIG. 1 BEAM MODEL

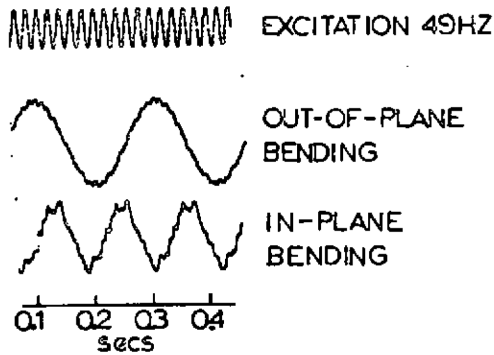

FIG. 2 MODEL RESPONSES

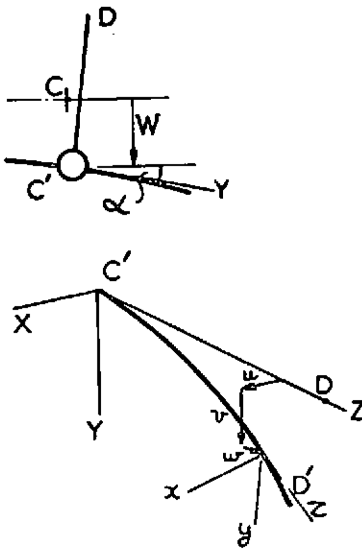

FIG. 3 MODEL DEFORMATION

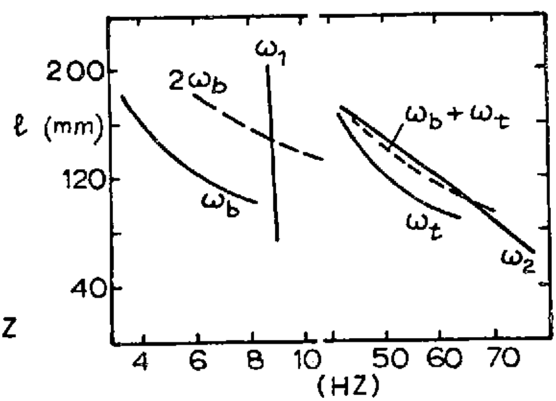

FIG. 4 MODEL NATURAL FREQUENCIES