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THE APPLICATION OF EIGENVECTOR DECOMPOSITION IN THE NEAR FIELD

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INTRODUCTION

There has been a considerable interest recently in the use of singular value decomposition in the processing of signals from arrays in order to achieve higher resolution. Two particular techniques which have received much attention are the Music algorithm developed by Schmidt(1), and the method proposed by Kumaresan and Tufts(2). These methods have mainly been applied to the separation of sources in the far field of the array, but as arrays become larger it is quite possible for the sources to be in the near field. This paper discusses the application of these techniques to the separation of sources in the near field, and it is shown that by use of a fairly simple process the same type of performance can be obtained in the near field as is obtained in the far field.

SYSTEM MODEL

The system being considered comprises a linear array of N equally spaced sensors receiving signals originating from a number of sources of unknown position. The sources are assumed to have a sinusoidal form given by the expression

$$a_m(t) = A_m \cos(\omega_m t + \phi_m) \tag{1}$$

where A_m, ω_m, and ϕ_m are the amplitude, frequency and phase of the source respectively.

The signal at the ith element due to the mth source will be

$$s_{mi}(t) = a_m(t) \cdot c_{mi} \tag{2}$$

c_{mi} is a factor which represents the effect of the propagation and is given by

$$c_{mi} = \frac{1}{r_{mi}} \exp(jkr_{mi}) \tag{3}$$

where k is the wave number $2\pi/\lambda$ and r_{mi} is the range from source m to element i.

In practice the values of r_{mi} are fairly similar and although the small differences have a very significant effect on the phase the effect on the amplitude can be ignored. We can then re-define c_{mi} as

$$c_{mi} = \exp(jkr_{mi}) \tag{4}$$

If there are M sources then the total signal received by element i is given by

$$s_i(t) = \sum_{m=1}^{M} s_{mi}(t) \tag{5}$$

It is convenient to express the signals received by the array at a particular instant as a column vector viz:

$$S^T(t) = \left[s_1(t), s_2(t) \;\ldots\ldots\; s_N(t) \right] \tag{6}$$

We can represent the space effect on each source as an N dimension column vector:

$$C_m = \begin{bmatrix} \exp(jkr_{m1}) \\ \exp(jkr_{m2}) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \exp(jkr_{mN}) \end{bmatrix} \tag{7}$$

Similarly the source signals can be represented as a signal vector

$$A^T(t) = \left[a_1(t), a_2(t) \;\ldots\ldots\; a_M(t) \right] \tag{8}$$

and the space vectors can be combined to form a matrix.

$$C = \begin{bmatrix} C_1 & C_2 & C_3 & \ldots\ldots & C_M \end{bmatrix} \tag{9}$$

The vector S(t) represents a snapshot at a particular instant in time

$$S(t) = A(t) \cdot C \tag{10}$$

In practice since the bandwidth is usually small compared with the centre frequency we express the received signal snapshot vector as a column vector of complex values representing the amplitude and phase of the received signal on each element at that instant viz:

$$X^T(t) = \left[x_1(t), x_2(t) \;\ldots\ldots\ldots \right] \tag{11}$$

Of course in addition to the signals there is almost certainly some accompanying noise.

If a succession of snapshots are taken then we form a data matrix containing the set of vectors X(k), the integer k representing the number of the snapshot.

$$D = \begin{bmatrix} X(1), X(2), \ldots, X(K) \end{bmatrix} \tag{12}$$

An important parameter in the analysis is the covariance matrix which is defined as

$$R = E\left[X^* X^T \right] \tag{13}$$

In practice we have to estimate the covariance matrix from the available data and the best estimate is given by

$$\hat{R} = \sum_{k=1}^{K} X^*(k)\, X^T(k) \qquad (14)$$

The the estimate of the covariance matrix is given by

$$\hat{R} = D^* D^T \qquad (15)$$

EIGENVECTOR DECOMPOSITION

The covariance matrix can be decomposed into a number of matrices each of which is the outer product of an eigenvector of the covariance matrix viz:

$$R = \sum_i \lambda_i U_i^* U_i^T \qquad (16)$$

where λ_i are the eigenvalues.

This can be expressed in matrix notation as

$$R = U^* \Lambda U^T \qquad (17)$$

where U is a matrix whose columns are the eigenvectors and Λ is a diagonal matrix of the eigenvalues.

It is fairly easy to show that the eigenvalues can be divided into two classes, those associated with the noise - the so called noise subspace - and those with the signal - the signal subspace. The eigenvalues of the eigenvectors in the noise space are equal to the noise power and those of the signal space are in general equal to the signal power of the appropriate signal plus the background noise. We can thus write equation 17 as

$$R = R_S + R_N \qquad (18)$$

In practice only an estimate of the covariance matrix is available and as the number of signals is not generally known apriori then the division into two classes is subject to error. However, assuming such a division is made, then it is possible to use the eigenvectors to determine both the position and power of the signals. Since the noise eigenvectors are orthogonal to the signal vectors the beam patterns obtained by multiplying the vectors by the steering vector will have zeros in the directions of the signals. The MUSIC algorithm makes use of this fact and by averaging the beampatterns resulting from the noise eigenvectors obtains an accurate estimate of the signal positions. A further simple process enables the powers to be obtained. The algorithm proposed by Kumaresan and Tufts is also based on the decomposition of the covariance matrix and on the division into the noise and signal subspaces. However this algorithm uses the signal subspace eigenvectors and attempts to determine a vector such that

$$R_S \cdot W = 0 \qquad (19)$$

One of the elements of the vector W is made equal to unity to avoid the trivial solution of W=0.

PARTITIONING TASK

It was mentioned earlier that the signal eigenvalues correspond to the signal powers. This is only true for widely separated signals. In Figure 1 we see the effect on the eigenvalues of moving one source across the range of angles while the other source remains stationary. As the two sources get close together the eigenvalues separate into a sum and difference of the signal powers and clearly one of the eigenvalues is tending towards zero. Thus the partitioning task can become very unreliable unless we have a very large amount of data.

RESULTS

In Figure 2 we see the beam patterns of the individual noise eigenvectors. In Figure 3 we show the beampatterns obtained using the MUSIC and the Kumaresan-Tufts algorithms. In both of the figures there were 8 elements spaced by half a wavelength and there were three sources in the far field. It can be seen that both algorithms accurately locate the positions of the sources. Using this data we can estimate the power of the signals and in Figure 4 we see a typical conventional angular spectrum together with the powers estimated from the algorithms. The striking ability of these methods to locate and estimate the power of the sources is well illustrated.

If the sources are in the near field then we need to use the appropriate steering vector. Figure 5(a) illustrates the effect of continuing to use the far field steering vector. It can be seen to have a very significant effect on the performance. However by first processing the data as though the sources are along a line normal to the array and at an appropriate distance (in effect focusing the array) Figure 5(b) shows that a considerable improvement can be obtained.

Of course in order to compute these curves we must have apriori knowledge of the expected distance. Without this it would be necessary to process the data for many different distances - a time consuming process.

REFERENCES

(1) R.O. Schmidt, 'Multiple Emitter Location and Parameter Estimation', Proc. R.A.D.C. Spectral Estimation Workshop (Rome, N.Y.) 1979.

(2) R. Kumaresan and D.W. Tufts, 'Estimating the angles of arrival of multiple plane waves', I.E.E.E. Trans. A.E.S. Vol. 19 No. 1 Jan. 1983 pp 134-139.

FIGURES

Figure 1 - Variation of eigenvalues with spacing

Figure 2 - Noise eigenvector beams

Figure 3 - MUSIC and KT algorithms

Figure 4 - Angular Power Spectrum

Figure 5 - Showing the effect of using an incorrect steering vector.

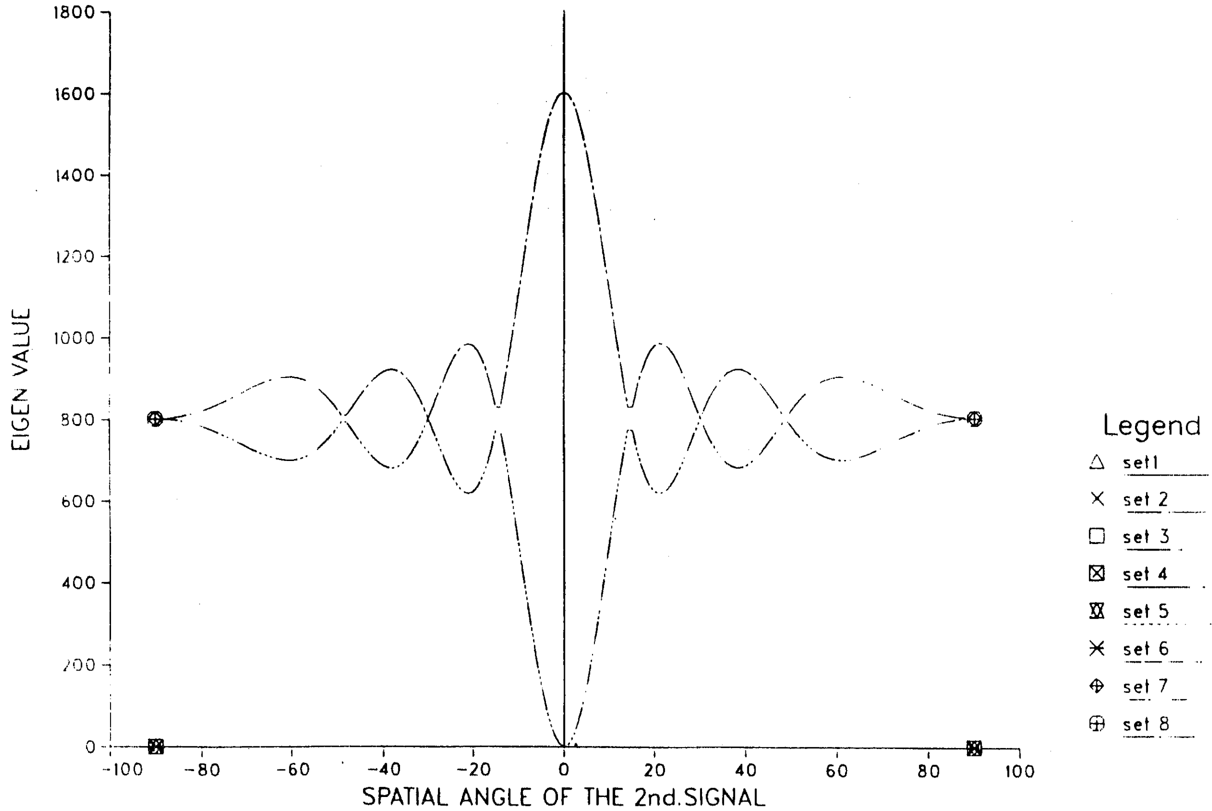

Figure 1. Variation of Eigenvalues with Spacing

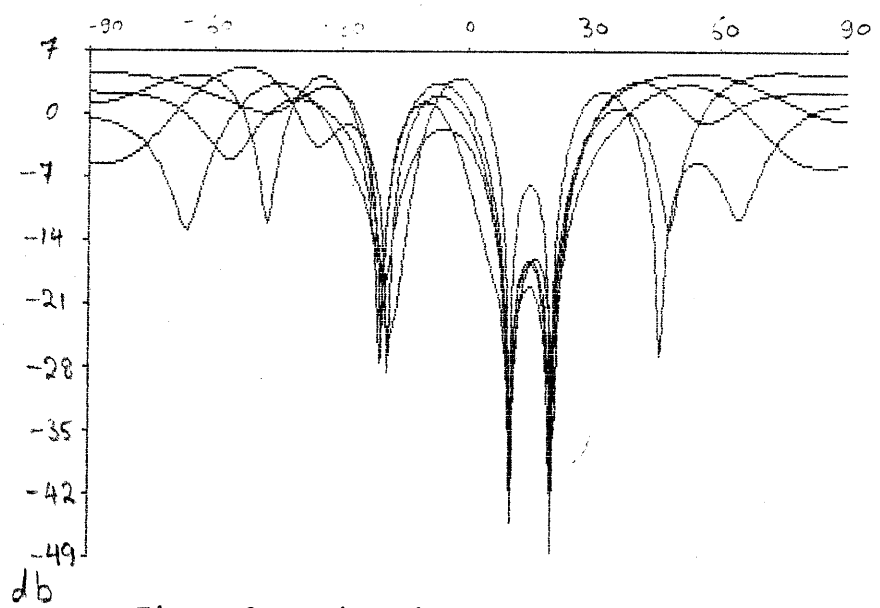

Figure 2. Noise Eigenvector Beams

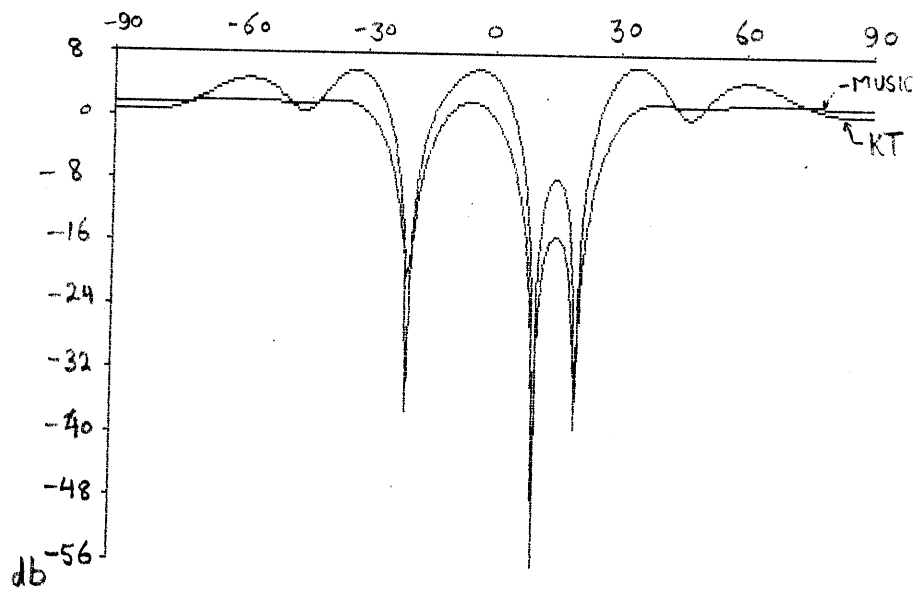

Figure 3. MUSIC and KT Algorithms

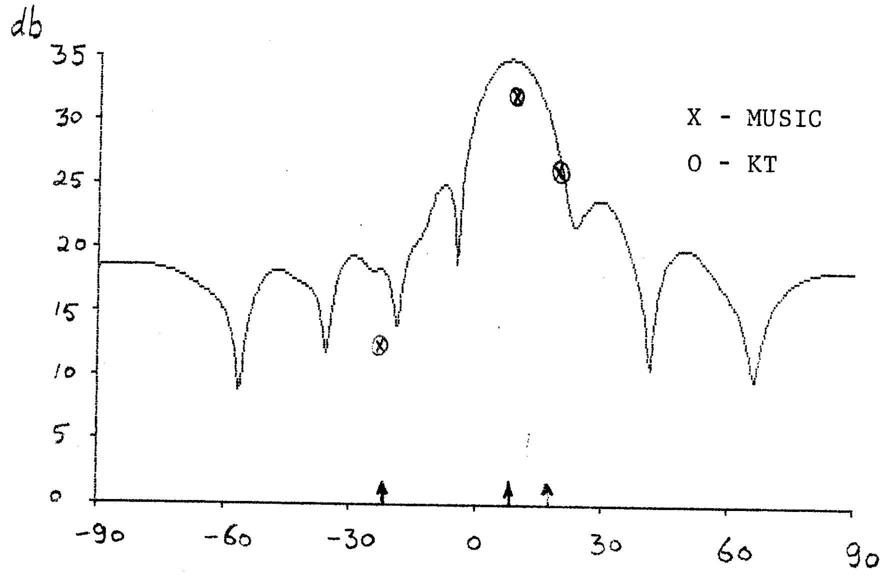

Figure 4. Angular Power Spectrum and Estimated Power

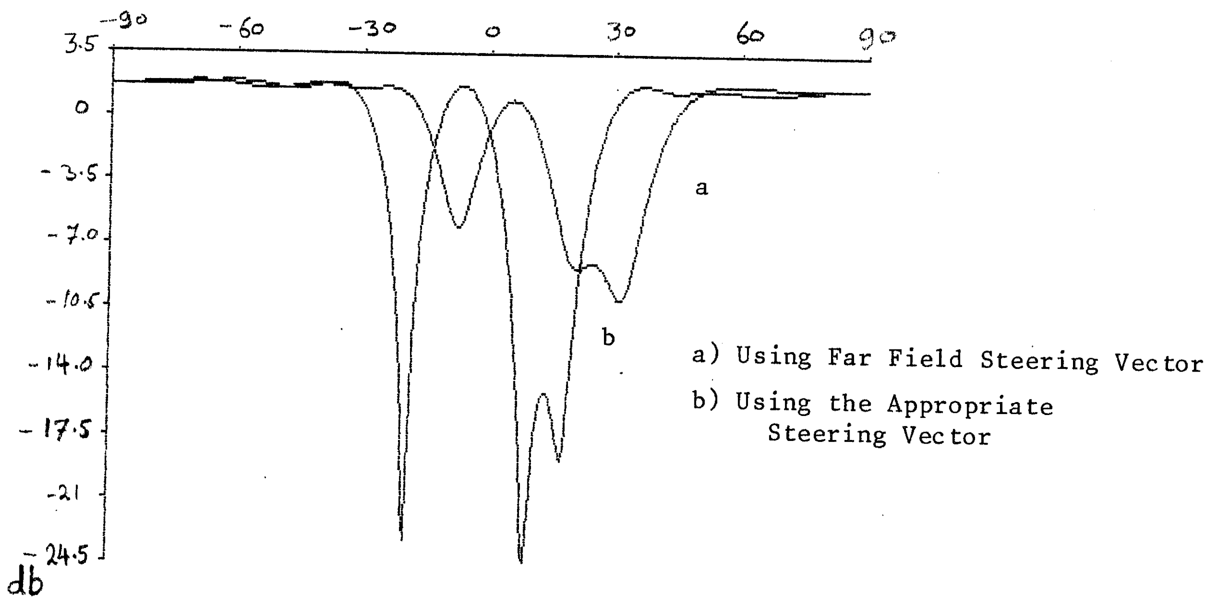

Figure 5. The Effect of using an Incorrect Steering Vector