Proceedings of the Institute of Acoustics

SENSOR ARRAY PROCESSING

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1. INTRODUCTION

Sensor array processing deals with the processing of signals from an array of sensors in order to extract information from these signals. Arrays of sensors are used in sonar, communication and radar systems, and the desired information may take many forms including the estimation of the direction from which the signals arrive, the modulation on the carrier of one or more of the signals as in a communication receiver, or to reduce interference from strong unwanted sources such as "jammers" in a military environment.

The subject of sensor array processing has received considerable attention in the literature although perhaps too many of the papers have dealt only with theoretical possibilities and with computer simulations. Some of the basic concepts which underline the work can be traced back many years and perhaps one of the most important early references is to the work of Baron de Pronys, published in 1795, on fitting exponential curves to measurement data (ref.1).

Some of the recent interest may be attributed to the possibilities that are being opened up by the rapid developments in microelectronics and computing. These developments are making it possible to consider the implementation in real time of some of the complex algorithms which have been proposed, and indeed there are now some practical systems in operation.

There is a very close relation between the work on sensor array processing (which corresponds to spacial sampling of a field), to the work on modern spectrum analysis (ref.2) in which the sampling is in the time domain. The two problems are intimately linked through the use of the Fourier transform.

No attempt will be made in this short paper to try to review the literature. Good bibliographies can be found, for instance, in references 2,3,4 and 5.

2. PROBLEM FORMULATION AND MODELLING

Many practical arrays of sensors are arranged in a linear fashion with the elements spaced equidistance along the array. However there are also important classes of arrays, such as circular and conformal arrays, which are not linear and some which although intended to be linear do not necessarily remain so. A towed array in sonar is one important application to which this applies. However it considerably simplifies the mathematics and makes the understanding of the principles of the methods much easier if the assumption of a linear array is made

Consider the diagram in figure 1 and assume one of the signals is coming from a source in the far field of the array in a direction theta relative to the normal to the array.
Using the first element as a reference we can represent the voltages on the elements due to this signal as follows:-

SOME PRACTICAL RESULTS ON A HIGH RESOLUTION SONAR SYSTEM

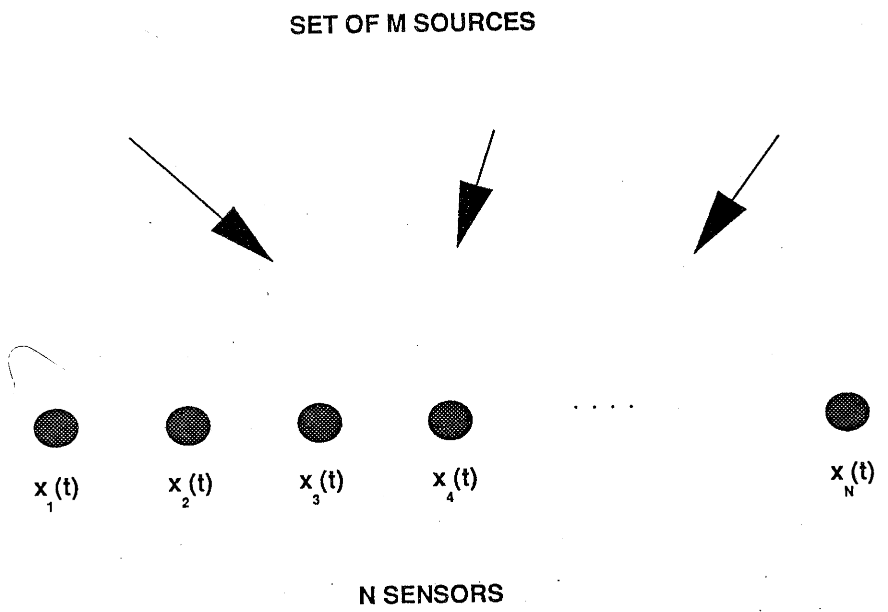

Figure 1

$$x_1(t) = s(t)$$
$$x_2(t) = s(t-\alpha)$$
$$x_3(t) = s(t-2\alpha)$$
$$\vdots$$
$$x_N(t) = s(t-(N-1)\alpha) \qquad \ldots(1)$$

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where $\alpha = \frac{d}{c}\sin(\theta)$

$d =$ The spacing between elements

$N =$ The number of elements

$c =$ The acoustic velocity

We can represent this set of signals as a vector

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ x_N(t) \end{bmatrix} \qquad \ldots(2)$$

If we are dealing with narrow band signals then complex notation can be used and the time delays can be represented by phase shifts. Thus

$$\vec{x}(t) = \vec{a}(\theta)s(t) \qquad \ldots(3)$$

where

$$\vec{a}(\theta) = \begin{bmatrix} 1 \\ \exp(-j\phi) \\ \exp(-j2\phi) \\ \cdot \\ \cdot \\ \exp(-j(N-1)\phi) \end{bmatrix} \quad \text{and} \quad \phi = \frac{2\pi d}{\lambda}\sin(\theta)$$

Assuming that there are a set of **m** sources present then the expression can be extended to

$$\vec{x}(t) = A\vec{s}(t) \qquad \ldots(4)$$

where $A = [\vec{a}_1(\theta_1), \vec{a}_2(\theta_2), \ldots, \vec{a}_m(\theta_m)]$

and $$\vec{s}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \cdot \\ \cdot \\ s_m(t) \end{bmatrix}$$

The matrix containing the total set of possible direction vectors is known as the array manifold.

$$A = \{\vec{a}(\theta_i) \quad \theta_i \in \Theta\} \qquad ....(5)$$

Normally there is noise present and we have to add a term to represent this.

$$\vec{x}(t) = A\vec{s}(t) + \vec{n}(t) \qquad ....(6)$$

Where $\vec{n}(t) = [n_1(t), n_2(t), ..., n_N(t)]^T$

The vector of element voltages at a particular time is called a snapshot. A number of such snapshots can be taken and a data matrix **D** can be formed which contains the set of snapshots.

$$D = [\vec{x}(1), \vec{x}(2), ..., \vec{x}(p)] \qquad ....(7)$$

where **p** is the number of snapshots.
The covariance matrix is defined as

$$R = E[\vec{x}\vec{x}^H]$$
$$= A(\theta)SA^H(\theta) + \sigma^2 I \qquad ....(8)$$

Where $S = E[\vec{s}(t)\vec{s}^H(t)]$

and it is assumed that the noise voltages on each element are independent and zero mean.

An estimate of the covariance matrix can be made from the **p** snapshots

$$\hat{R} = \frac{1}{p}\sum_{t=1}^{t=p}[\vec{x}(t)\vec{x}^H(t)]$$
$$= \frac{1}{p}[D.D^H] \qquad ....(9)$$

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3. CONVENTIONAL BEAMFORMING

In a beamformer the outputs of the individual elements are weighted to form a common output as is shown in figure 2.

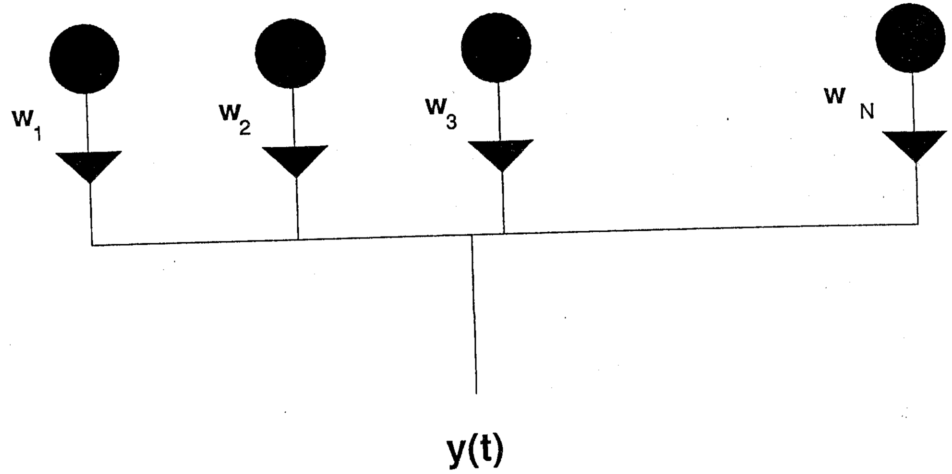

Figure 2: Conventional Beamforming

$$y = \vec{w}^T\vec{x} \qquad ....(10)$$

where $\vec{w} = [w_1, w_2, ..., w_N]^T$

The power output is given by

$$P = E[y^* y] \qquad ....(11)$$
$$= E[\vec{w}^T\vec{x}\vec{x}^H\vec{w}^*]$$
$$= \vec{w}^T R \vec{w}^*$$

where $R = E[\vec{x}\vec{x}^H]$

f we let $\vec{w} = \vec{a}^{*}(\theta)$

Then $P(\theta) = \vec{a}^{H}(\theta)R\vec{a}(\theta)$ ....(12)

This is the Angular Power Spectrum for the Conventional Beamformer. It has a maximum corresponding to the direction vector used and the array is said to be steered in this direction.

A typical curve for the conventional beamformer is shown in figure 3. It should be noticed that this is a power response and that the first sidelobes are about 13db down on the main lobe. The resolution capability of the conventional beamformer is defined as the 3db beamwidth and this, expressed in radians, is approximately the reciprocal of the length of the array measured in wavelengths. For the array in the diagram the length is 4 wavelengths so the beamwidth is approximately 15 degrees

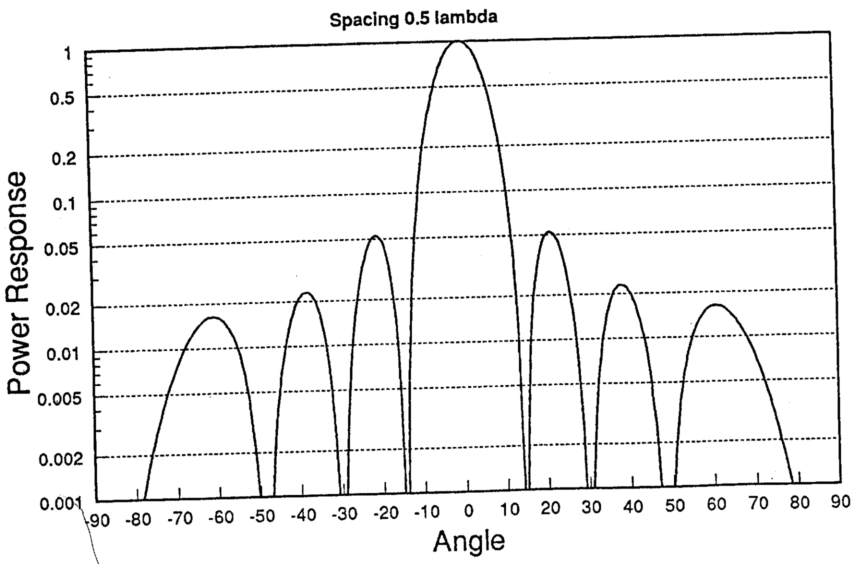

Figure 3

In practise we do not know **R** exactly and an estimate has to be used as defined in equation 9. Alternatively the estimated power output for a given set of snapshots can be expressed without calculating the covariance matrix viz:

$$\hat{P}(\theta) = \frac{1}{p}\sum_{t=1}^{t=p} |\vec{a}^{H}(\theta)\vec{x}(t)|^{2} \qquad ....(13)$$

Further the output of a conventional beamformer is more often expressed as a voltage rather than power so avoiding the computational problems introduced by squaring. The conventional beamformer is a very robust method of separating the signals coming from different directions which of course explains its widespread use. However the sidelobes of the beam allow strong interfering signals to break through and also the resolution is limited. Shading of the array using such as Dolph-Chebyshev or Binomial weighting can improve the situation as regards the side-lobes but there are practical limitations to the extent that this can be applied.

4. HIGH RESOLUTION AND ADAPTIVE METHODS

4.1 Introduction:
The basic property of an adaptive system is its ability to change its parameters to meet the changing nature of the environment in which it is operating. Thus instead of using the fixed weighting of the conventional beamformer we can adjust the weights to achieve some particular goal. This adaptation can be done in a continuous manner or cease after an initial learning phase. Capons method (ref. 6) described below is a typical example of such a system in which the gain of the array for a given direction is fixed by a constraint while the output power is minimised.

A major group of methods which improve the resolution of the array such as the MUSIC algorithm are based on eigen analysis.

The action of squaring which is implicit in the calculation of the covariance matrix and its eigenvectors can lead to accuracy problems in a digital computer if the number of bits available to represent each value is too low. For this reason a number of methods have been developed which avoid this squaring operation. Associated with these methods is the use of the systolic array and has led to what is called algorithmic engineering (Ref 9).

4.2 Capons Minimum Variance Method:
Basically we wish to constrain the weighting of the array to form a beam in a particular direction while at the same time to minimise the mean square output from the array i.e., we wish to minimise **P** subject to

$$\vec{w}_{opt}^{H}\vec{a}(\theta) = 1 \qquad ....(14)$$

We tackle this using Lagrange multipliers by defining a cost function **H**.

$$\textit{Let} \quad H(w) = \vec{w}^{H}R\vec{w} + \lambda(1 - \vec{w}^{H}\vec{a}(\theta)) \qquad ....(15)$$

λ is the Lagrange multiplier

$$\frac{\delta H(w)}{\delta \vec{w}} = 2R\vec{w} - \lambda\vec{a}(\theta) = 0$$

$$thus \quad \vec{a}(\theta) = 2R\vec{w}_{opt}$$

$$\vec{w}^H_{opt}\vec{a}(\theta)\lambda = 2\vec{w}^H_{opt}R\vec{w}_{opt}$$

$$\lambda = 2P_{opt}$$

$$\vec{w}_{opt} = \frac{\lambda}{2}R^{-1}\vec{a}(\theta)$$

$$= P_{opt}R^{-1}\vec{a}(\theta) \qquad \text{....(16)}$$

$$But \quad \vec{a}^H(\theta)\vec{w}_{opt} = 1$$

$$\vec{a}^H(\theta)P_{opt}R^{-1}\vec{a}(\theta) = 1$$

$$\therefore \quad P_{opt} = \frac{1}{\vec{a}^H(\theta)R^{-1}\vec{a}(\theta)} \qquad \text{....(17)}$$

This is the so called Capon Estimate of the angular spectrum. In direction finding the angular spectrum is also referred to as the Direction Function. It is a function of theta and we will define it as

$$P_{MVM}(\theta) = \frac{1}{\vec{a}^H(\theta)R^{-1}\vec{a}(\theta)} \qquad \text{....(18)}$$

The angular spectrum function formed by the Capon Method is directly referenced to receiver noise power so that the peaks represent an estimate of the signal power from that direction permitting measurement of relative source strength. The residual background ripple is low and relatively well behaved, and provided the array manifold is known it is not necessary for the array sensors to be equally spaced.

On the other hand its resolution is less than some of the other high resolution techniques which we will discuss below, ill conditions may arise when calculating the inverse of the covariance matrix, it fails to resolve correlated sources and its computation load is relatively high.

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4.3 Maximum Entropy Method (MEM):
This algorithm has strong similarity to the LPC algorithm used in speech coding but as the method is not used to a great extent in array processing the derivation of this algorithm will not be included. It can be likened to the constraint method in which the constraint vector is the unit vector defined below and which fixes the weight of the first element to zero allowing the other weights to take any value. The effect of this is to minimise the output and provided there are enough elements will place nulls at the position of all the strong signals. The inversion of this response gives an estimate of the angular spectrum but the amplitude of the peaks are not directly related to the power of the sources.

The MEM Angular Spectrum is given by:

$$P_{MEM}(\theta) = \frac{e_1^H R^{-1} e_1}{|e_1^H R^{-1}\vec{a}(\theta)|^2} \qquad \text{....(19)}$$

Where e_1
denotes the first unit vector i.e,

$$e_1 = (1, 0, ..., 0)^T \qquad \text{....(20)}$$

The main advantage of this method is the low computation load but bias and line splitting may occur giving false targets and the array sensors need to be equally spaced. The method is useful in adaptive arrays for cases where the wanted signal itself is weaker than the background noise. The interferences are first removed and then the signal recovered from the noise by post-processing. Used in this manner it is often referred to as the power inversion algorithm.

5. EIGENVECTOR BASED METHODS

5.1 Eigenvector Analysis:
Analogous to the way in which we expand a waveform into its Fourier Series, we can expand the covariance matrix in terms of its eigenvectors viz

$$R = \sum_{i=1}^{N} \lambda_i \vec{e}_i \vec{e}_i^H$$

$$= E\Lambda E^H \qquad \text{....(21)}$$

The minimum eigenvalue of **R** has a multiplicity of **(N-m')** where **m'** is the rank of S. [m'<= m].If all the signals are uncorrelated then m'=m.

We can then partition **R** as follows:

$$R = E_s\Lambda_s E_s^H + E_n\Lambda_n E_n^H \qquad \text{....(22)}$$

where E_s is N by m'

E_n is N by $(N-m')$

The columns of E_S span a subset of the space spanned by $A(\theta)$ i.e

$$\Lambda_n = \sigma^2 I$$

$$Span\{E_s\} \subset span\{A(\theta)\} \qquad \ldots.(23)$$

5.2 MUSIC (MUltiple SIgnal Classification):
Although a number of workers were working independently on these methods the algorithm is usually credited to Schmidt (ref. 7).
Assume S has full rank (i.e., the sources are uncorrelated) then

$$Span\{E_s\} = Span\{A(\theta)\}$$

$$Span\{E_n\} \quad is \quad perp \quad to \quad Span\{A(\theta)\}$$

$$E_s \equiv Signal \quad subspace$$

$$E_n \equiv Noise \quad subspace$$

Thus

$$E_n^H \vec{a}(\theta_i) = 0 \qquad i = 1 \ldots. d \qquad \ldots.(24)$$

From equation 24 it can be seen that the direction vectors of the sources are orthogonal to the noise eigenvectors and hence we can define what is known as the Music Spectrum

$$P_{music}(\theta) = \frac{1}{\vec{a}^H(\theta)\hat{E}_n\hat{E}_n^H\vec{a}(\theta)} = \frac{1}{\sum_{i=M+1}^{N} |\vec{a}^H(\theta)e_i|^2} \qquad \ldots.(25)$$

Where e_i

is the ith eigenvector of R.

It should be noticed that this only locates the direction of the sources. It requires a further step to estimate the source powers.
The main advantages of this algorithm are that it has very high resolution, the background ripple is very low, the array sensors need not be equally spaced and to some extent it can resolve correlated sources.
Its main disadvantage is the high computation load.

5.3 Minimum Norm Method (MNM):
This method was proposed by Kumeresan and Tufts (ref. 8) and the angular spectrum is defined as:

$$P_{MNM}(\theta) = \frac{1}{|A^H(\theta)d|^2} \qquad \ldots.(26)$$

where d is a linear combination of all the eigenvectors of the noise subspace and hence is orthogonal to the signal eigenvectors.

The method has very high resolution with a computational load which is less than the MUSIC algorithm and it can resolve sources with some correlation. The main disadvantage is that the estimates can be biassed and the background level is high.

5.4 Other Eigenvector Methods:
A number of other eigenvector methods have been published recently offering some advantages when compared to the MUSIC algorithm which now is often taken as a standard of performance.

A typical example of these is the ESPRIT algorithm (ref. 10). In the MUSIC algorithm it is necessary to know the array manifold exactly and in practical systems this can prove a problem. The ESPRIT algorithm is an attempt to reduce this dependence. Essentially the array is assumed to comprise a number of doublets which can be obtained by moving an arbitrary array through a fixed amount. It is not necessary to know the array manifold, only that vector through which it is moved and the assumption that the array manifold of the two arrays are identical except for the factor due to this movement.
This simplifies the calculations and produces some very good results without knowledge of the manifold.

Another extension is the CLOSEST algorithm (ref. 11). This is useful when it is desired to use high resolution methods to separate a group of sources in a cluster. The method selects one (or a group) of the noise sub-space vectors which are closest in some sense to the wanted direction and because of this provides superior resolution to the MUSIC algorithm without introducing bias.
A number of such algorithms exist and time will tell as to which will prove to have long term advantages.

6. MULTIDIMENTIONAL SIGNAL SUBSPACE METHODS

It was shown earlier that

$$Span\{E_s\} \subset span\{A(\theta)\}$$

Thus there must exist a non-singular m by m' matrix **T** such that

$$E_s = A(\theta)T \qquad ...(27)$$

Given an estimate of E_s then we can attempt to get obtain the least square fit of the signal subspace and the model subspace.

$$[\hat{\theta},\hat{T}] = \arg\min \| \hat{E}_s - A(\theta)T \|_F^2 \qquad ....(28)$$

The symbol $\| . \|_F$ denotes the Frobenius Matrix Norm defined as the root of the sum of the squared moduli of all the matrix elements.
Methods based on this approach are becoming of more importance as processing capability improves. An excellent treatment of these methods is given in reference 12.

7. PARAMETRIC METHODS

In these methods a signal model is assumed and the unknown parameters e.g., number of signals, directions, amplitudes and phases are chosen to give a best fit to the measured data. These methods have been known for many years and have been used very successfully in astronomy. (CLEAN algorithm). There has been a resurgence of interest recently and a method proposed by Ira Clark called IMP has been used with good results. (ref. 13)

8. SOME PRACTICAL RESULTS

In many of the papers published in this field results are obtained by simulation on a computer to show that the method works and the desired effects are produced. However there is a dearth of practical results. In a companion paper (ref.15) some early experimental results on a practical sonar system are presented.

9. CONCLUSION

This article has attempted to give an overview of the subject of sensor array processing and and in a companion paper (Ref.15) to present a few practical results illustrating the performance of the various algorithms. It is not possible in a short work of this nature to do justice to the extensive work that has, and is, being carried out in this subject. I hope that this introduction has served to whet the readers appetite for further study.

10. ACKNOWLEDGEMENTS

In a summary paper of this nature the author has obviously made considerable use of the work of many others. I should like to acknowledge the many interesting and informative discussions I have had with many of the authors quoted as well as many others.
I should also like to thank my colleague Tahseen Rafik for his help in the preparation of this paper.

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