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HIGHER ORDER SPECTRA TO CHARACTERIZE ACOUSTIC AND VIBRATION SIGNALS

J W A Fackrell, P R White and R J Pinnington

Institute of Sound and Vibration Research, University of Southampton

1 Introduction

Recently there has been a great deal of interest in the Higher Order properties of signals. These properties have potential uses in fields as diverse as Economics [1], Plasma Physics [2], Speech Processing [3] and Underwater Acoustics [4]. These Higher Order properties may be able to supplement information obtained from traditional methods, and may prove useful in condition monitoring and source identification. There has been a good deal of theoretical development in this field, mostly carried out by statisticians, but there has not been much work done on the applications of the theory. We are investigating the origins of higher order spectral content in real signals.

Traditionally, most spectral analysis has concentrated on the Second Order Spectrum - the Power Spectrum. This is because of computational convenience, and the fact that the magnitude of the 2nd Order Spectrum represents a useful physical property of the signal - the power.

The Power Spectrum is sensitive to the 1st and 2nd order properties (characterized by the mean and variance) of the incoming signal, but many real-life signals may have higher order properties as well - skew (the Third Order Moment) and kurtosis (the Fourth Order Moment). Thus there is a possibility that a study of the third and fourth order spectra of a signal may reveal information about that signal that cannot be obtained from conventional Power Spectrum analysis. In this work we have concentrated on the 3rd-order Spectrum - the Bispectrum.

2 The Bispectrum

Higher Order spectral measures are usually found by extending the definitions of the familiar Second-Order measures. The Bispectrum of a continuous signal can be expressed in terms of the 3rd order cumulants[1] of the signal. For our purposes it is more convenient to express the (continuous) Bispectrum as a triple product of Fourier Transforms.

$$B(f_1, f_2) = X(f_1)X(f_2)X^*(f_1 + f_2) \tag{1}$$

[1]Cumulants are related to the signal's moments - the 1st order cumulant is the mean, and the 2nd order cumulant is the auto-correlation function. A discussion of cumulants can be found in [6].

The third term $X^*(f_1 + f_2)$ in equation 1 can be re-written as the Fourier Transform of a third non-independent frequency f_3, which must satisfy $f_1 + f_2 + f_3 = 0$. For random noise, the phase of the signal at three different frequencies will generally be uncorrelated, and as a result the Bispectral contributions will disappear with averaging [2]. However, if three frequencies have phases which are related in some way, then the Bispectrum will not average to zero. This is the key to the mechanism of the Bispectrum. Non-linear interactions between frequencies give rise to related frequency triplets, and a non-zero Bispectrum. This effect is called quadratic phase coupling.

Whereas the 2nd order Spectrum can be plotted in 2 dimensions as power spectral density versus frequency, the Bispectrum is plotted in three dimensions with two frequency axes f_1 and f_2 and the Spectral content rising out of this frequency plane. It is not necessary to compute the Bispectrum for all combinations of f_1 and f_2 because there are a number of symmetry lines in the Bispectral domain, which substantially reduce the number of calculations required. For the discrete Bispectrum, the non-redundant region is called the Principal Domain, and can be divided into two triangles, called the Inner Triangle (IT) and the Outer Triangle (OT) [4](see Figure 1). Certain properties of the signal show themselves by their form in these triangles, and there are a number of tests devised by Hinich [1], [5] which make use of the form of the Bispectrum in one or other of the triangles.

A test for skewness can be carried out by summing the modulus squared Skewness function (see equation 3 below) for each point in IT. If the signal is not skewed then this sum will be χ^2 distributed, with twice as many degrees of freedom as there are points in IT. By calculating the deviation of the sum from the χ^2 distribution with these degrees of freedom, at some confidence level, a decision can be made on the skewness of the signal [7]. In the case of Gaussian signals, which have no statistical properties above order two, all spectra of order above two (including the Bispectrum) will be zero, so this test can also be said to be a test for non-Gaussianity. If the Skewness function is statistically constant in IT then the process is linear. The same test can be carried out in the OT to give a test for stationarity.

For a finite sequence of data, estimation techniques must be used to give reliable predictions. The incoming data signal x(t) is sampled at a frequency high enough to prevent aliasing. The sampled sequence x_n of length N is then divided into K segments (which can overlap) and the L-point Discrete Fourier Transform $X_i(\omega)$ is formed for each segment i, now with discrete frequencies ω. Within each segment the *raw* Bispectrum is then computed from a triple periodogram product.

$$B_i(\omega_1, \omega_2) = X_i(\omega_1) X_i(\omega_2) X_i^*(\omega_3)$$

The Bispectral estimate is then formed by taking the average Bispectrum over all the K segments.

$$B_K(\omega_1, \omega_2) = \frac{1}{K} \sum_{i=1}^{K} B_i(\omega_1, \omega_2) \qquad (2)$$

This estimate is asymptotically consistent provided that the number of segments is at least as large as the FFT size used [1] (for non-overlapping segments). However, the variance of this estimator is proportional to the triple product of the spectra $S(\omega_1)S(\omega_2)S(\omega_3)$, and so this estimate is sensitive to the variance of the signal. A normalised Skewness estimator can be formed by dividing equation 2 by the triple product of spectra [5].

$$s^2(\omega_1,\omega_2) = 2\frac{| B_K(\omega_1,\omega_2) |^2}{S(\omega_1)S(\omega_2)S(\omega_3)} \qquad (3)$$

This expression is valid for the situation where the number of segments is identical to the FFT size, and there are no overlaps.

An alternative normalisation, called the bicoherence function, can be formed [2] from

$$b^2(\omega_1,\omega_2) = \frac{| E[X(\omega_1)X(\omega_2)X(\omega_3)] |^2}{E[| X(\omega_1)X(\omega_2) |^2]E[| X(\omega_3) |^2]} \qquad (4)$$

The bicoherence function only takes values between 0 and 1, with a value close to 1 indicating a strong quadratic phase coupling. This function may prove a useful measure in the future, as it is easier to interpret than the Skewness function. However, the statistics of the variance of the Skewness function are better defined, and so statistical tests usually use the Skewness function.

3 Experiments

The Bispectrum estimation routines written in MATLAB were tested by sampling some real continuous skewed noise. A continuous source was used because there are practical limitations on the use of discrete linear models to generate skewed signals [8]. A White Noise Generator was used as a signal source, and the signal was subsequently Low Pass (LP) filtered, squared, and sampled to give a skewed sequence x_n. For a fixed sample rate f_s, the signal must be bandlimited at $f_0 < \frac{f_s}{4}$ to prevent aliasing, because the squaring process generates signal components at $2f_0$.

Data were captured for two Low Pass Filter settings with a fixed sampling rate. The first signal was LP filtered with $f_0 = \frac{f_s}{4}$ resulting in a signal bandlimited to $f_s/2$ after squaring, and so the sampled signal was unaliased. The second signal was LP filtered to $f_0 = \frac{f_s}{2}$, resulting in a signal bandlimited to f_s after squaring, so this signal *was* aliased. The time histories, Power Spectra, and the normalised Skewness functions of these captured signals are shown in Figure 2. Only Skewness points larger than 10.6 are shown in these plots, because these are significant at the 0.5 per cent level [7].

For the properly sampled signal, it is evident that the OT is very small, whereas for the under sampled signal it is evident that the OT is more highly populated. The statistical tests of Hinich also indicate that the 2nd data set has a much larger OT, and therefore, if the signals are both assumed stationary, the tests *indicate* that the second time history is aliased. Both signals have a significant IT component because they are derived from a nonlinear filtering operation (squaring) on a Gaussian process (White Noise).

It is also worth noting that in this case the existence of aliasing could be predicted just as well from the Power Spectra. The unaliased signal has a Power Spectrum which has a very low level at the folding frequency $f_s/2$, whereas the aliased signal has a large Spectrum right up to the folding frequency.

Current attention is now focussing on the Bispectral features of real mechanical systems. An experimental rig has been set up with an electrically powered air compressor mounted on a perspex surface (see Figure 3). It is possible to insert vibration isolators between the compressor and its mounts, and measurements of acceleration and sound level are possible for each configuration. The purpose of these experiments is to investigate how Bispctral components are generated, how these components are attenuated with distance from source, and whether the Bispectral components in some way characterize the machine.

Initial results indicate that the vibration on the compressor is highly sinusoidal, and has a small Bispectrum. The vibration levels on the perspex sheet however, show significant Bispectral content. This may indicate a non-linear transmission path between the source and measurement position. We are continuing our experiments to see whether the shape of the Bispectrum changes as the measurement position changes, and whether the Bispectral content diminishes more quickly or less quickly with distance than the Power Spectrum.

We are also interested in finding out if there are correlations between the Bispectra measured at different plate positions for a particular vibration isolator. If these correlations exist, they may have uses in condition monitoring and related subjects.

4 Conclusion

It is evident from the discussion presented above, that the Bispectrum can reveal information about a system that the Power Spectrum cannot. However, the analysis and interpretation of these results is not as straightforward as for the Power Spectrum, and more work is warranted on the origins of Bispectral content. Our experimental work is continuing, and we hope to find some correlation between the various experimental set-ups and the measured Bispectra.

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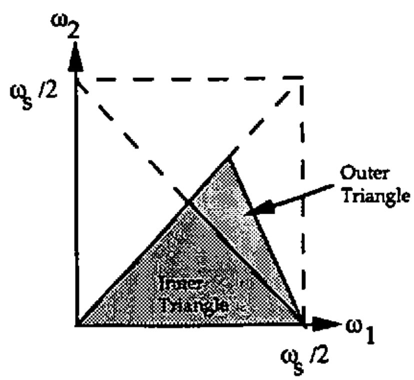

Figure 1: Principal Domain of the Discrete Bispectrum

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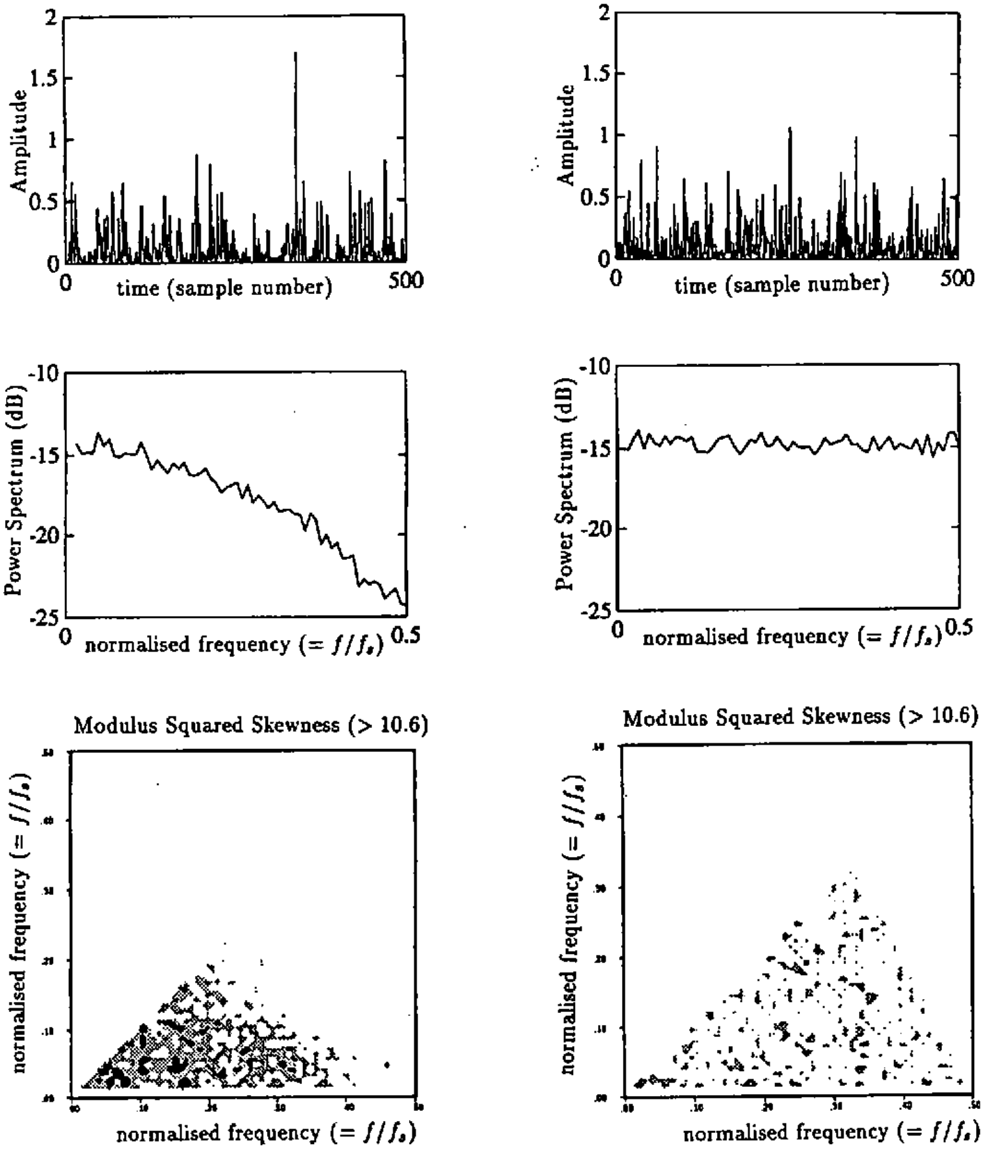
Figure 2: a) Unaliased Skewed Noise b) Aliased Skewed Noise



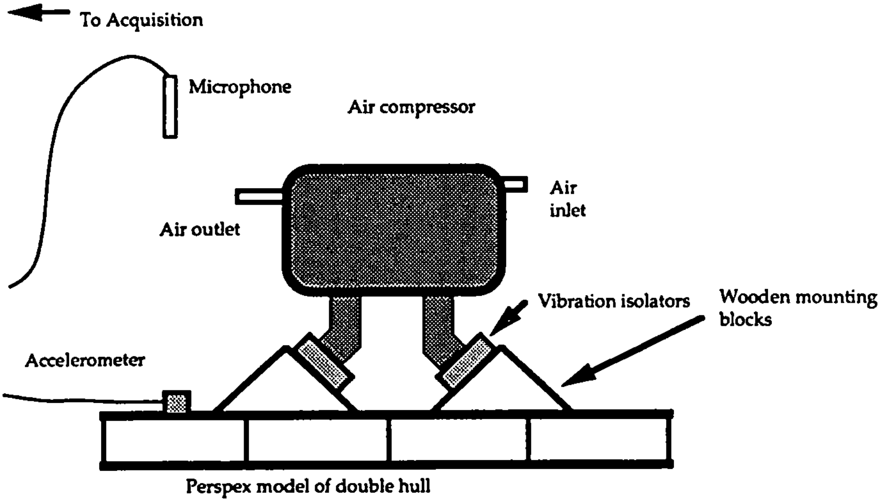


Figure 3: Experimental Set-Up